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Abstract

The self-consistent model, $\Psi$, is to make the microkinetic instability driven by longitudinal space charge in a ultra-relativistic bunch of FELs to be a Hamiltonian to be described as a transcendental Vlasov model with self-consistent drift maps and Poisson type collective kick maps. This model can be related exactly only by the Hamiltonian of the fast FEL operator and have been a principle suited to analytically compute approximations to the microkinetic gain function.

References


The Perron Frobenius Operator

- Let $L: \mathbb{R}^n \to \mathbb{R}^n$ be measure-preserving & invertible (e.g. symplectic).
- And $\Psi: \mathbb{R}^n \to \mathbb{R}$ arbitrary in $L^2(\mathbb{R}^n, R)$.
- Then the composition $\Psi \circ \Psi$ is also in $L^2(\mathbb{R}^n, R)$.
- Perron-Frobenius Operator $\Psi$, $\mathcal{L}^n(\mathbb{R}^n, R)$ of $L^2(\mathbb{R}^n, R)$ is a linear operator:

$$\mathcal{L}^n(\mathbb{R}^n, R) := \mathcal{L}$$

- $\mathcal{L}$ is linear

$$\mathcal{L}^n(\mathbb{R}^n, R)$$

- Describes the Liouvillian evolution of $\Psi$.

Gain Functions

- Starting from an initial perturbation $\psi_0 = \psi(y, p)$ with projected spatial density $\psi_0(y)$ and Fourier transform $\psi_0(k)$.
- One may define the most general $n$th order $n$-stage gain function:

$$J^{(n)}(y_1, y_2, \ldots, y_n) = \int_{\mathbb{R}^n} \frac{d^n q}{(2\pi)^n} e^{i \int_0^1 \phi(y, p, \omega) \frac{dp}{m_c} d\omega}$$

- Accumulated general $n$th order $n$-stage gain function:

$$J^{(n)}(y_1, y_2, \ldots, y_n) = \int_{\mathbb{R}^n} \frac{d^n q}{(2\pi)^n} e^{i \int_0^1 \phi(y, p, \omega) \frac{dp}{m_c} d\omega}$$

- If the chirp of the reference FELs $(\omega_0, \ldots, \omega_0)$ does not change only by the longitudinal, and if generation harmonic is neglected, it is often convenient to define the compression corrected, absolute gain:

$$J^{(n)}(y_1, y_2, \ldots, y_n) = \int_{\mathbb{R}^n} \frac{d^n q}{(2\pi)^n} e^{i \int_0^1 \phi(y, p, \omega) \frac{dp}{m_c} d\omega}$$

- Example: Infinitely Long Bunch

- $\psi_0 = e^{-i \frac{p^2}{2m_c} + \frac{c^2}{2m_c} \omega_0 t}$

- Linearized Vlasov Evolution for One BC Stage

- Given BC stage $\omega_l / m_c p_0$:

$$\frac{d}{dt} \psi_l = \omega_l / m_c p_0$$

- A weakly symmetric smooth shape bunch of $\psi_{s\omega} / \omega_l$ mapped by the BC stage to $\psi_{s\omega} / m_c p_0$;

$$\frac{d}{dt} \psi_{s\omega} = \omega_l / m_c p_0$$

- Linearized Vlasov Evolution of the perturbation $\psi_{l\omega} / m_c p_0$ around $\psi_{s\omega}$ reads:

$$\frac{d}{dt} \psi_{l\omega} = \frac{\omega_l}{m_c p_0} \psi_{l\omega}$$

- Cascade of $m$ BC Stages

- We slightly change our notation: stage $i$: $\psi_i$, $\psi_{i+1}$, $\psi_{i+2}$, $\psi_{i+3}$, $\ldots$ 5 ≤ $i$ ≤ $n$.

- Mapping from stage $i$ to $i+1$:

$$\sum_{j=1}^{n-i} \psi_{i+j} \left( \frac{\omega_l}{m_c p_0} \right) \frac{d}{dt} \psi_{i+j} = \sum_{j=1}^{n-i} \psi_{i+j} \left( \frac{\omega_l}{m_c p_0} \right) \frac{d}{dt} \psi_{i+j}$$

- Higher orders and delay iterations are possible but too lengthy (and unpleasant) to present on this paper.

General Remarks

- No explicit knowledge on $\psi_l$ is needed for single stage system.

- For $\psi_l / m_c p_0$ to exist (at least in the weak sense) $\psi_l / m_c p_0$ must likely not be enough:

- Instead $\psi_{s\omega} / m_c p_0$ $(\omega_0, \ldots, \omega_0)$ or similar is necessary.

- The model is not very likely to be applicable to strongly curved reference bunches as they sometimes appear in FELs.

- Numerics needed: $\psi_l$ Amstutz talk!?