

# Unconventional transport in 2D materials with strong Rashba coupling

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*In collaboration with:*

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*Valuable discussions with:*

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SAPIENZA  
UNIVERSITÀ DI ROMA

## Purpose of the talk

clarify whether and how spin-orbit coupling affects DC charge transport

## Outline

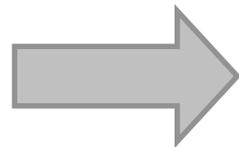
- ▶ Motivations
- ▶ Model and regimes
- ▶ Results
  - Single-particle properties
  - Conductivity and mobility
- ▶ Perspectives

Simple analytical expressions!

# Spin-orbit (SO) coupling in solids

- weak anti-localization
- anomalous Hall effect
- spin Hall effect, spin relaxation
- topological phases
- Majorana fermions
- ...

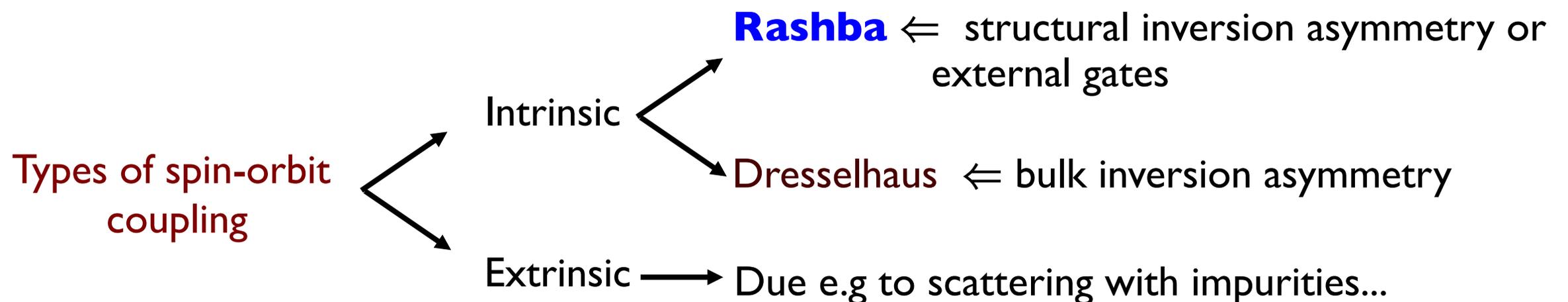
SPINTRONICS = spin transport electronics



“understand and control the transport of spin-polarized currents and to eventually apply this knowledge in information technologies”

D. Awschalom, Physics (2009)

Intense efforts to engineer structures and materials with strong spin-orbit coupling



**In this talk: Rashba spin-orbit coupling in charge transport**

# Emerging new materials ...

- Surface alloys

Adatoms: Bi, Pb

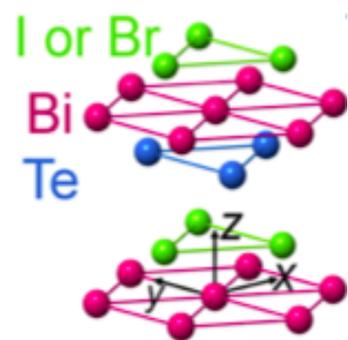


C. Ast et al., *Phys. Rev. Lett.* (2007); K. Yaji et al., *Nature Comms.* (2009);

Tunability by changing stoichiometry

Spin-orbit coupling up to 200 meV

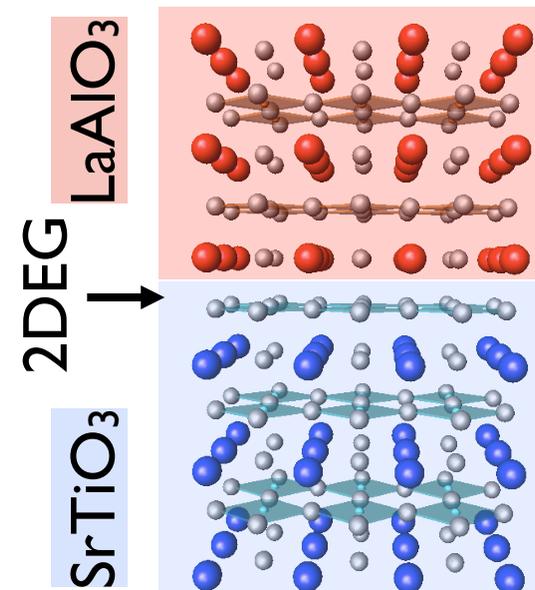
- Surfaces of BiTeX, X=Cl, I, ...



Spin-orbit coupling  
up to 100 meV

After Sakano et al. *Phys. Rev. Lett.* (2013); Ereemeev et al. *ibid.* (2012); A. Crepaldi *ibid.* (2012); ...

- Oxides heterostructures



A. Ohtomo & . Huang, *Nature* (2004); A. Caviglia et al., *Nature* (2008); ...

Gate tunable

Spin-orbit coupling estimates range from  
5 to 20 meV;

- Other systems:
  - HgTe quantum wells
  - Organometal compounds
  - Ferroelectric materials
  - ...

...with strong (*tunable*)  
*Rashba* coupling

Common features are:

- 2-dimensional
- strong spin-orbit coupling,  $E_0$
- tunable carrier density, small  $E_F$

Very different from traditional III-V semiconductors  
where SO coupling is a small perturbation!

Need for a theoretical description of transport  
non-perturbative in  $E_0/E_F$

## Model

$$H = \int d\mathbf{r} \Psi^\dagger(\mathbf{r}) \left[ \frac{p^2}{2m} + \alpha \hat{z} \cdot (\mathbf{p} \times \vec{\sigma}) + V_{\text{imp}}(\mathbf{r}) \right] \Psi(\mathbf{r})$$

Rashba Hamiltonian

+

Disorder

- **Rashba Hamiltonian**

$$H_{\mathbf{p}}^{\text{R}} = \frac{p^2}{2m} + \alpha |\mathbf{p}| S$$

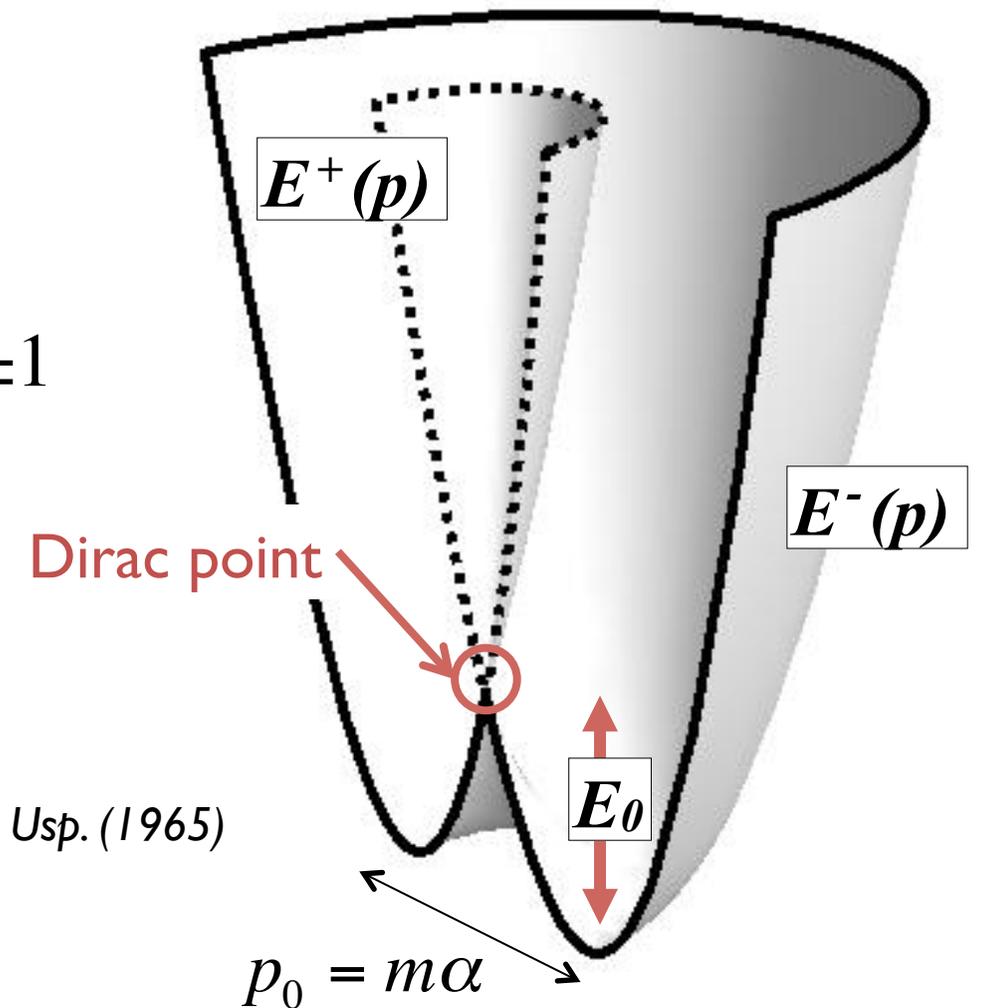
Helicity:  $S = [\hat{p} \times \vec{\sigma}]_z$      $S|\mathbf{p}s\rangle = s|\mathbf{p}s\rangle$      $s = \pm 1$

Two bands with opposite helicities

$$E^s(p) = \frac{p^2}{2m} + s\alpha |\vec{p}|$$

Spin-orbit coupling strength:  $E_0 = \frac{m\alpha^2}{2}$

E.I. Rashba, *Sov. Phys. Usp.* (1965)



- **Disorder** due to static impurities with density  $n_i$ :

$$\langle V_{\text{imp}}(\mathbf{r}) V_{\text{imp}}(\mathbf{r}') \rangle_{\text{imp}} = n_i v_{\text{imp}}^2 \delta(\mathbf{r} - \mathbf{r}')$$

Amount and strength of impurity scattering defined by:

$$\Gamma_0 = \frac{m n_i v_{\text{imp}}^2}{2}$$

elastic scattering rate at zero SOC

- Elastic s-wave scattering

- Inelastic scattering (phonons, e-e) negligible

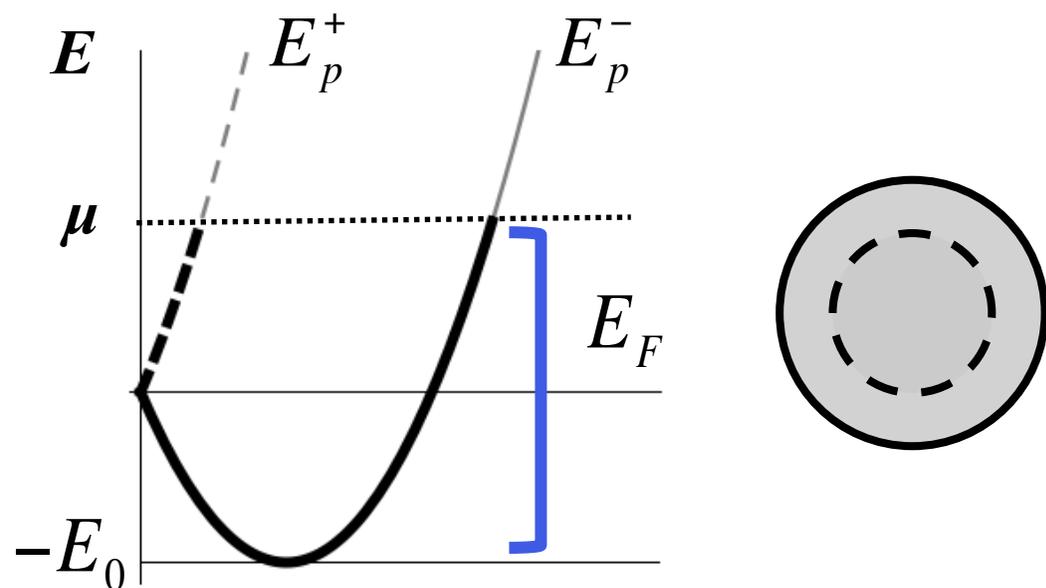
Work on-going with Capone's group !

# Two charge-transport regimes

Relevant energy scales:  $\Gamma$ ,  $E_0$  and  $E_F$

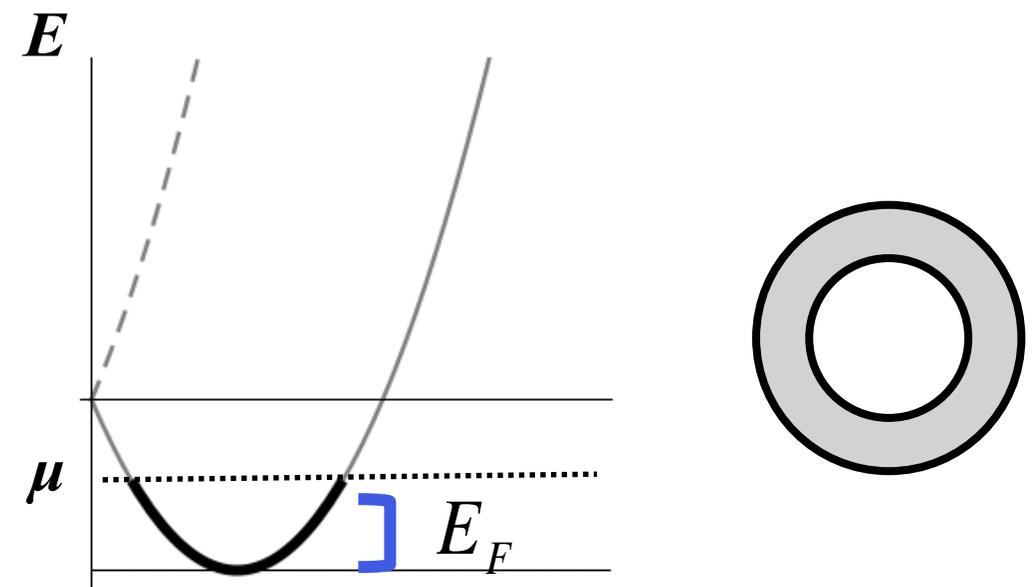
Diluted impurities  $\Rightarrow \Gamma < E_F, E_0$

High-density (HD) regime:  $E_F > E_0$



Fermi surface (FS) = two circles with opposite helicities

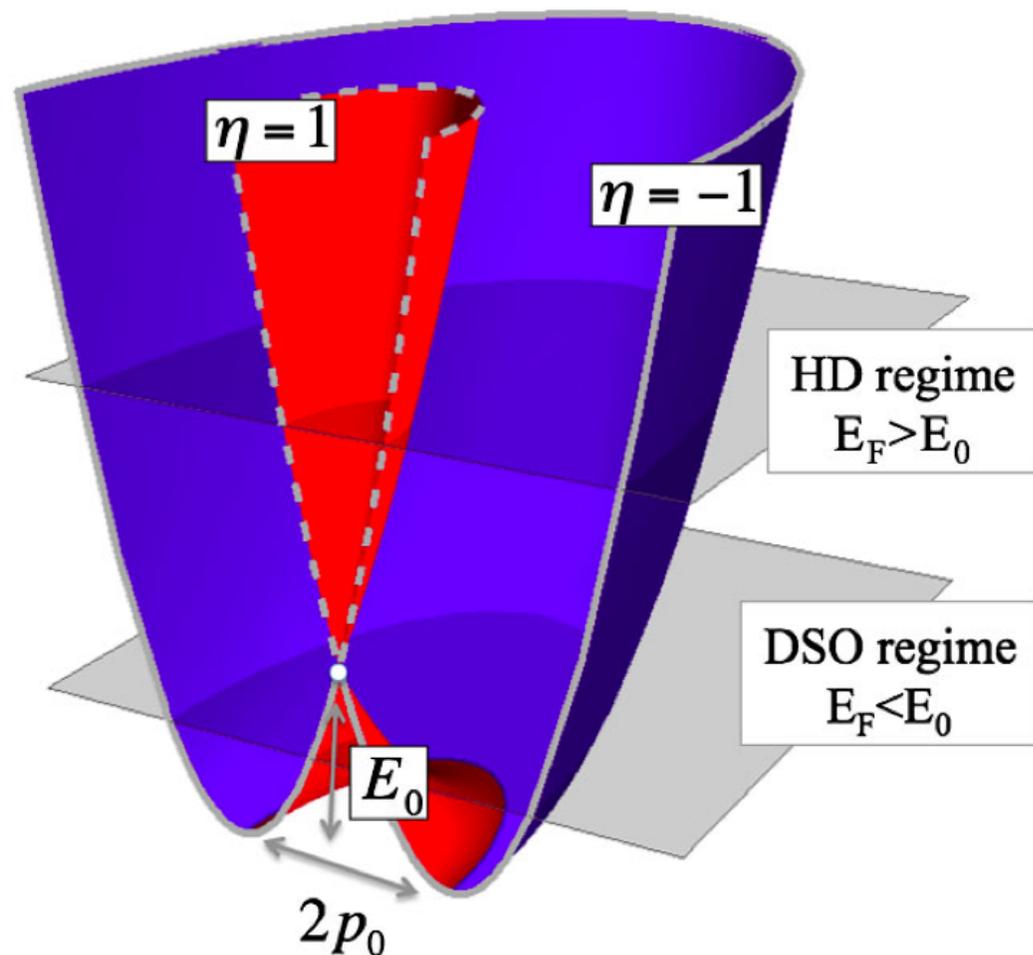
Dominant spin-orbit (DSO) regime:  $E_F < E_0$



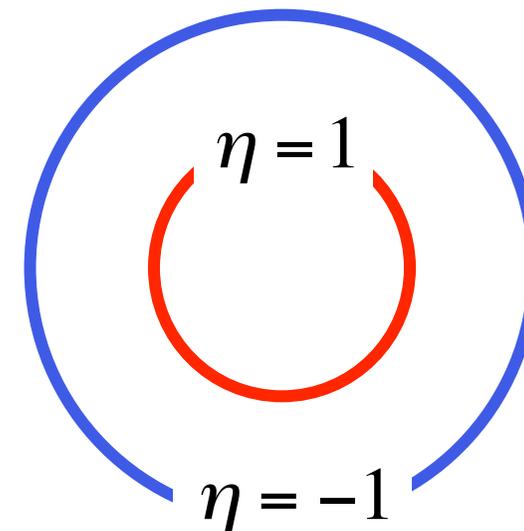
FS = two circles with equal helicities but opposite velocities

Sign-change in the quasi-particle velocity suggest a universal classification of the states in the two-regimes!

Transport helicity  $\eta = (\hat{v}_{\mathbf{p}s} \cdot \hat{p}) s$  Average velocity of helicity state  $s$



In both regimes we have FS with opposite transport helicities:



- Unified classification of the states in the two regimes;
- Simple and compact description of transport.

To different eigenvalues of the transport helicity correspond very different transport properties!

# Single-particle properties

# Green's function

- Diagonal Green's function in the helicity basis

$$G^R(\mathbf{p}, \omega) = \begin{pmatrix} g_+^R(p, \omega) & 0 \\ 0 & g_-^R(p, \omega) \end{pmatrix}$$

$$g_s^R(p, \omega) = [\omega - E_p^s + \mu - \Sigma^R(\omega)]^{-1}$$

- Spin-independent self-energy!

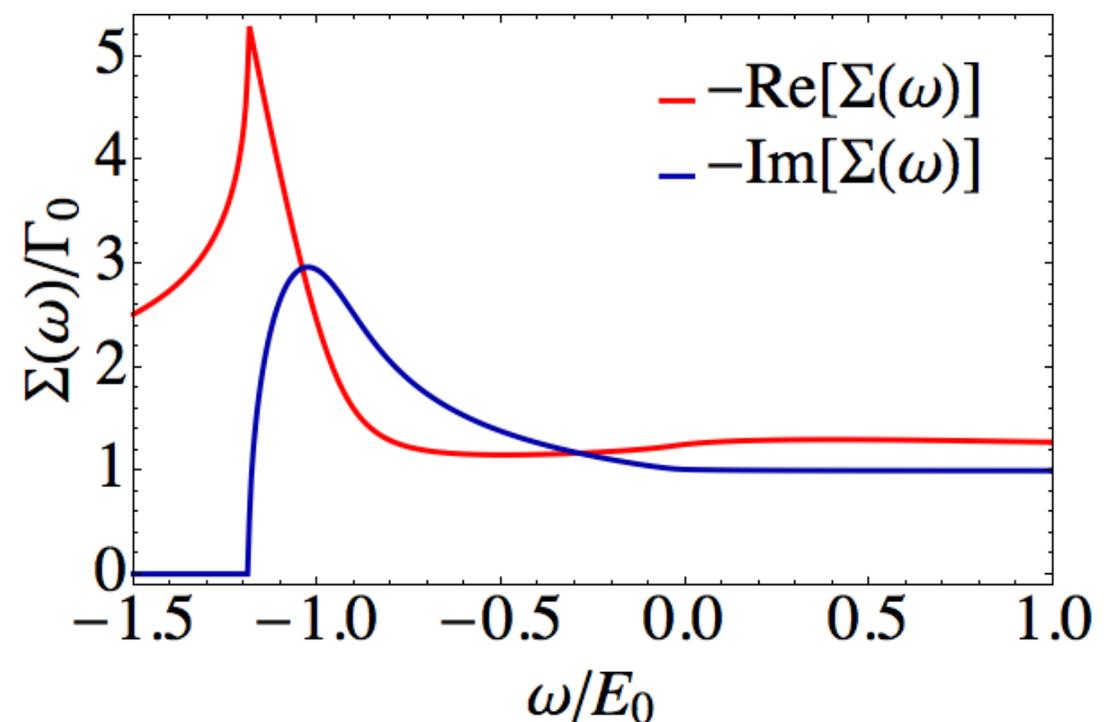
$$\Sigma^R(\omega) = \frac{n_i v_0^2}{\mathcal{V}} \sum_{\mathbf{p}, s} g_s^R(p, \omega)$$

Diagrammatic perturbation theory in Matsubara frequencies

Self-consistent Born approximation (SCBA)

$$\Sigma(i\omega_n) = \text{Impurity average} \left[ \text{Diagram: a solid line } G(i\omega_n) \text{ with a dashed loop containing a crossed vertex} \right]$$

Self-consistency close to band-edge

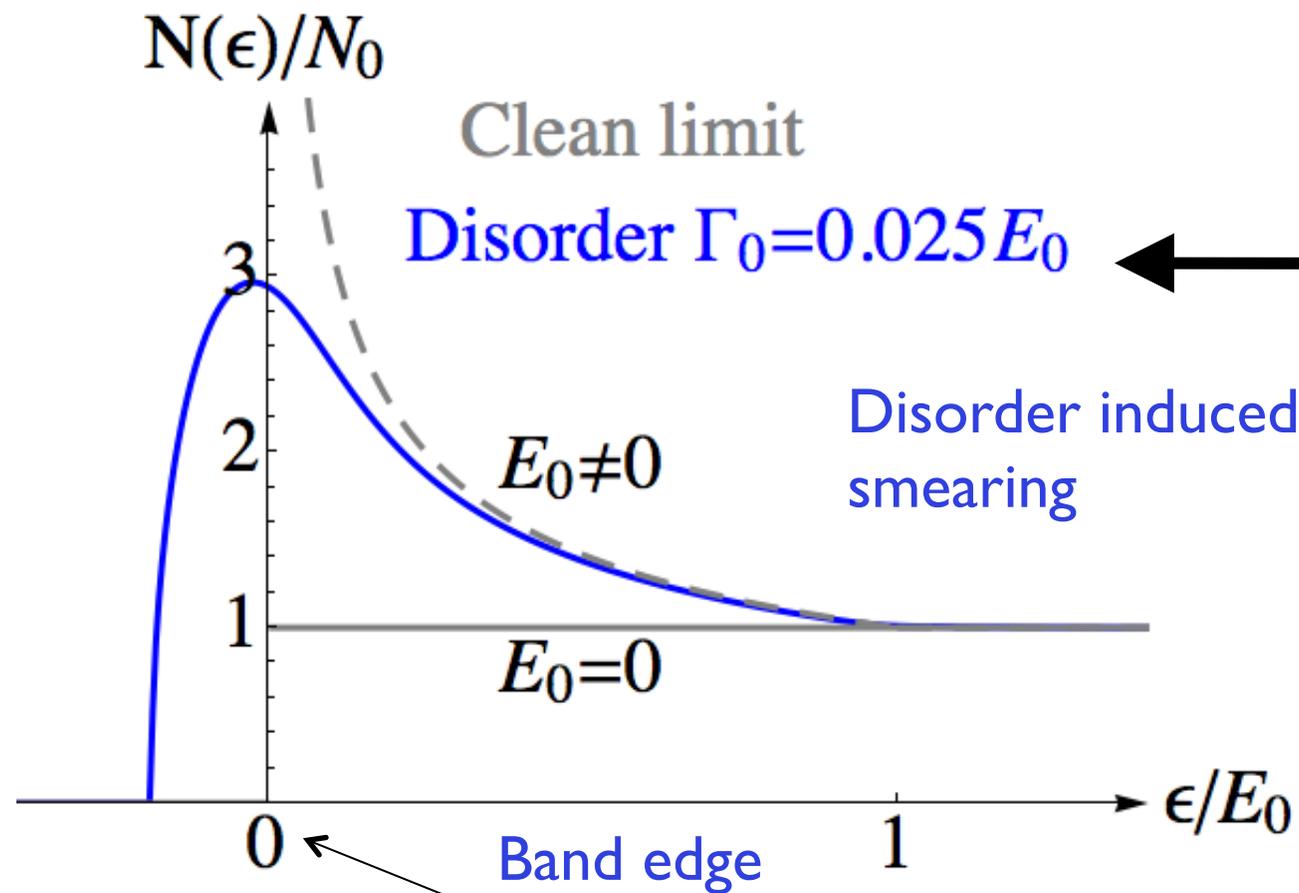


DOS and elastic scattering rate:

$$\left. \begin{matrix} \Gamma \\ N(E_F) \end{matrix} \right\} \propto -\text{Im}[\Sigma^R(0)]$$

To lowest order in  $\frac{\Gamma}{E_F}$

## Density of states (DOS)

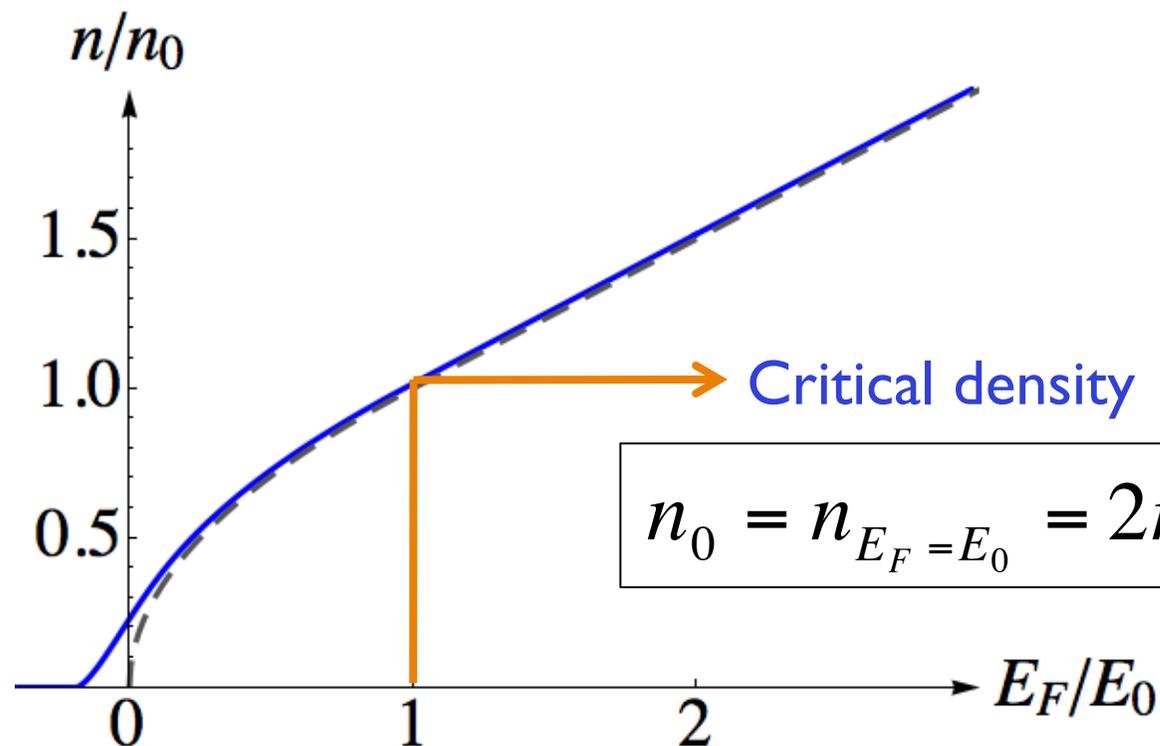


Van Hove singularity due to Rashba coupling

$$N(\varepsilon) = \begin{cases} N_0 & E_F > E_0 \\ N_0 \sqrt{E_0/\varepsilon} & E_F < E_0 \end{cases}$$

$$N_0 = m/\pi$$

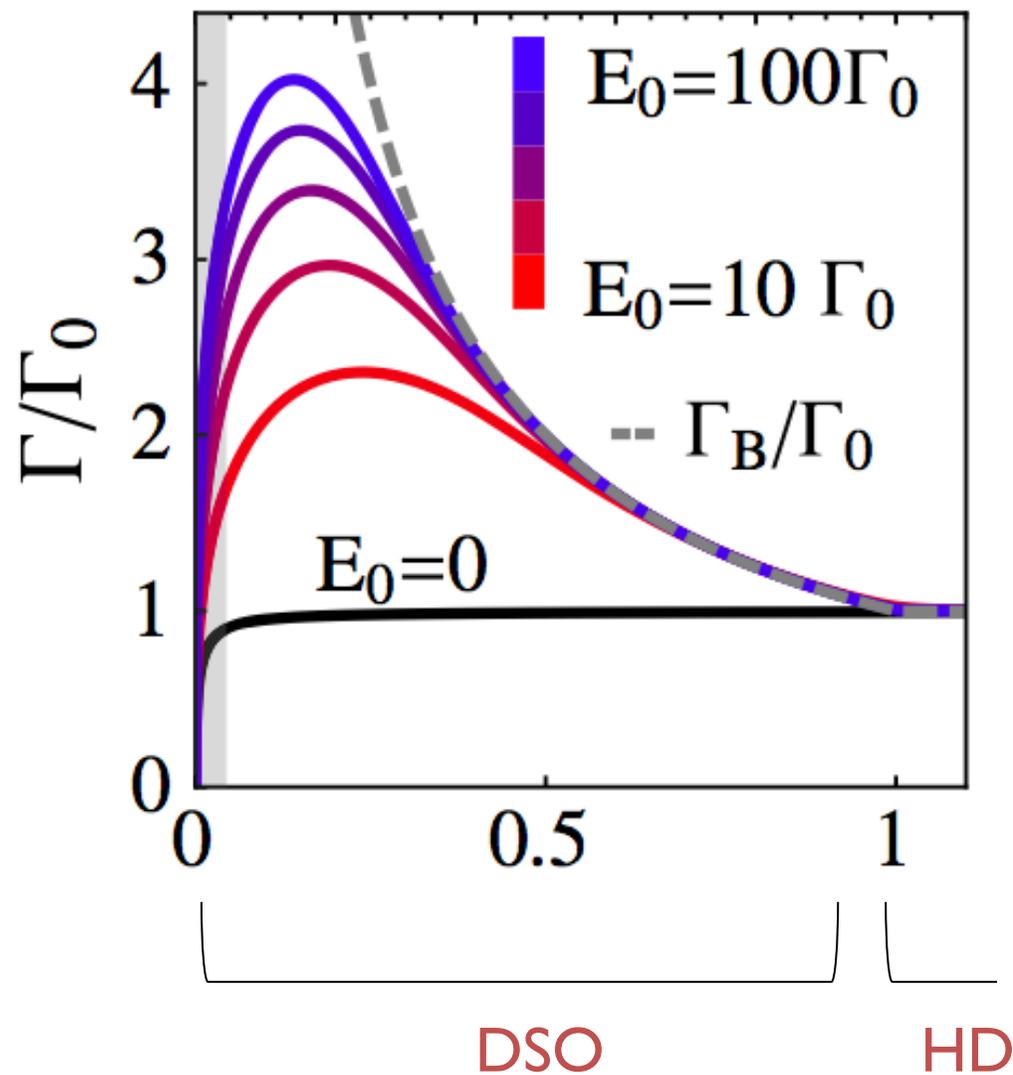
## Charge density



$$n = \begin{cases} N_0(E_F + E_0) & E_F > E_0 \\ 2N_0 \sqrt{E_F E_0} & E_F < E_0 \end{cases}$$

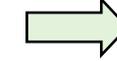
Analytical formulae in clean limit !

# Elastic scattering rate

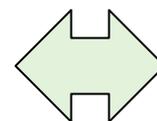


Shaded region : corrections to SCBA important

Van Hove singularity



Clearly different behavior in the HD and DSO regimes!

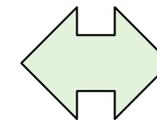


Neglect broadening of spectral functions

$$|g_s^R(p, 0)|^2 \simeq \delta(E_p^s - \mu)\pi/\Gamma$$

Well-defined quasi-particles

Fully quantum approach



Boltzmann approach

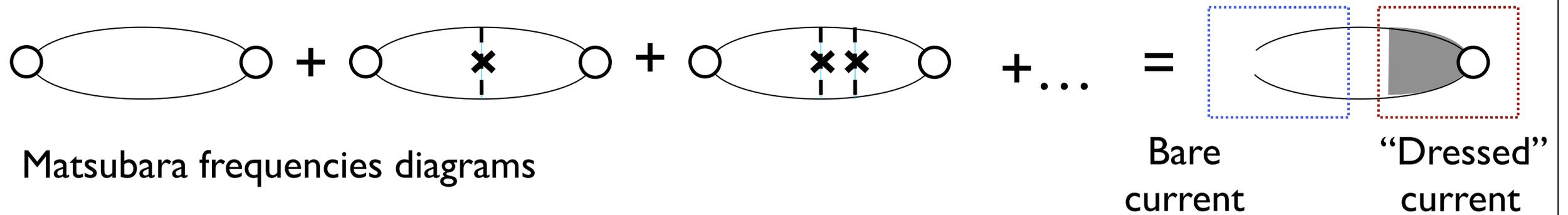
Simple analytical formulae

HD regime :  $\Gamma_B \cong \Gamma_0 \Rightarrow \tau \cong \tau_0$

DSO regime:  $\Gamma_B \cong \Gamma_0 \frac{n_0}{n} \Rightarrow \tau \cong \tau_0 \frac{n}{n_0}$

# Transport properties

## Kubo formula



$$\sigma \simeq \frac{1}{2\pi\mathcal{V}} \sum_{\mathbf{p}} \text{Tr} [j_x(\mathbf{p}) G^R(\mathbf{p}, 0) J_x^{RA}(\mathbf{p}) G^A(\mathbf{p}, 0)]$$

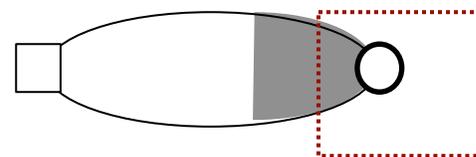
A or R = advanced or retarded argument

$$\bigcirc = j_x(\mathbf{p}) = e(p_x/m + \alpha\sigma_y)$$

$$\text{shaded semi-circle} = J_x^{RA}(\mathbf{p}) = e(p_x/m + \tilde{\alpha}^{RA}\sigma_y)$$

Same vertex also leads to vanishing spin-Hall effect !!!

Spin-Hall bubble



Only the “anomalous” current is renormalized

Boltzmann approach:

$$\sigma_{\text{dc}}^B = \frac{e^2 v_F}{4\pi} \sum_{\eta} \tau_{\eta}^{\text{tr}} p_{\eta} = \sum_{\eta} \sigma_{\eta}$$

Incoherent sum of contributions of the two FS corresponding to different  $\eta$

Transport scattering times  $\leftarrow$

$$\text{Fermi momenta : } p_{\eta} = |mv_F - \eta p_0|$$

$$v_F = \sqrt{2mE_F}$$

# Conductivity

In the absence of spin-orbit coupling  
within our assumptions:

$$\sigma = \sigma_{\text{Drude}} \equiv \frac{e^2 n \tau}{m}$$

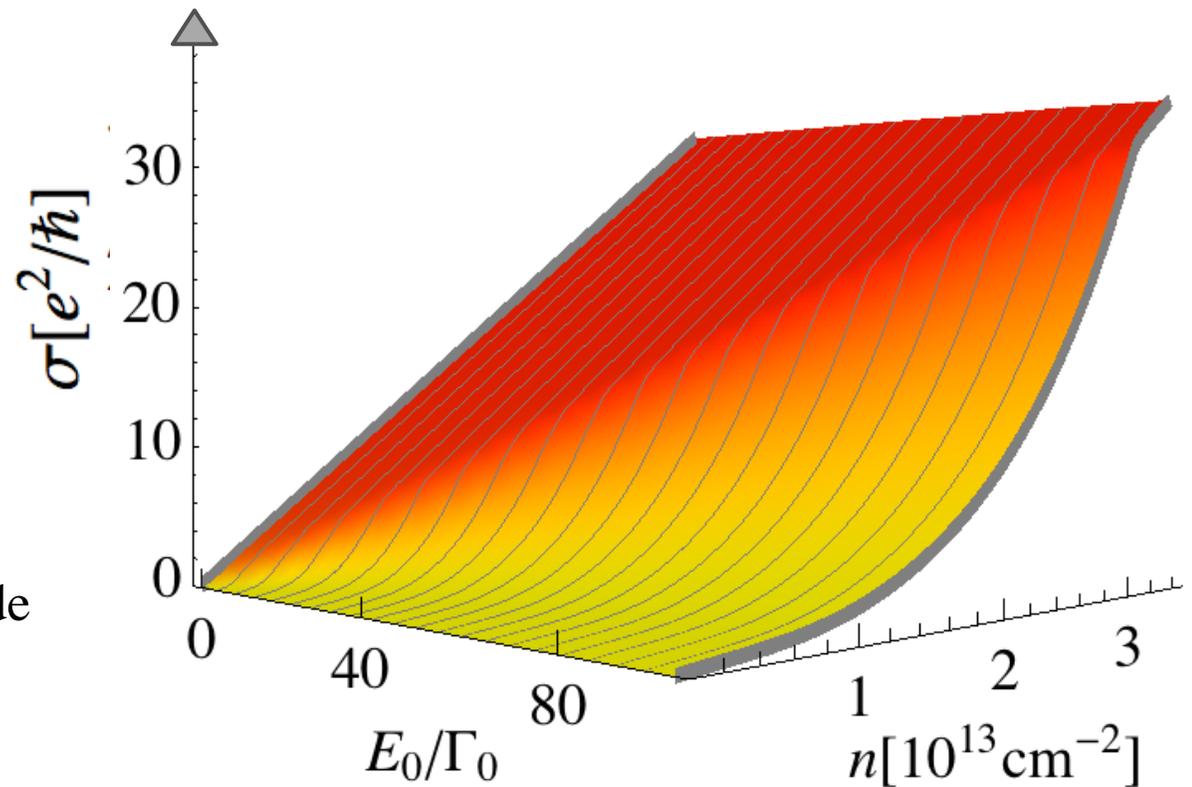
# Conductivity as a function of density and Rashba coupling



$\sigma = 0$

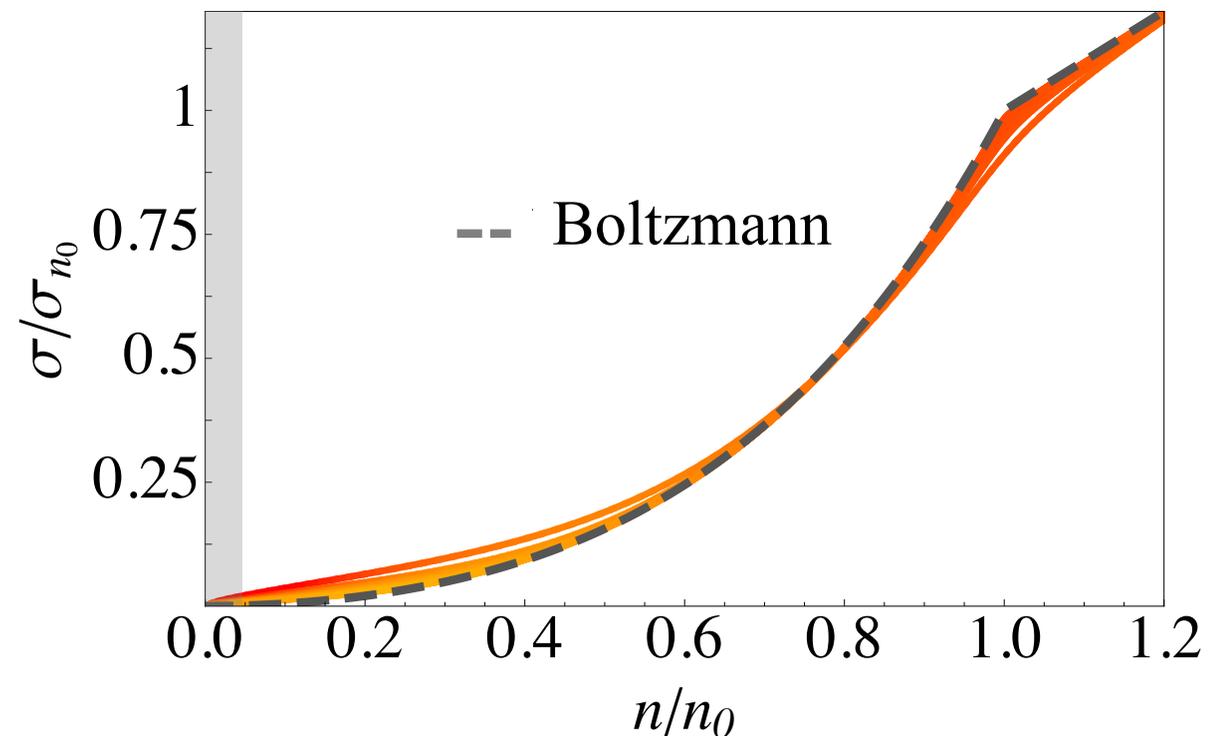
$\sigma = \sigma_{\text{Drude}}$

$$\sigma_{\text{Drude}} \equiv \frac{e^2 n \tau}{m}$$



- If  $E_0 > \Gamma_0$  at low-doping conductivity becomes sublinear and deviates from Drude law

- By appropriate rescaling universal behavior obtained



## Two charge-transport regimes

Analytical results within the approximation of well-defined quasi-particles

$$n > n_0 \quad \Rightarrow \quad \sigma_{\text{Drude}} \equiv \frac{e^2 n \tau}{m} \quad \tau \simeq \tau_0$$

$$n < n_0 \quad \Rightarrow \quad \sigma_{\text{DSO}} \equiv \frac{e^2 n_0 \tau_0}{2m} \left( \frac{n^2}{n_0^2} + \frac{n^4}{n_0^4} \right) \quad \tau \simeq \tau_0 \frac{n}{n_0}$$

Remarkably simple formula!

In both cases coincide with Boltzmann results!

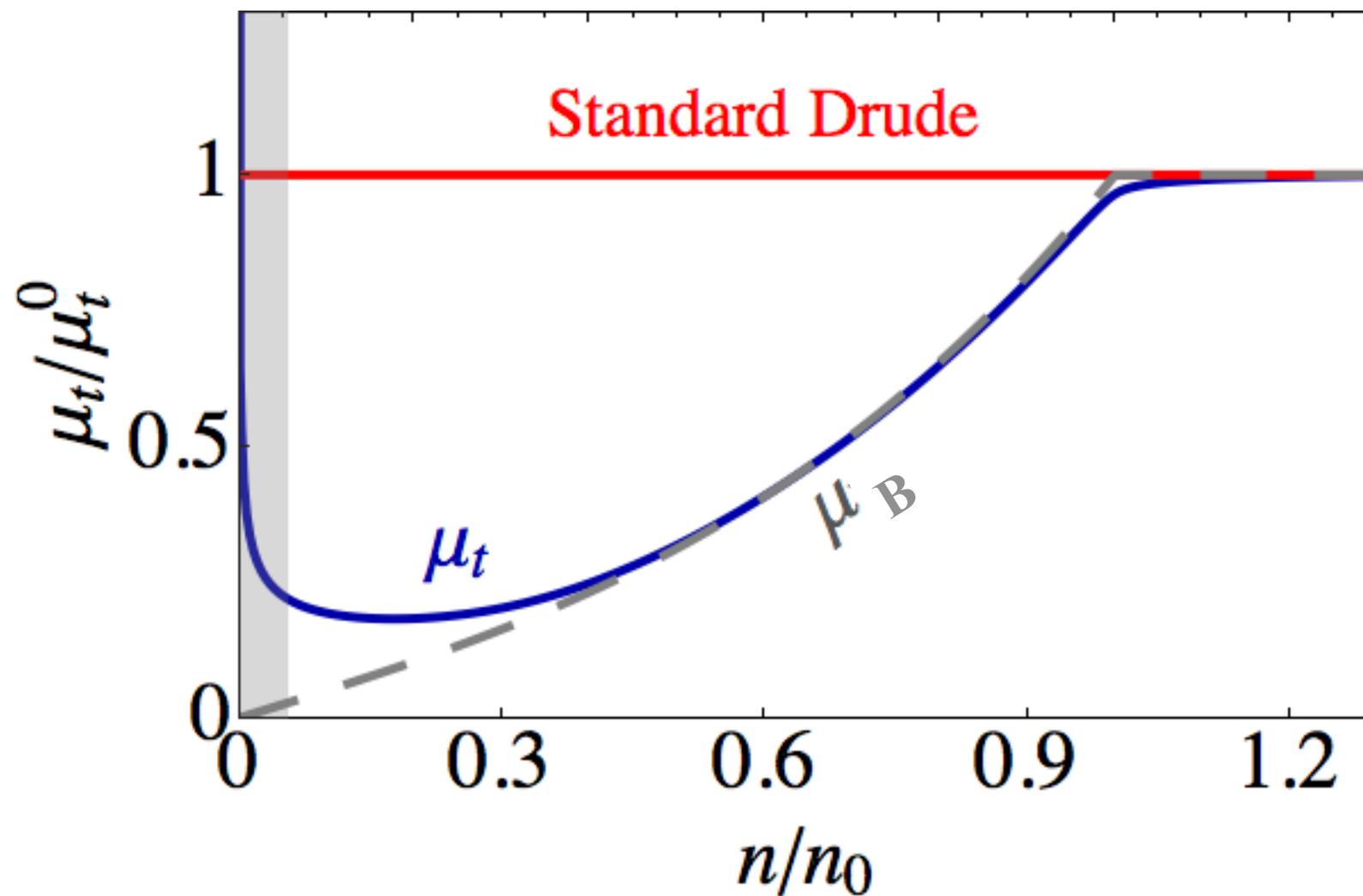
Decrease of the  
conductivity due to BOTH

- Increase of the scattering rate
- Non-zero anomalous vertex

$$\sigma_{\text{DSO}} = \frac{1}{2} \left( 1 + \frac{n^2}{n_0^2} \right) \sigma_{\text{Drude}}$$

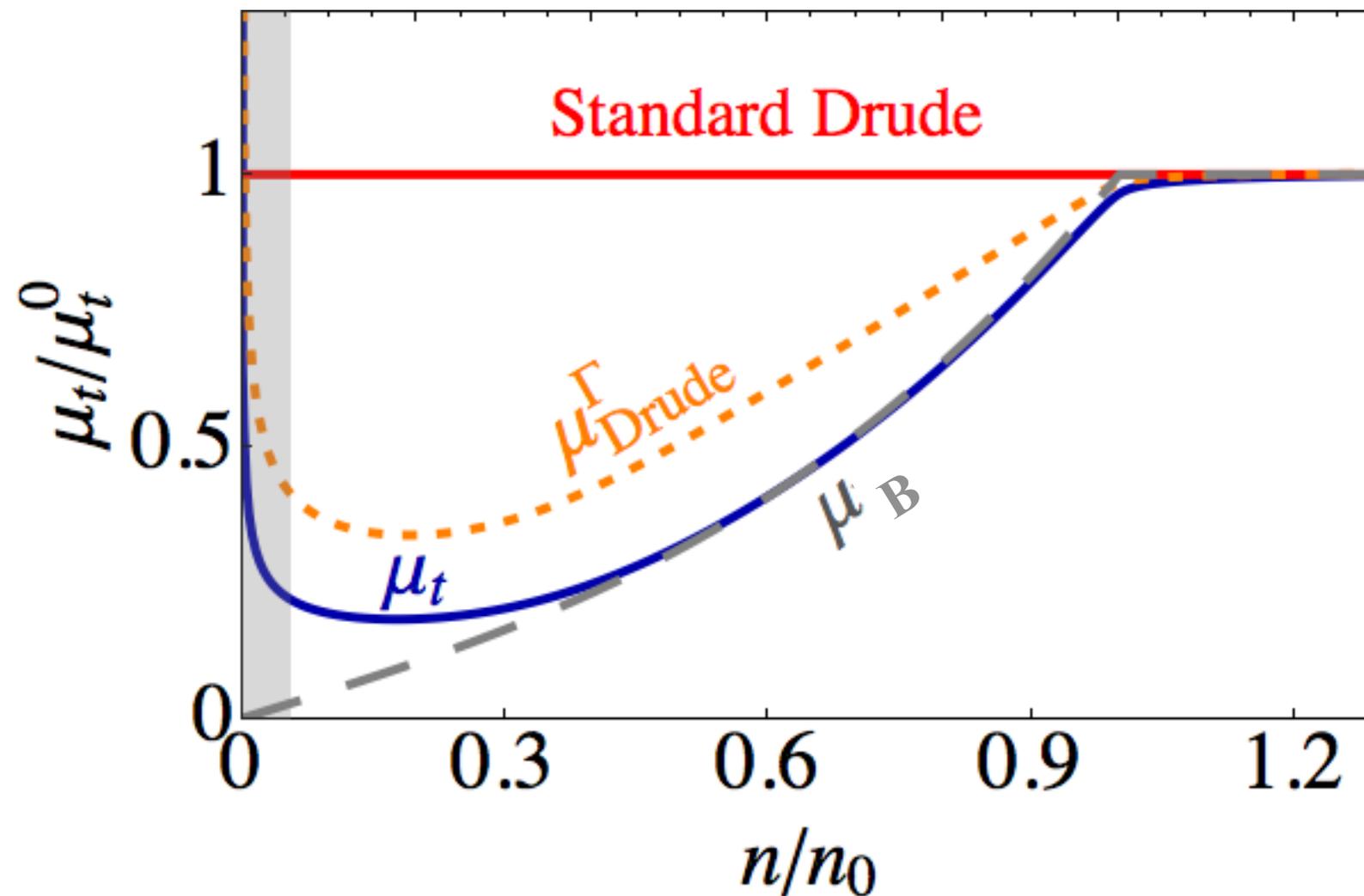
**Mobility,  $\mu_t$**  Drift velocity per unit electric field  $\mu_t = v_{\text{drift}}/E$

Related to the conductivity, via  $\mu_t = \frac{\sigma}{en}$  Drude limit  $\mu_t^0 = \frac{e\tau_0}{m}$



**High accuracy of Boltzmann !**

## Two-fold origin mobility modulation



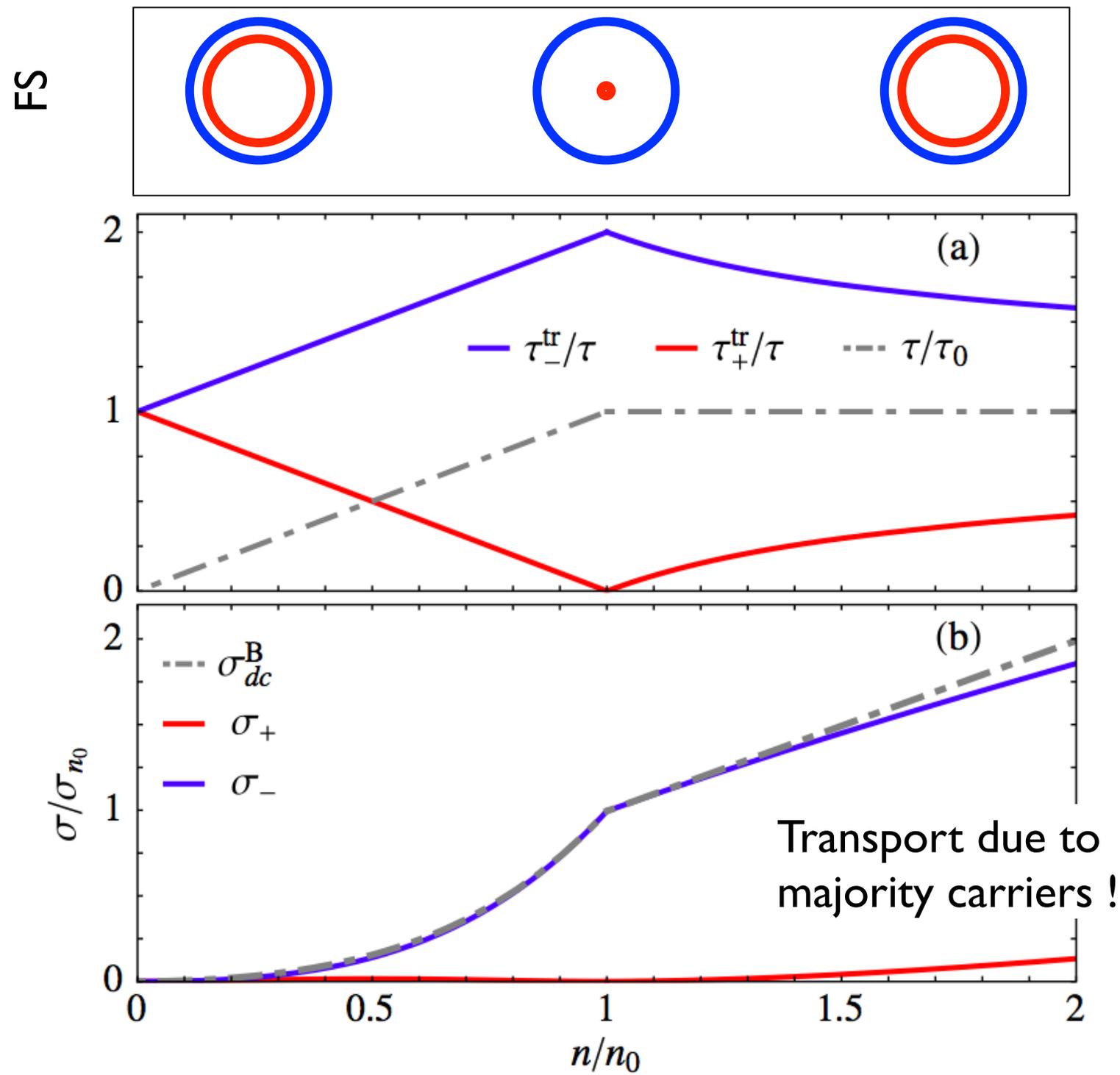
$$\mu_{\text{Drude}}^{\Gamma} = \frac{e\tau}{m}$$

Strong dependence of the mobility on doping due to BOTH

- Increase of the scattering rate
- Non-zero anomalous vertex

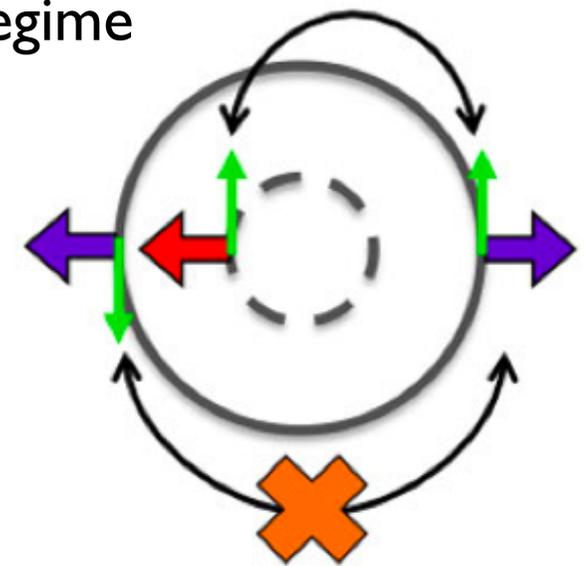
# Two types of charge carriers

Different transport properties across different density regimes

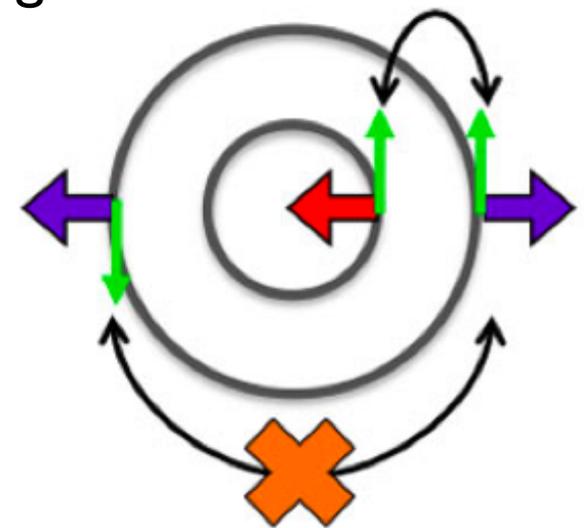


Suppression of backscattering within the same chiral band

HD regime



DSO regime

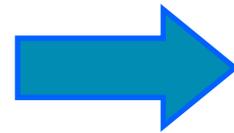


# Conclusions

- In the HD regime: no effect of SOC on transport !!!
- In the DSO regime:

Van-Hove singularity

Non-zero anomalous vertex



Reduction of the mobility and of the longitudinal conductivity

**Analytical universal formulae to describe the conductivity which could be readily used to fit experiments**

- Measure  $E_0$  in a plain DC transport experiment!
- Use Rashba to control DC transport

**ON-GOING WORK: EFFECT OF RASHBA SPIN-ORBIT COUPLING IN STRONGLY CORRELATED ELECTRONIC SYSTEMS**

In collaboration with:

A. Amaricci, M. Capone and L. Fanfarillo @ SISSA

G. Sangiovanni @ Wurzburg