Technical challenges producing Round Photon Beams in future Storage Ring based Diffraction Limited Light Sources

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I. Round Beams – Why?

“A significant fraction of the beamline users at Swiss light source (SLS) prefer “round beam“ rather than flat beam, ...“, M. Aiba, et al., TUPJE045, IPAC2015

- Imaging applications would profit, better match to optics, circular zone plates
- Monochromators without entrance slit and dispersion into the vertical plane prefer flat beams

What users really prefer most is radiation optimized for their own experiment.

Round Beam Workshop, SOLEIL, June 14th - 15th, 2017
https://www.synchrotron-soleil.fr/fr/evenements-mini-workshop-round-beams
I. Round Beams – Why?

Reduced electron density from round electron beam has advantages:
- smaller Touschek losses increase life time
- decreased intra beam scattering (IBS) lowers beam blow up

Can all beamlines cope with a round electron beam?
Elettra today: $\varepsilon_x = 7 \text{ nm-rad} \quad \varepsilon_y = 70 \text{ pm-rad} \quad \text{Elettra 2.0: } \varepsilon_x = \varepsilon_y = 154 \text{ pm-rad}$
Spectral brilliance and coherent fraction

Liu Lin: “Towards Diffraction Limited Storage Ring Based Light Sources”

- **Spectral brilliance**: Flux density in phase space

\[
B(\lambda) \propto \frac{F(\lambda)}{(\epsilon_{x,e^-} \otimes \epsilon_r(\lambda))(\epsilon_{y,e^-} \otimes \epsilon_r(\lambda))}
\]

- **Photon flux** [photons/s/0.1% bw]

- **Electron beam emittance**

- **Photon limiting emittance**

\[
\epsilon_r = \sigma_r \sigma_r' = \frac{\lambda}{4\pi}
\]

for Gaussian beam

\[
\epsilon_r = \sigma_r \sigma_r' \approx \frac{\lambda}{2\pi}
\]

for undulator beam

- **Coherent fraction for undulator radiation**

\[
f_{coh} = \frac{(\lambda/2\pi)^2}{(\epsilon_{x,e^-} \otimes \epsilon_r(\lambda))(\epsilon_{y,e^-} \otimes \epsilon_r(\lambda))}
\]

- **Diffraction limited storage ring**

\[
\epsilon_{x,y} \approx \epsilon_r(\lambda) = \frac{\lambda}{2\pi}
\]

\[
\epsilon_{x,y} \approx 100 \text{ pm.rad}
\]

diffraction limit for 2 keV

\[
\epsilon_{x,y} \approx 20 \text{ pm.rad}
\]

diffraction limit for 10 keV
Diffraction Limit: low emittance is not all!
Phase space matching

Highest brilliance from undulator of length $L$ is achieved when

$$\beta^\text{opt}_{x,y} \approx \frac{L}{\pi}$$

$$\beta^\text{opt}_{x,y} \sim 1 - 2m$$


IV. CONCLUSIONS

In this paper we have described three different coherent mode representations of partially coherent undulator radiation. We began with the well-known Gaussian-Sehell decomposition in terms of Gauss-Hermite modes, which is valid provided the electron beam emittance is much larger than the natural radiation emittance $\lambda/4\pi$. In this largely incoherent case the specifics of the single-electron undulator field are unimportant. We then refined our analysis to include the situation when the electron beam emittance $\varepsilon_x$ in one direction is arbitrary, and found that the modes along $y$ are determined by solving a matrix

Figure 8(a) shows that the coherence is maximized when $\beta_z \approx L_u/\pi$ (or $\hat{\beta}_z \approx 1$), which indicates that the “natural” Rayleigh range of undulator radiation $Z_R \approx \beta_z \approx L_u/\pi$. Unfortunately, it is nearly impossible for lattice designers to make the beta functions in both $x$ and $y$ be simultaneously that small, and typically $\beta_x > 3L_u/\pi$.

Liu Lin: “Towards Diffraction Limited Storage Ring Based Light Sources”
The best approximation therefore appears to be:

<table>
<thead>
<tr>
<th>Method</th>
<th>$\sigma_R$</th>
<th>$\sigma_R'$</th>
<th>$\varepsilon_R = \sigma_R \sigma_R'$</th>
<th>$\beta_R = \sigma_R / \sigma_R'$</th>
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</thead>
<tbody>
<tr>
<td>Kim (NIM 1986)†</td>
<td>$\sqrt{\lambda/L}$</td>
<td>$\sqrt{\lambda L/4\pi}$</td>
<td>$\lambda/4\pi$</td>
<td>$L/4\pi$</td>
</tr>
<tr>
<td>Kim (PAC 1987)</td>
<td>$\sqrt{\lambda/2L}$</td>
<td>$\sqrt{2\lambda L/4\pi}$</td>
<td>$\lambda/4\pi$</td>
<td>$L/2\pi$</td>
</tr>
<tr>
<td>Borland (IPAC 2012)</td>
<td>$\sqrt{\lambda/2L}$</td>
<td>$\sqrt{2\lambda L/2\pi}$</td>
<td>$\lambda/2\pi$</td>
<td>$L/\pi$</td>
</tr>
<tr>
<td>Hettel &amp; Borland (PAC 2013)</td>
<td>$\sqrt{\lambda/2L}$</td>
<td>$\sqrt{2\lambda L/2\pi}$</td>
<td>$\lambda/2\pi$</td>
<td>$L/\pi$</td>
</tr>
<tr>
<td>Hettel (IPAC 2014)</td>
<td>$\sqrt{\lambda/2L}$</td>
<td>$\sqrt{2\lambda L/2\pi}$</td>
<td>$\lambda/2\pi$</td>
<td>$L/\pi$</td>
</tr>
<tr>
<td>Huang (IPAC 2013)</td>
<td>$\sqrt{\lambda/4L}$</td>
<td>$\sqrt{\lambda L/2\pi}$</td>
<td>$\lambda/4\pi$</td>
<td>$L/2\pi$</td>
</tr>
<tr>
<td>Lindberg &amp; Kim (PRSTAB 2015)</td>
<td>$\sqrt{\lambda/4L}$</td>
<td>$\sqrt{\lambda L/2\pi}$</td>
<td>$\lambda/4\pi$</td>
<td>$L/\pi$</td>
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<tr>
<td>Liu (IPAC 2017)</td>
<td>$\sqrt{\lambda/2L}$</td>
<td>$\sqrt{2\lambda L/2\pi}$</td>
<td>$\lambda/2\pi$</td>
<td>$L/\pi$</td>
</tr>
</tbody>
</table>
II. Round Beam Locally – Flat-Round-Transformation

Application of the Emittance Adapter to SOLEIL and MAX IV

Pascale Brunelle
on behalf of the Round Beam Project Team
II. Round Beam Locally – Flat-Round-Transformation

\[
\left( \varepsilon_x \right)_{\text{apparent}} \sim \sqrt{\left( \varepsilon_x \cdot \varepsilon_z \right)_{\text{ring}}}
\]

\[
n_x = 18.127, \quad n_z = 11.142
\]

\[
\varepsilon_x = 9.6 \, \text{nm rad}
\]

Adaptation to the nominal optics
II. Round Beam Locally – Flat-Round-Transformation

Summary
Pascale Brunelle

For the two SOLEIL beamlines tested as example, it has been demonstrated that:

→ The gain in flux density at the sample is mainly due to the reduction of the horizontal electron beam size at source.
→ A reduction by a factor \( \sim 3 \) is obtained on the present SOLEIL and MAX-IV storage rings with a 10 T solenoid field (perfect adaptation is obtained with a solenoid field of \( \sim 140 \) T for an undulator length of 2 m).

Round Beam Workshop, SOLEIL, June 14\textsuperscript{th} - 15\textsuperscript{th}, 2017
https://www.synchrotron-soleil.fr/fr/evénements/mini-workshop-round-beams
Ultimate synchrotron radiation source with horizontal field wigglers

A. Bogomyagkov, E. Levichev, P. Piminov, S. Sinyatkin

Budker Institute of Nuclear Physics
Novosibirsk

Low Emittance Rings 2014 Workshop
17-19 September 2014 INFN-LNF
III. Radial Damping Wiggler Fields

Straight section: damping wigglers with horizontal field

Wigglers with horizontal field:
- $B = 2.3$ T
- $\lambda = 4.8$ cm
- $N_\lambda = 42$
- $L_{\text{wiggler}} = 2.04$ m
- $N_{\text{total}} = 20$
- $L_{\text{total}} = 40.8$ m
III. Radial Damping Wiggler Fields

Parameters of the ring

\[ \text{Ring} = 4 \times 6 \times \left[ 5 \times \text{FiveCell} + \text{Straight} \right] \]

20 straight are sections empty
4 straight sections are occupied by damping wigglers

<table>
<thead>
<tr>
<th></th>
<th>Wigg OFF</th>
<th>Wigg ON</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy, GeV</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>Circumference, m</td>
<td></td>
<td>1379</td>
</tr>
<tr>
<td>Chromaticity h/v</td>
<td></td>
<td>-184/-251</td>
</tr>
<tr>
<td>Betatron tunes h/v</td>
<td></td>
<td>84.52/91.772</td>
</tr>
<tr>
<td>Horizontal Emittance, pm rad</td>
<td>64</td>
<td>3</td>
</tr>
<tr>
<td>Vertical Emittance, pm rad</td>
<td>0.6</td>
<td>8.6</td>
</tr>
<tr>
<td>Energy spread</td>
<td>(4 \times 10^{-4})</td>
<td>(1.2 \times 10^{-3})</td>
</tr>
<tr>
<td>Momentum compaction</td>
<td>(7.8 \times 10^{-5})</td>
<td>(7.8 \times 10^{-5})</td>
</tr>
<tr>
<td>Damping times h/v/s, msec</td>
<td>210/210/105</td>
<td>10/10/5</td>
</tr>
<tr>
<td>Wiggler field, T</td>
<td>0</td>
<td>2.33</td>
</tr>
</tbody>
</table>

If you want 10% to 20% coupling: vertical dispersion helps
• Has been used to create slightly enlarged vertical emittance – coupling on the %-level (BESSY II, ALS, …) – less sensitive to impact of IDs on β-coupling
• A vertical chicane in a straight section easily produces sufficient dispersion and vertical emittance on the 20%-level
IV. Round Beam in Möbius Accelerator

In the Möbius Accelerator transverse particle coordinates are exchanged every turn by a set of skew quadrupole magnets sharing the natural emittance equally among the two planes. (R. Talman, PRL 74, 1590 (1995) and M. Aiba, et al., TUPJE045, IPAC2015, Richmond, VA, USA)
IV. Round Beam in Möbius Accelerator

Uncoupled storage ring:
IV. Round Beam in Möbius Accelerator

Uncoupled storage ring:

insertion
IV. Round Beam in Möbius Accelerator

Orbit and optics repeat every second turn:

- rotator – complete exchange of horizontal and vertical motion
- transverse particle coordinates are exchanged every turn by a set of skew quadrupole
IV. Round Beam in Möbius Accelerator

Orbit and optics repeat every second turn:

rotator – complete exchange of horizontal and vertical motion

transverse particle coordinates are exchanged every turn by a set of skew quadrupole
Award-winning Norwegian architectural firm, Snøhetta, unveiled an innovative proposal for the Max-Lab in Lund, Sweden. The circular shape is twisted and raised to create a dynamic form based on a Möbius strip that becomes an actual volume, not just a ribbon.
Off-axis injection impossible with this really strong coupling of the horizontal and vertical plane.

Except – you have a large circumference like PETRA IV and can exchange transverse coordinates twice per revolution (see talk of Ilya Agapov, “Round beams at Petra IV”, NOCE, Arcidosso, Italy, September, 2017)

25/25 pm lattice
E. Wilson „Linear Coupling“, CERN 85-19, p. 114, with time dependent skew quadrupole:

\[ \ddot{x} + \omega_x^2 x = -y \cdot k \cdot (e^{i\omega t} + e^{-i\omega t}) / 2 \]
\[ \ddot{y} + \omega_y^2 y = -x \cdot k \cdot (e^{i\omega t} + e^{-i\omega t}) / 2 \]

**Ansatz – small coupling:**

\[ x(t) = X(t) \cdot e^{i\omega_x t} \]
\[ y(t) = Y(t) \cdot e^{i\omega_y t} \]

\( X(t) \) and \( Y(t) \) are slowly varying functions – second time derivatives as well as fast oscillating terms are ignored

\[ 2i\omega_x \dot{X} = -Y \cdot k \left[ e^{i(\omega - \Delta \omega) t} + e^{-i(\omega + \Delta \omega) t} \right] \]
\[ 2i\omega_y \dot{Y} = -X \cdot k \left[ e^{-i(\omega - \Delta \omega) t} + e^{i(\omega + \Delta \omega) t} \right] \]

\( X(t) \) and \( Y(t) \) are slowly varying functions – second time derivatives as well as fast oscillating terms are ignored

Coupled first order turned into uncoupled second order differential equation:

\[ \dddot{X} - i(\Delta \omega - \omega) \dot{X} + \frac{k^2}{16 \omega_x \omega_y} X = 0 \]

On resonance – the fast oscillation \( x(t) \) shows a harmonic modulation and beating with energy exchange to the vertical plane occurs

**General resonance condition:**

\[ Q_x - Q_y = n \pm \omega / \omega_0 \]

with the revolution frequency, \( \omega_0 \), and the frequency of the skew gradient, \( \omega \).

Identical results for coupling created by constant or time dependent fields in solenoids.
VI. EMITTANCE SHARING – COUPLING RESONANCE

linear coupling due to skew quadrupole gradient:

\[ Q_x - Q_y = n, \quad n = \text{integer} \]

on resonance emittance sharing - \( \varepsilon_y = \varepsilon_x = \varepsilon_0 / 2 \)

with equal damping times, \( T_x = T_y \), in both planes.

Comparison of solutions from multi particle tracking and first modeling attempts with analytical solutions based on moment mapping.
VI. EMITTANCE SHARING – COUPLING RESONANCE

linear coupling due to skew quadrupole gradient:
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with equal damping times, \( T_x = T_y \), in both planes

\[ \varepsilon_y = \varepsilon_x = 2/3 \varepsilon_0 \]

Comparison of solutions from multi particle tracking and first modeling attempts with analytical solutions based on moment mapping.

With \( T_x = T_y / 2 \) and on resonance \( \varepsilon_y = \varepsilon_x = 2/3 \varepsilon_0 \)

Elettra 2.0:

\[ \varepsilon_y = \varepsilon_x = 22.2\text{ms}/(22.2\text{ms} + 14.6\text{ms}) \varepsilon_0 = 154\text{pm rad} \]
Comparison of solutions from multi particle tracking and first modeling attempts with analytical solutions based on moment mapping.

With $T_x = T_y/2$ and on resonance $\varepsilon_y = \varepsilon_x = 2/3 \varepsilon_0$

Elettra 2.0:

$\varepsilon_y = \varepsilon_x = 22.2\text{ms}/(22.2\text{ms} + 14.6\text{ms}) \varepsilon_0 = 154\text{pm}\cdot\text{rad}$

Compensation of the coupling resonance in the BESSY II storage ring – as expected: damping dominates for very small coupling coefficients, and width depends on coupling strength: „power broadening“, will be helpful later on.
VII. EMITTANCE SHARING – BY EXCITING THE COUPLING RESONANCE

For better control of the coupling and in case the storage ring can not be operated at the coupling resonance the resonance can be excited artificially. With a time dependent sinusoidal varying skew gradient the resonance condition is:

\[ Q_x - Q_y = n \pm \frac{\omega}{\omega_0} \]

with the revolution frequency, \( \omega_0 \), and the frequency of the skew gradient, \( \omega \).

Neighboring currents in opposite directions.
Full coupling and emittance sharing achievable – little power broadening, sensitive to tune jitter.
The required frequency, $F_{sq}$, for the skew quadrupole is on the order of 100 kHz. Striplines are not required. Simpler design could look like this:

skew quadrupole with four wire arrangement and currents flowing in alternating directions

$$\left| \frac{\partial B_x}{\partial x} \right| = \frac{4 \cdot \mu \cdot I}{\pi \cdot a^2} = \frac{1.6 \cdot 10^{-6} \cdot I [A]}{a^2 [m^2]} [T/m]$$

A time dependent solenoid might even be simpler to construct.

quite similar to our non-linear injection kicker magnet

(T. Atkinson, et al., THPO024, IPAC2011)
VIII. EMITTANCE SHARING – Static Skew Quadrupole Magnet

Beam injected at +10 mm – the first 100 turns

β_x = 10.00 m  β_y = 2.00 m

dB_x/dx = -0.00224 T/m

100
with skew quadrupole magnet the required acceptance exceeds physical vertical aperture if \( X_{\text{inj}} > \text{vertical aperture} \cdot (\beta_{X_{\text{inj}}}/\beta_{Y_{\text{ap}}})^{1/2} \)

Inject with smaller initial amplitude, \( X_{\text{inj}} \), because of small dynamic aperture
VI. EMITTANCE SHARING – Time dependent Skew Quadrupole Magnet

Required acceptance – non-linear skew quadrupole magnet:

Beam injected at +10 mm – the first 100 turns

\[ B_x(x,y=0) \]

\[ B_x = 10.00 \text{m} \quad \beta_y = 2.00 \text{m} \]

\[ I = 0.035 \text{A} \]

\[ dB_x/dx = -0.00224 \text{T/m} \]
VI. EMITTANCE SHARING – Time dependent Skew Quadrupole Magnet

Required acceptance – non-linear skew quadrupole magnet:

Beam injected at +10 mm – the first 1,000,000 turns

\[ B_x(x,y=0) \]

\[ \beta_x = 10.00 \text{m} \]
\[ \beta_y = 2.00 \text{m} \]

Required acceptance fits vertical acceptance – with non-linear skew quadrupole magnet

Current: 0.035 A
\[ dBx/dx = -0.00224 \text{T/m} \]
VIII. EXCITING THE COUPLING RESONANCE – OPERATIONAL ASPECTS

Level of excitation as large as not yet to cause injection losses

Level of excitation will still be too small to broaden the coupling resonance considerably
Small resonance width – stability of the tunes sufficient? → active tune stabilization

$I = 0.835 \, \text{A}$
$\frac{dBx}{dx} = -0.00224 \, \text{T/m}$
Tune shift with amplitude would help – resonance condition only fulfilled for small amplitudes, stronger excitation could be used

\[ \Delta v_x = 0.02 \rightarrow \Delta F_x = 11.4 \text{ kHz} \rightarrow \text{much larger than the natural resonance width} \]
\[ \equiv \frac{4}{\tau_{\text{trans}}} \] or width due to non-linear chromatic effects
Tune shift with amplitude would help – resonance condition only fulfilled for small amplitudes, stronger excitation could be used.

\[ \Delta \nu \sim 0.018 \, @ \, +5\text{mm} \rightarrow \Delta F = 21 \text{kHz} \] – much larger than the natural resonance width \( 4/\tau_{\text{trans}} \) or width due to non-linear chromatic effects.
Beam injected at +10 mm – with tune shift with amplitude - the first 100 turns

4 times larger skew gradient – fast filamentation of the injected beam due to non-linearity which creates $2 \cdot 10^{-2}$ tune shift for 10mm horizontal amplitude
Beam injected at +10 mm – with tune shift with amplitude - over 100,000 turns

4 times larger skew gradient – relaxed aperture requirement due to tune shift with amplitude
Beam injected at +10 mm – with tune shift with amplitude – over 100000 turns

Even with a much larger skew gradient the injected beam remains within the vertical acceptance
VIII. EXPERIMENTAL STUDIES AT THE APS (MOPMA013, IPAC2015)

Measured top-up injection efficiency vs. tune separation $\Delta$ at different $\kappa$ (legend)

\[ |\kappa| = \frac{1}{2\pi\beta p} \frac{\partial B_x}{\partial x} L \sqrt{\beta_x \beta_y} \]

Measured beam size (raw data) vs. tune separation $\Delta$ at different $\kappa$ (legend)
The resonant coupling sets in for small horizontal oscillation amplitude. Stronger skew gradients will not cause losses of injected particles and the “power broadening” can be made as large as desirable and acceptable by the amplitude dependent tune shift. Tune drift and jitter become less important.

This could be tried out at MAXIV with the tune of the ring set to the coupling resonance and the result would be equal emittances in both planes of \( \sim 200 \text{ pm} \cdot \text{rad} \). Elettra 2.0 will reach \( \sim 154 \text{ pm} \cdot \text{rad} \).
Five techniques for the production of round beams have been proposed:

- **Local flat-to-round beam transformation** (A. Chao and P. Raimondi, SLAC-PUB-14808)
- **Radial wiggler fields** (A. Bogomyagkov, et al., „Ultimate synchrotron radiation source with horizontal field wigglers“, LER Workshop, September 2014, Frascati, Italy)
- **The Möbius accelerator** (R. Talman, PRL 74, 1590 (1995) and M. Aiba, et al., TUPJE045, IPAC2015, Richmond, VA, USA)
- **Excitation of the coupling resonance with time dependent coupling fields** (this presentation)
- **Sitting on the coupling resonance and tune shift with amplitude**

Last two techniques require careful tune stabilization or adjustment of excitation frequency and strength of coupling fields (skew gradients or solenoid field)

**Operating on the coupling resonance will be the method of choice for Elettra 2.0**
## IX. SUMMARY

<table>
<thead>
<tr>
<th>Technical Approach</th>
<th>Injection</th>
<th>Emittance Control</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local Emittance Adapter</td>
<td>off- and better, on-axis</td>
<td>no</td>
<td>large</td>
</tr>
<tr>
<td>Radial Damping Wigglers</td>
<td>off-axis</td>
<td>yes</td>
<td>large</td>
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<tr>
<td>Möbius Accelerator</td>
<td>on-axis*</td>
<td>no</td>
<td>moderate challenging*</td>
</tr>
<tr>
<td>Coupling Resonance Excitation</td>
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<td>(no)</td>
<td>moderate</td>
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<tr>
<td>On Coupling Resonance</td>
<td>on-axis*</td>
<td>(no)</td>
<td>challenging* trivial</td>
</tr>
<tr>
<td></td>
<td>off-axis, tune shift with amplitude</td>
<td>(no)</td>
<td></td>
</tr>
</tbody>
</table>

* vertical aperture dependent, inject closer to axis and accumulate beam without swap-out