Esercitazione 7

Potenziali e campi (capitolo 10 Griffiths)
23 Maggio 2016

Esercizio 1

Example 10.1

Find the charge and current distributions that would give rise to the potentials

$$V = 0, \quad \mathbf{A} = \begin{cases} \frac{\mu_0 k}{4c}(ct - |x|)^2\,\hat{\mathbf{z}}, & \text{for } |x| < ct, \\ 0, & \text{for } |x| > ct, \end{cases}$$

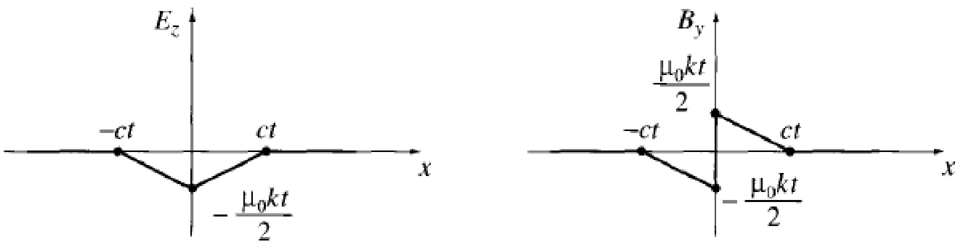
where k is a constant, and $c = 1/\sqrt{\epsilon_0 \mu_0}$.

Solution: First we'll determine the electric and magnetic fields, using Eqs. 10.2 and 10.3:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} = -\frac{\mu_0 k}{2}(ct - |x|)\,\hat{\mathbf{z}},$$

$$\mathbf{B} = \nabla \times \mathbf{A} = -\frac{\mu_0 k}{4c}\frac{\partial}{\partial x}(ct - |x|)^2\,\hat{\mathbf{y}} = \pm\frac{\mu_0 k}{2c}(ct - |x|)\,\hat{\mathbf{y}},$$

(plus for $x > 0$, minus for $x < 0$). These are for $|x| < ct$; when $|x| > ct$, $\mathbf{E} = \mathbf{B} = 0$

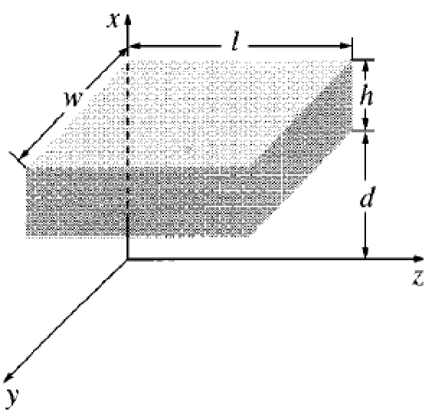


Problem 10.2 For the configuration in Ex. 10.1, consider a rectangular box of length l, width w, and height h, situated a distance d above the yz plane (Fig. 10.2).

(a) Find the energy in the box at time $t_1 = d/c$, and at $t_2 = (d + h)/c$.

(b) Find the Poynting vector, and determine the energy per unit time flowing into the box during the interval $t_1 < t < t_2$.

(c) Integrate the result in (b) from t_1 to t_2 and confirm that the increase in energy (part (a)) equals the net influx.



Esercizio 2

Problem 10.4 Suppose $V = 0$ and $\mathbf{A} = A_0 \sin(kx - \omega t)\,\hat{\mathbf{y}}$, where A_0, ω, and k are constants. Find **E** and **B**, and check that they satisfy Maxwell's equations in vacuum. What condition must you impose on ω and k?

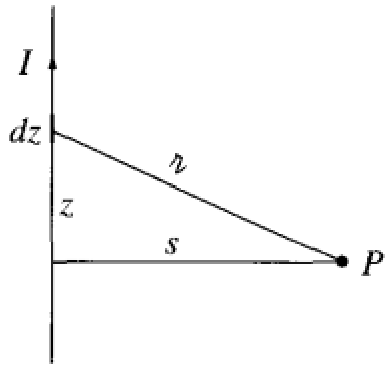
Esercizio 3

Problem 10.9

(a) Suppose the wire in Ex. 10.2 carries a linearly increasing current

$$I(t) = kt,$$

for $t > 0$. Find the electric and magnetic fields generated.



Esercizio 4

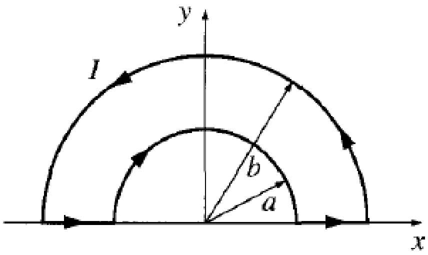


Figure 10.5

Problem 10.10 A piece of wire bent into a loop, as shown in Fig. 10.5, carries a current that increases linearly with time:

$$I(t) = kt.$$

Calculate the retarded vector potential **A** at the center. Find the electric field at the center. Why does this (neutral) wire produce an *electric* field? (Why can't you determine the *magnetic* field from this expression for **A**?)

Esercizio 5

Problem 10.3 Find the fields, and the charge and current distributions, corresponding to

$$V(\mathbf{r}, t) = 0, \quad \mathbf{A}(\mathbf{r}, t) = -\frac{1}{4\pi\epsilon_0}\frac{qt}{r^2}\,\hat{\mathbf{r}}.$$

Problem 10.5 Use the gauge function $\lambda = -(1/4\pi\epsilon_0)(qt/r)$ to transform the potentials in Prob. 10.3, and comment on the result.

Esercizio 6

Problem 10.13 A particle of charge q moves in a circle of radius a at constant angular velocity ω. (Assume that the circle lies in the xy plane, centered at the origin, and at time $t = 0$ the charge is at $(a, 0)$, on the positive x axis.) Find the Liénard-Wiechert potentials for points on the z axis.

Esercizio 7

Problem 10.18 Suppose a point charge q is constrained to move along the x axis. Show that the fields at points on the axis to the *right* of the charge are given by

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0}\frac{1}{\imath^2}\left(\frac{c+v}{c-v}\right)\hat{\mathbf{x}}, \quad \mathbf{B} = 0.$$

What are the fields on the axis to the *left* of the charge?