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Deconvolving mirror surface properties from X-ray probe beams

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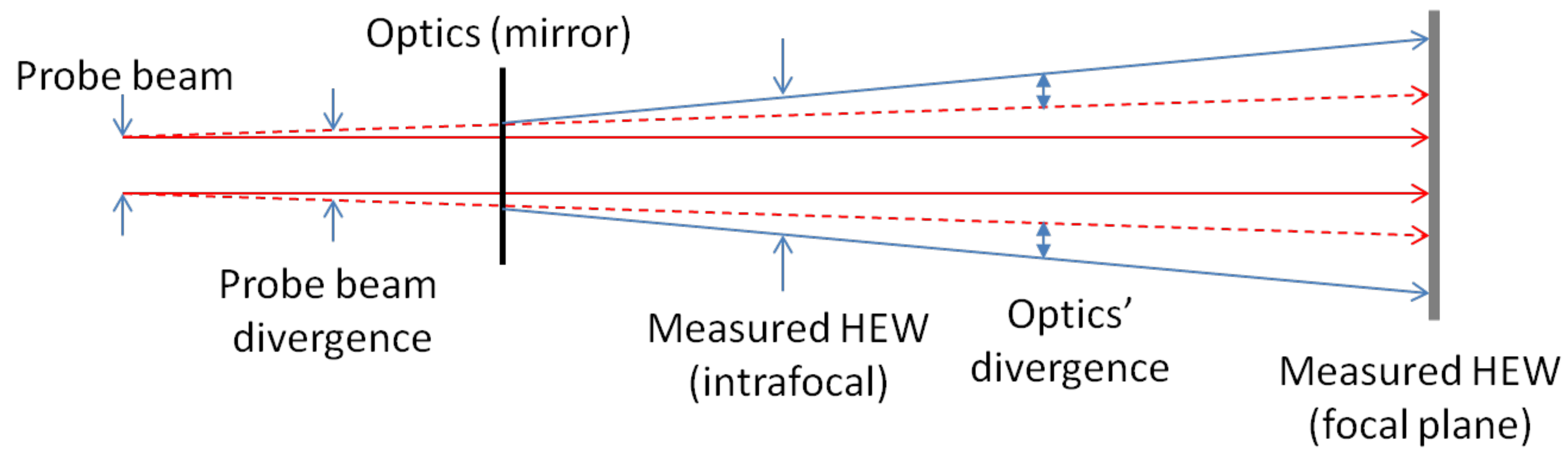
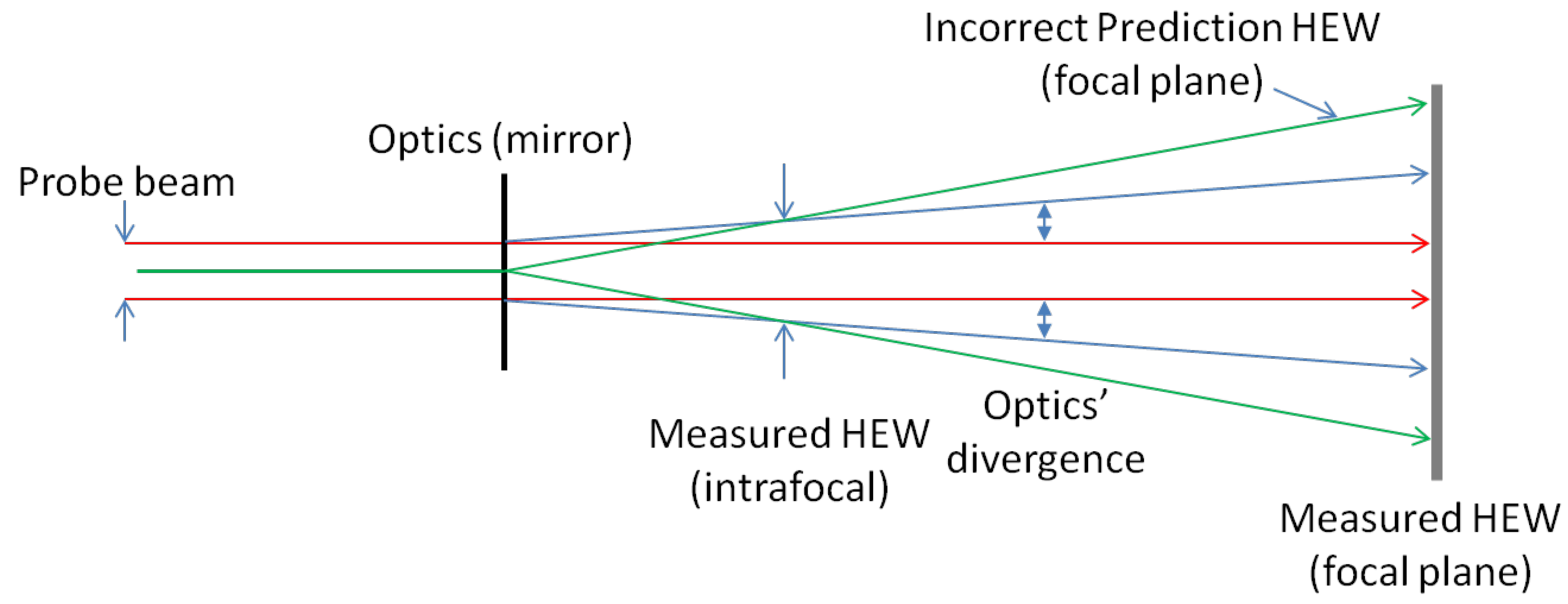
For large future space-based X-ray missions it will be important to be able to characterize the properties of very large mirror surfaces (of the order of hundreds of square meters). Due to time and manufacturing constraints it is practically impossible to do so fully using traditional tactile or optical profilometry methods. We present a study into the measurement of a mirror's surface quality based on the properties of a reflected X-ray probe beam. This type of measurement leads to a convolution of the beam with the surface properties. In order to retrieve the actual surface properties, one needs to disentangle the beam contribution from the observed reflected image: we have developed a numerical approach and show some of the results.

Introduction

An X-ray beam reflecting off a mirror gains an additional divergence from scattering caused by imperfections in the mirror.

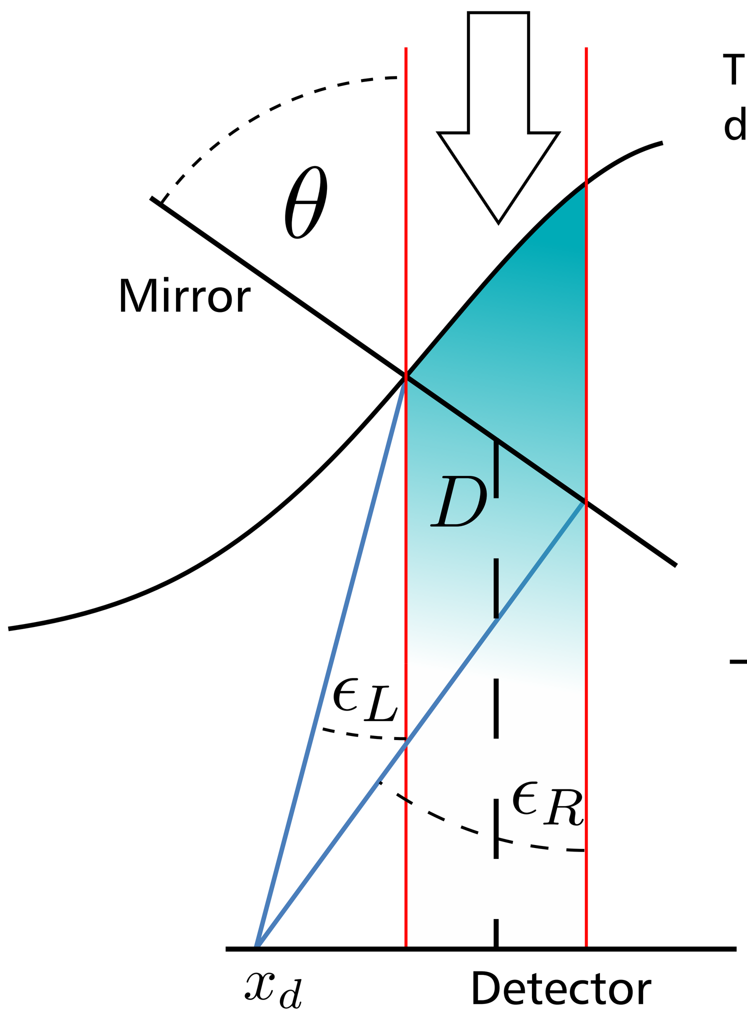


Based on intra-focal measurements, we want to subtract the HEW contribution of the incident beam from the HEW of the reflection to obtain the HEW of the mirror.



It is necessary to subtract the beam profile from the measured HEW to obtain a good estimate of the mirror's HEW.

Reflecting a collimated Top-Hat off a Gaussian mirror

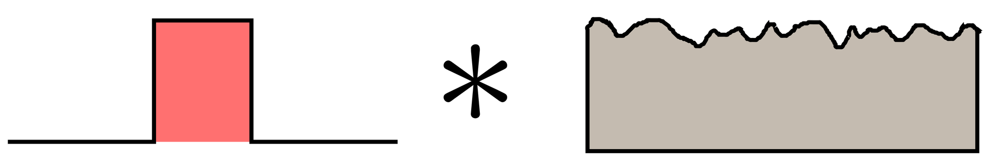


Schematic drawing of geometry. Each point on the detector integrates a different range of the slope distribution.

The figure is represented by a Gaussian slope distribution:

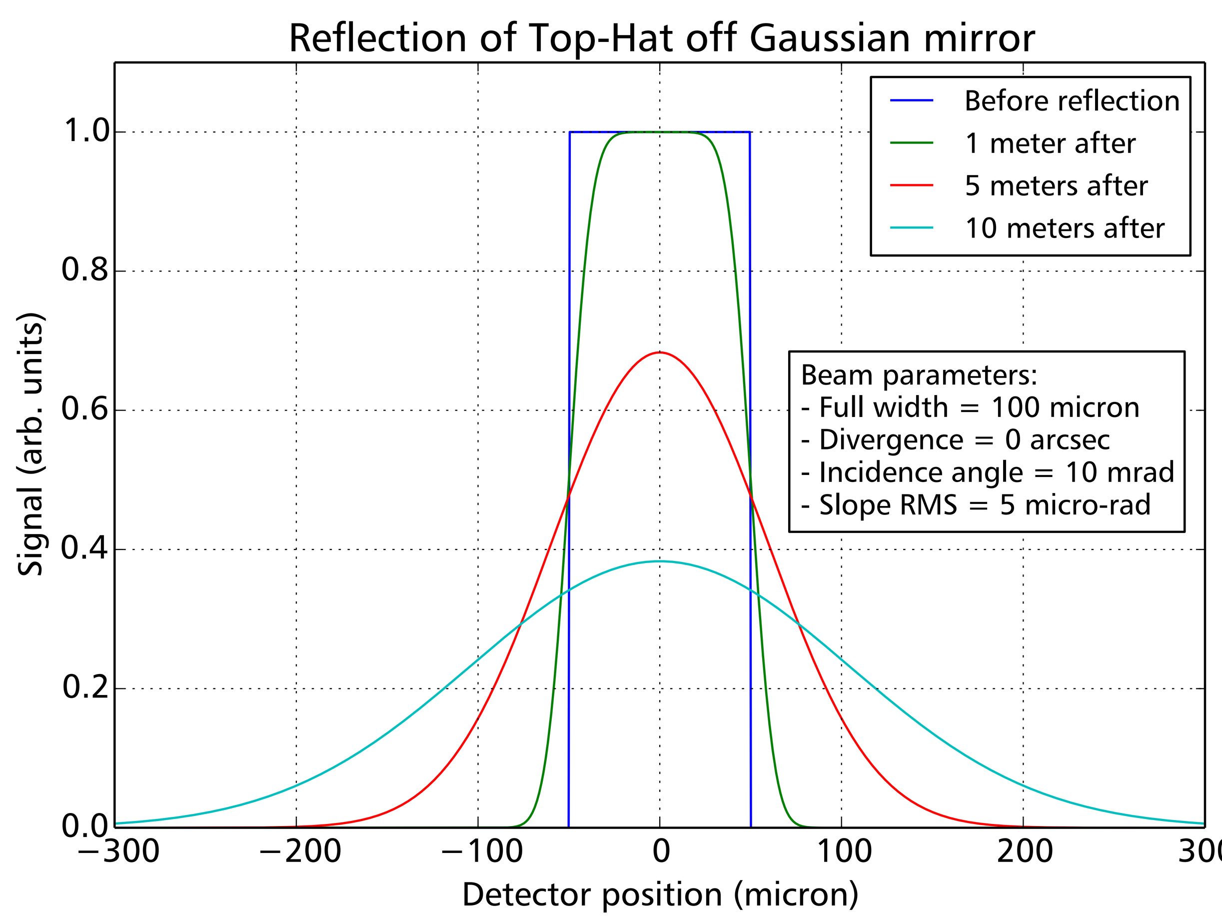
$$G_{2\mu}(\epsilon) = \frac{1}{\sqrt{2\pi}2\mu} \exp\left\{-\frac{\epsilon^2}{2(2\mu)^2}\right\}$$

where ϵ is the scattering angle with respect to the incidence angle θ and μ is the slope RMS.

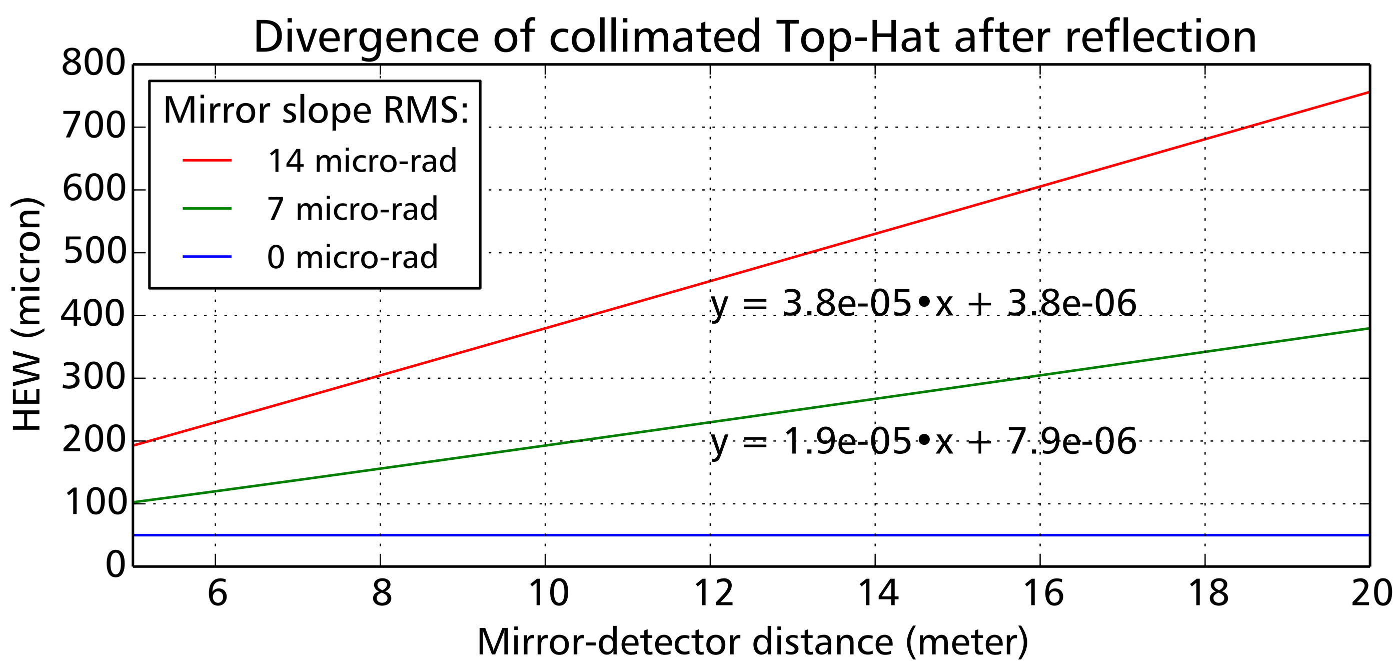


Convolving the beam profile with the figure:

$$I(x_d) = \int_{\epsilon_R}^{\epsilon_L} d\epsilon \cdot G_{2\mu}(\epsilon)$$



Plot of beam profiles before (blue) and after (green, red, teal) reflection. The reflected beam convolves into a broadening Gaussian.

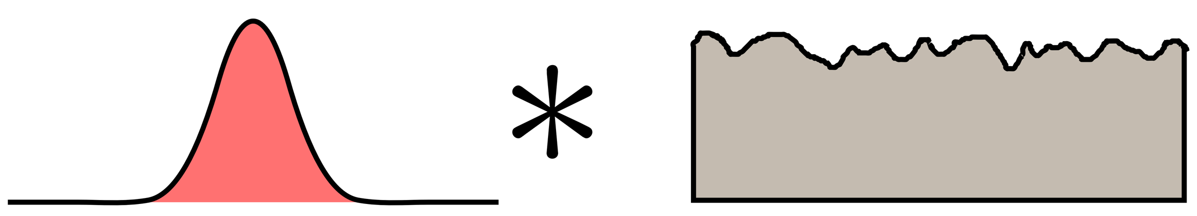


HEW of reflected Top-Hat as function of propagation distance. Doubling the slope RMS doubles the divergence.

Reflecting a divergent Gaussian off a Gaussian mirror

The extended convolution equation has the following form and solution:

$$I(x_d) = \int_{\epsilon_R}^{\epsilon_L} d\epsilon \cdot G_{2\mu}(\epsilon - \delta(\epsilon))\, G_{w_0}(\epsilon)$$

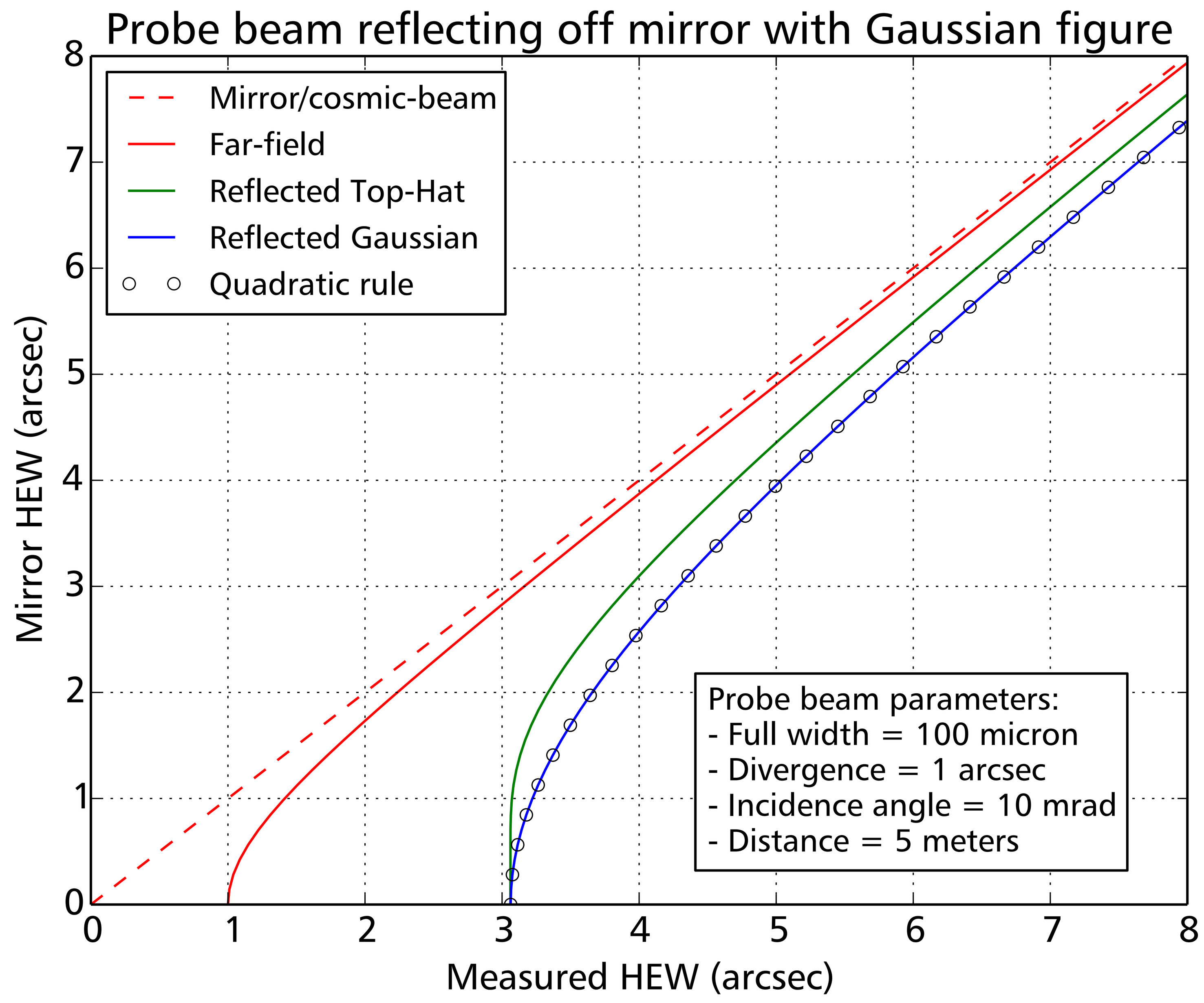


$$\simeq \frac{1}{\sqrt{2\pi}w} \exp\left\{-\frac{x_d^2}{2w^2}\right\}$$

The width parameter is given by the equation:

$$w = \sqrt{(2D\mu)^2 + (w_0 + D\delta)^2}$$ (the 'quadratic rule')

Measured/mirror HEW for Top-Hat and Gaussian



Mirror HEW as function of measured HEW for Top-Hat and Gaussian beam profiles.

In the far-field, the beam width (on the mirror) vanishes and only the divergences from the mirror and the beam remain:

$$HEW = \frac{\gamma w}{D} \rightarrow \gamma\sqrt{4\mu^2 + \delta^2} = \sqrt{HEW^2_{\text{mirror}} + HEW^2_{\text{beam}}}$$

with $\gamma = 2\sqrt{2} \cdot \text{erf}^{-1}\left(\frac{1}{2}\right) \simeq 1.35$ which converts beam width/sigma to HEW.

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