

Bender Model

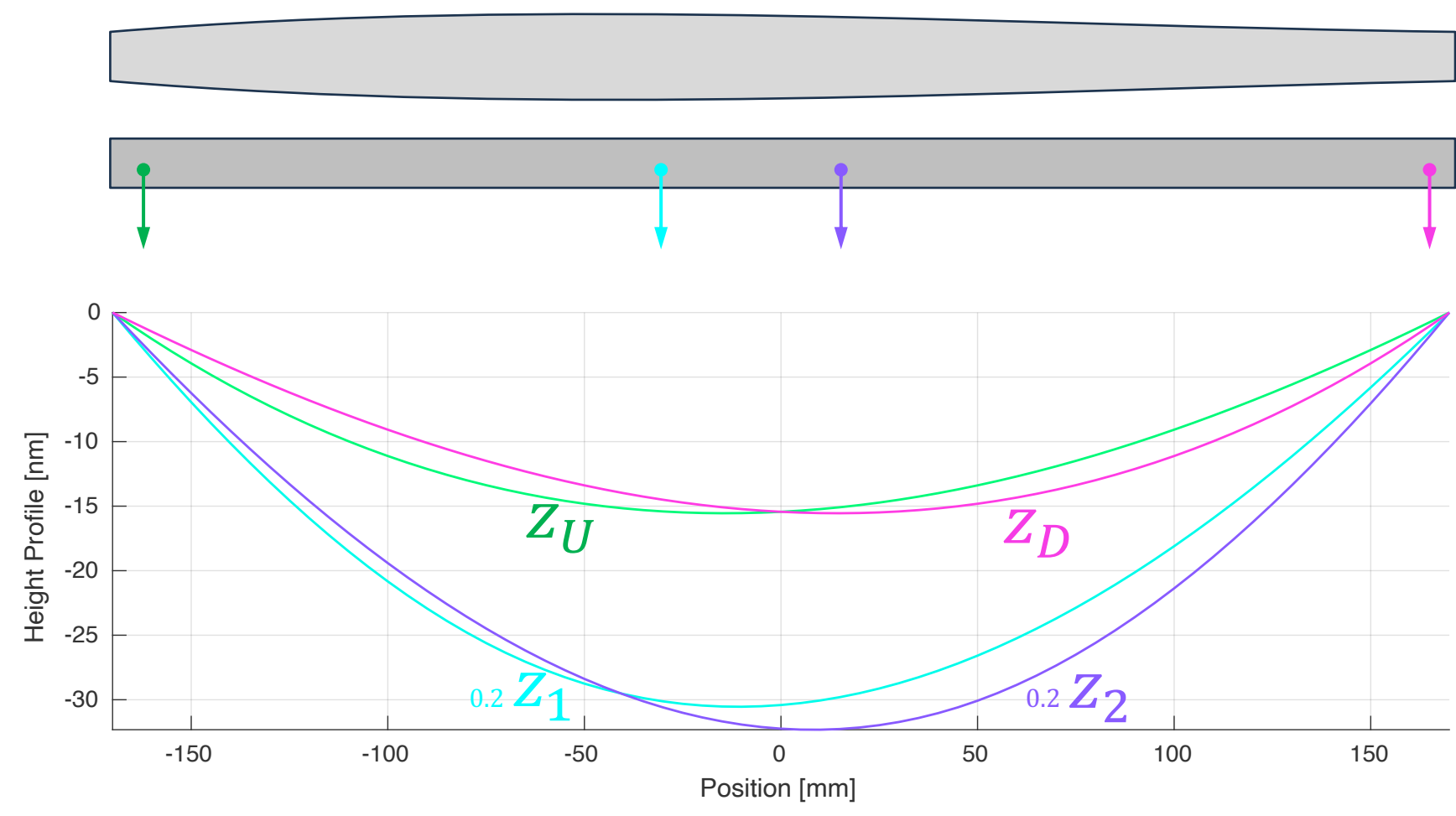
- The set of profiles one can generate onto a mirror substrate is given by a base of functions with dimension N equal to the number of actuators.

$$z(x) = \sum_{n=1}^N f_n Z_n(x) + z_R(x)$$

unit-force deformation response \leftarrow $f_n Z_n(x)$ \leftarrow Force-independent residual error $z_R(x)$

- $Z_n(x)$ can be calculated from the mirror width profile $b(x)$ and the positions a_n of the actuators.

$$Z_n(x) = \frac{12}{Eh^3} \int_{-\frac{L}{2}}^x \int_{-\frac{L}{2}}^{x'} \frac{1}{b(x'')} [(x'' - a_n)\theta(x'' - a_n)] dx'' dx' + c_{0n} + c_{1n}x$$



Example of base functions for a torpedo shaped mirror with four actuators

- The forces f_n "seen" by the mirror are obtained by a simple linear least square fit, with **unique solution**.

$$\begin{bmatrix} z(x_1) \\ z(x_2) \\ \vdots \\ z(x_L) \end{bmatrix}_{MEAS} = \begin{bmatrix} Z_U(x_1) & \dots & Z_D(x_1) \\ Z_U(x_2) & \dots & Z_D(x_2) \\ \vdots & & \vdots \\ Z_U(x_L) & \dots & Z_D(x_L) \end{bmatrix} \begin{bmatrix} f_U \\ \vdots \\ f_D \end{bmatrix} \rightarrow \{f_U, \dots, f_D\}$$

Forces as seen by the mirror, obtained from measured profile

- Forces depend on motor positions, but there is usually crosstalk between actuators, due to deformations of the mechanics.

$$\begin{bmatrix} f_U \\ f_D \end{bmatrix} = \begin{bmatrix} f_{0U} \\ f_{0D} \end{bmatrix} + \begin{bmatrix} A_{UU} & A_{UD} \\ A_{DU} & A_{DD} \end{bmatrix} \begin{bmatrix} m_U \\ m_D \end{bmatrix}$$

Force seen by mirror \leftarrow f_U, f_D \leftarrow Motor position m_U, m_D
Force seen by mirror at rest \leftarrow f_{0U}, f_{0D}
Matrix with **scale coefs** and **crosstalk coefs** (usually 1-3%)
Obtained by linear fitting of 3 or more measurements.

Contributions to error

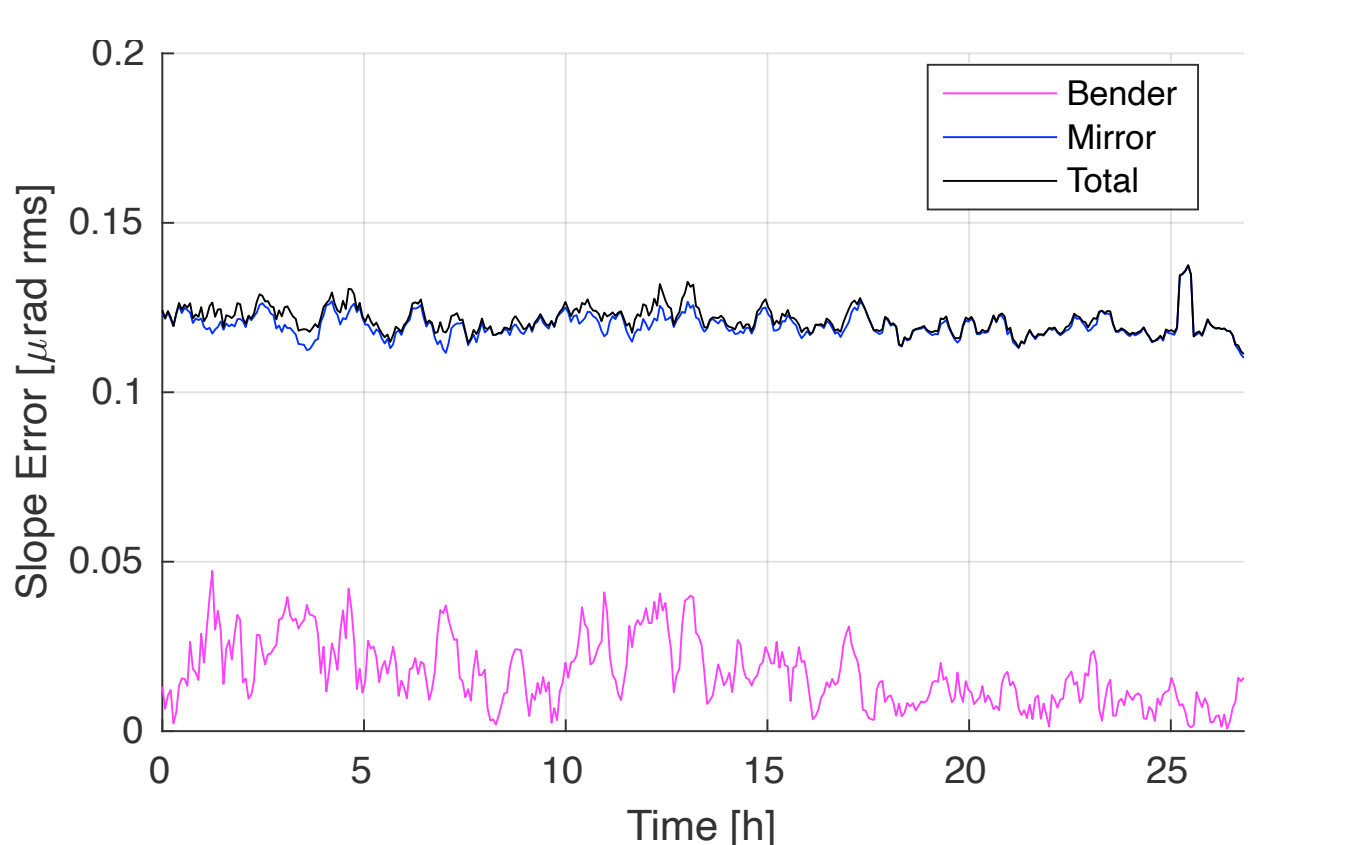
$$z_{MEAS}(x) = \sum_{n=1}^N f_n Z_n(x) + z_R(x) + z_M(x; f_n)$$

Measured profile.

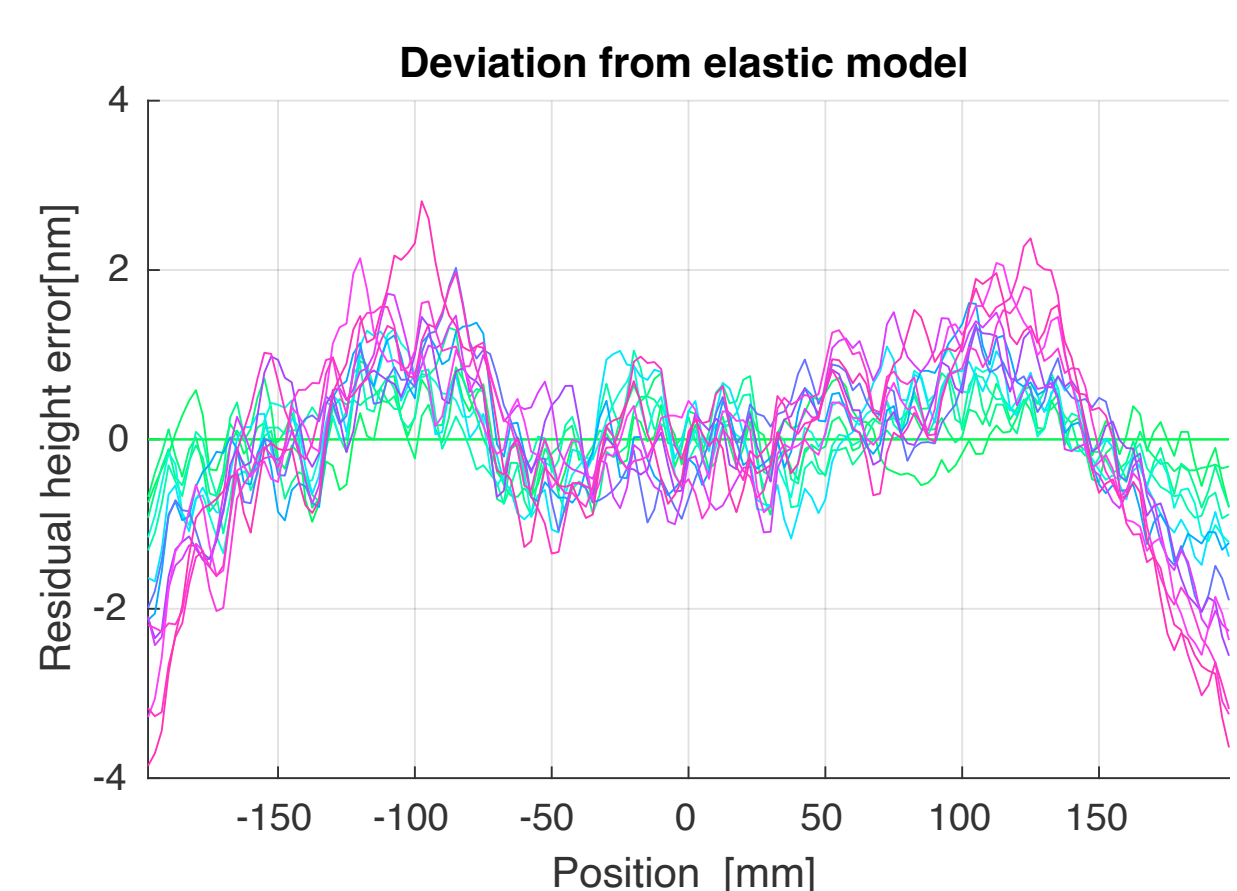
Active Optics contribution. surface errors that, regardless of their origin, can be compensated by tuning the actuators

Residual profile. Independent of forces. Cannot be compensated by active optics.

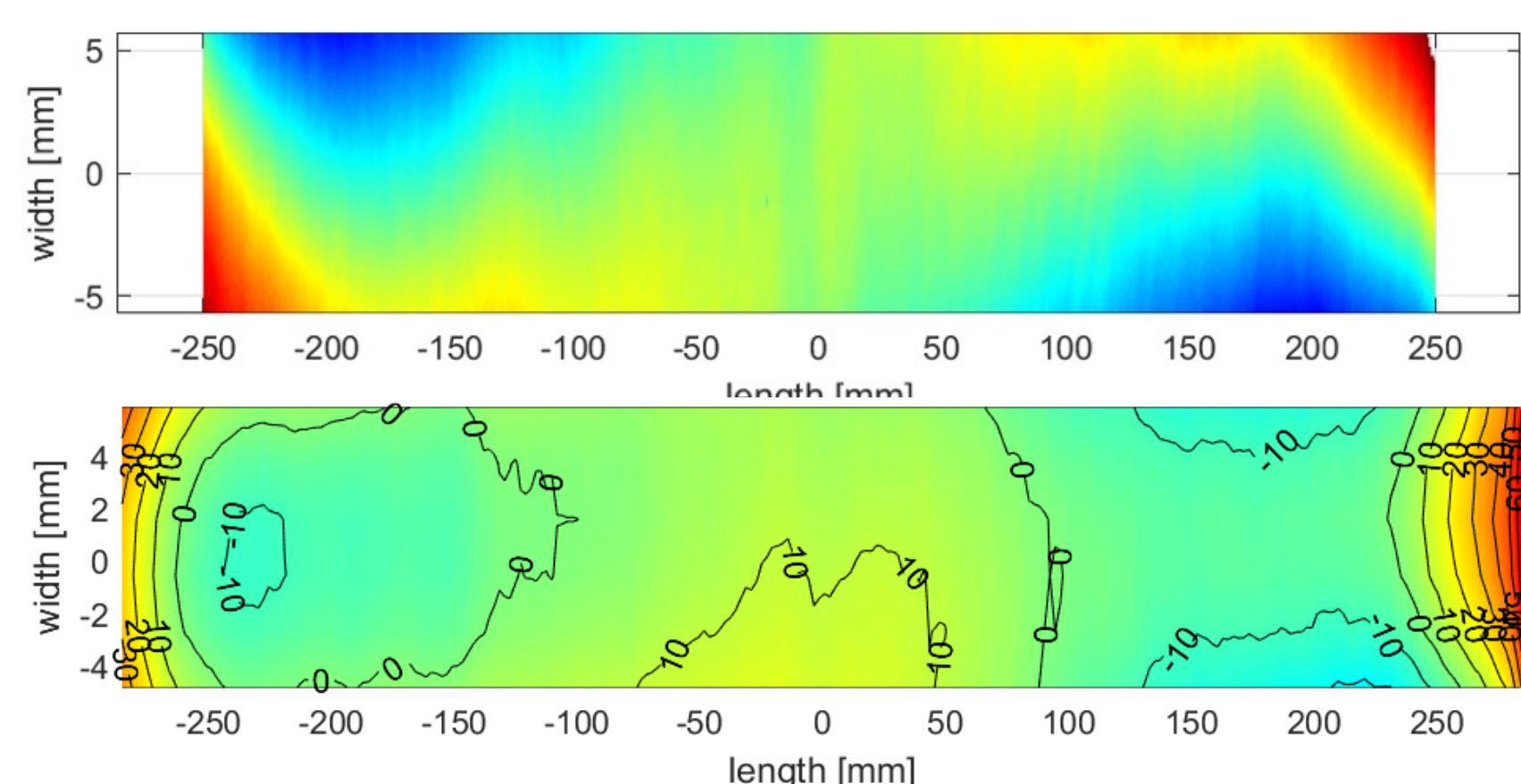
Model errors
- Twist, cooling, parasitic forces.
- Departure from the deformation model.
- measurement errors.
- Tolerance of figures.



Example of **stability test** showing slope error vs time and the contributions of the bender and mirror. **Repeatability** is analyzed equally.



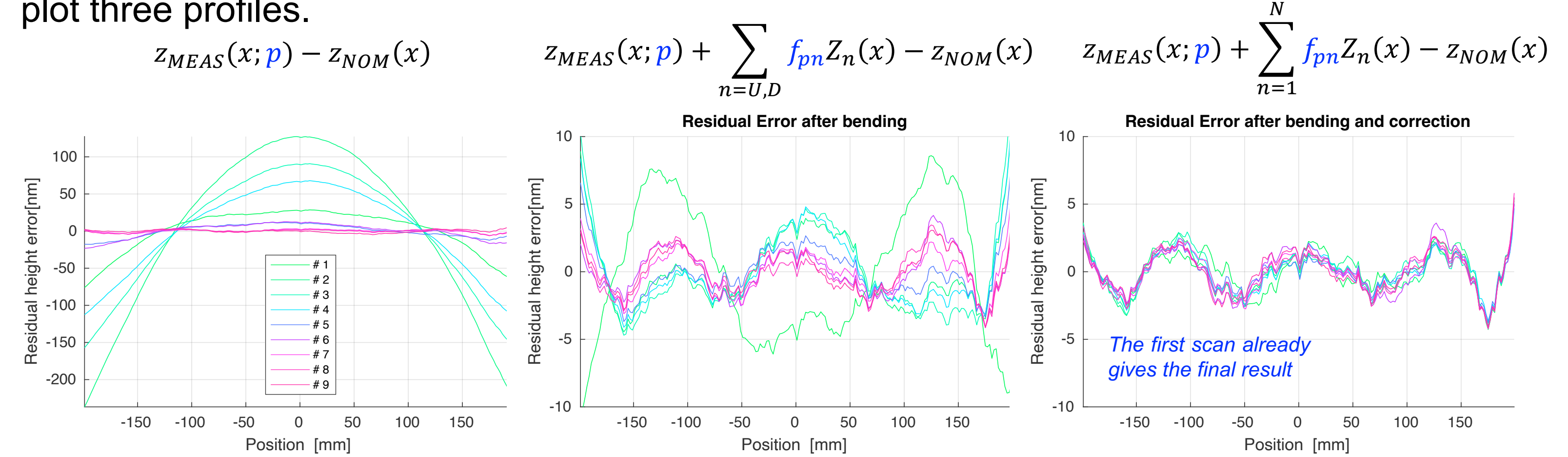
Twist introduces an m-shaped error that contributes to "departure" error.



For strong bending curvature benders show errors near the clamps also due to anticlastic bending

Figure optimization

The optimization of the mirror system involves several iterations (index p), for each of them we plot three profiles.



Total error with respect to the nominal ellipse. Error that would be achieved if bending forces were well adjusted. Error that would be achieved if all actuators were well adjusted. Differences are explained by:
- measurement errors and noise,
- model errors.

Ellipse Map

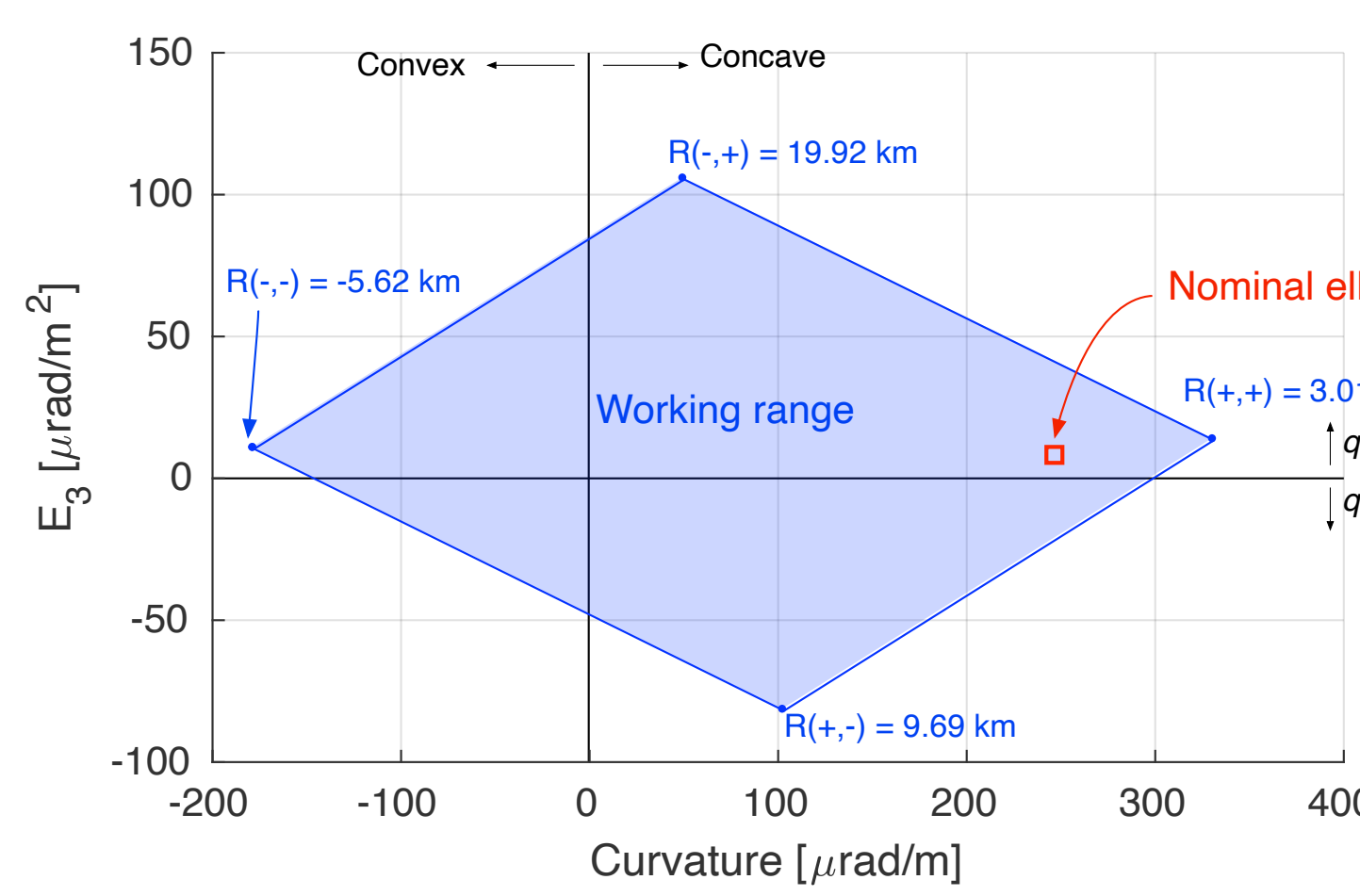
A representation in which each ellipse is a point, and that gives a measure on the distance to a given target ellipse.

$$z_{MEAS}(x) = E_0 + E_1x + E_2x^2 + E_3x^3 + E_R(x)$$

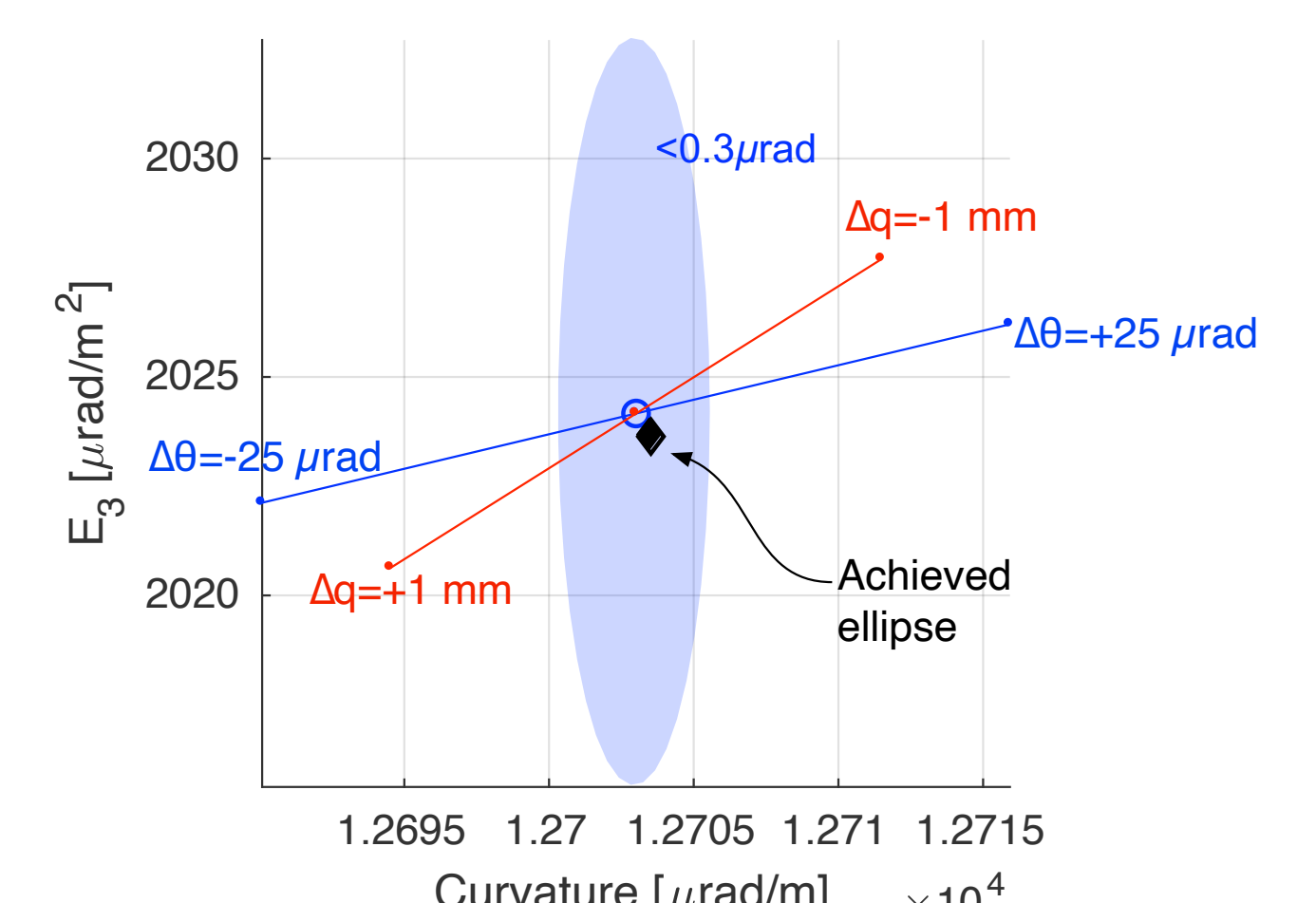
Mean curvature \leftarrow E_2 \leftarrow "Eccentricity" \leftarrow E_3

Error contribution

$$\epsilon_2 = \frac{\Delta E_2 L}{\sqrt{3}} \quad \epsilon_3 = \frac{\Delta E_3 L^2}{2\sqrt{5}}$$



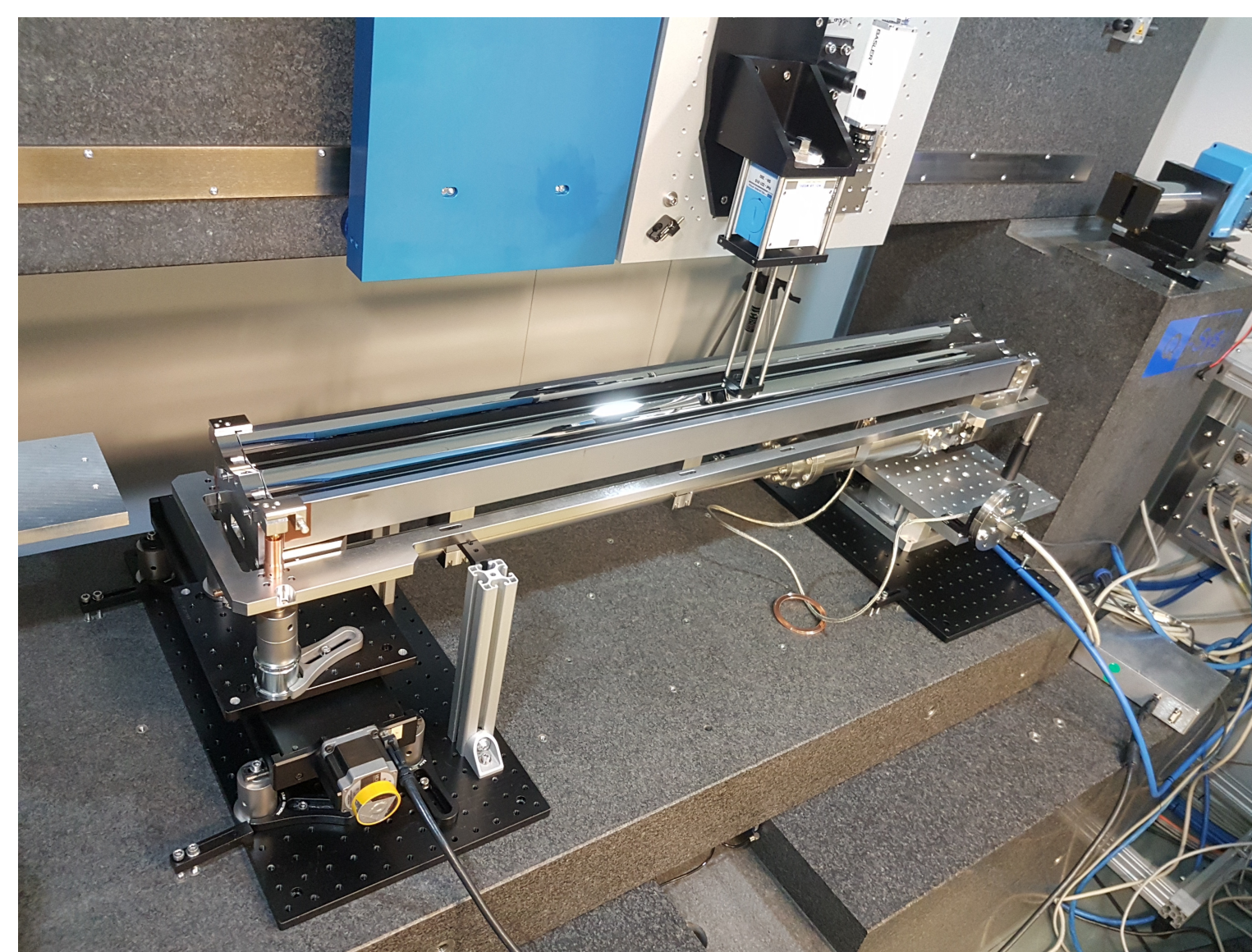
Representation of the range of a mirror bender.



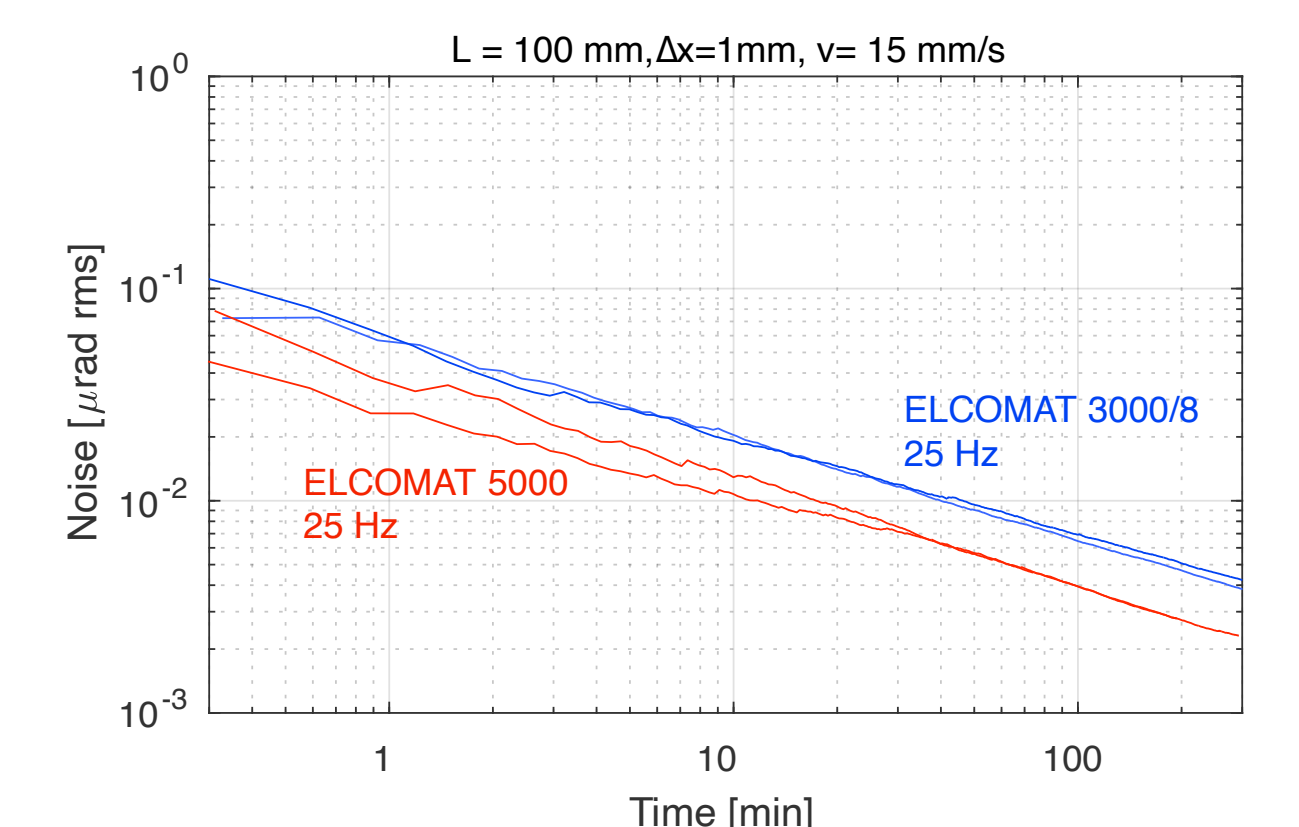
Representation of the target ellipse accuracy as compared to other tolerances at the beamline.

ALBA-NOM

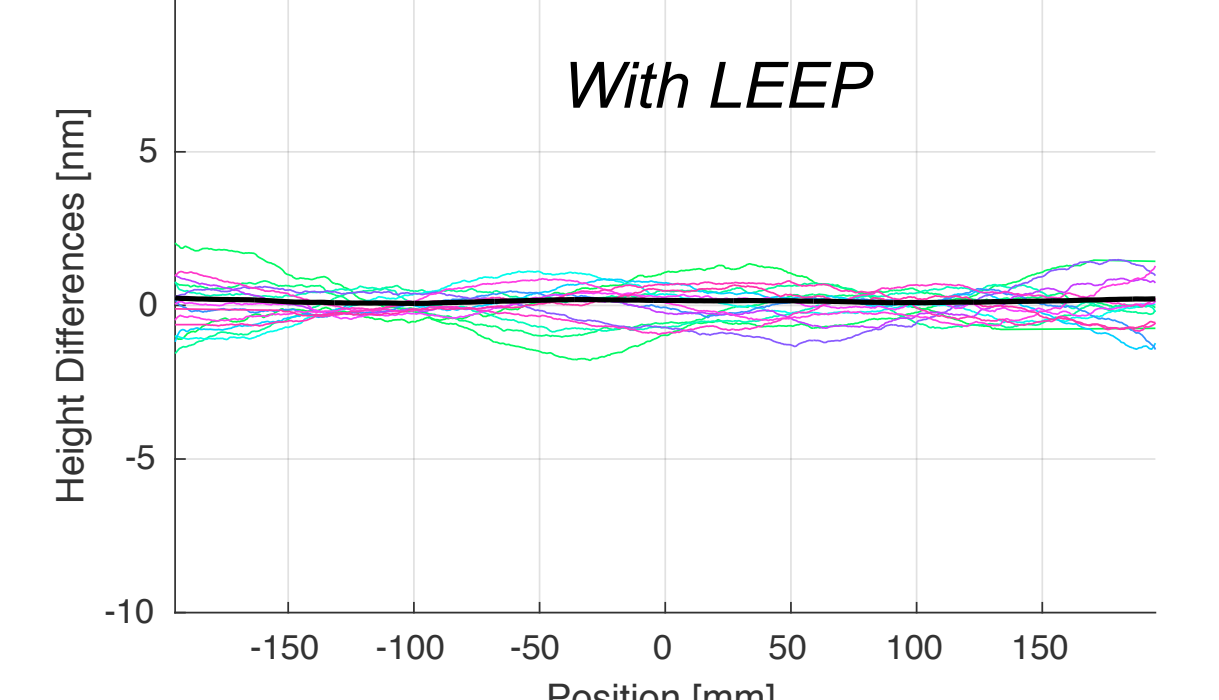
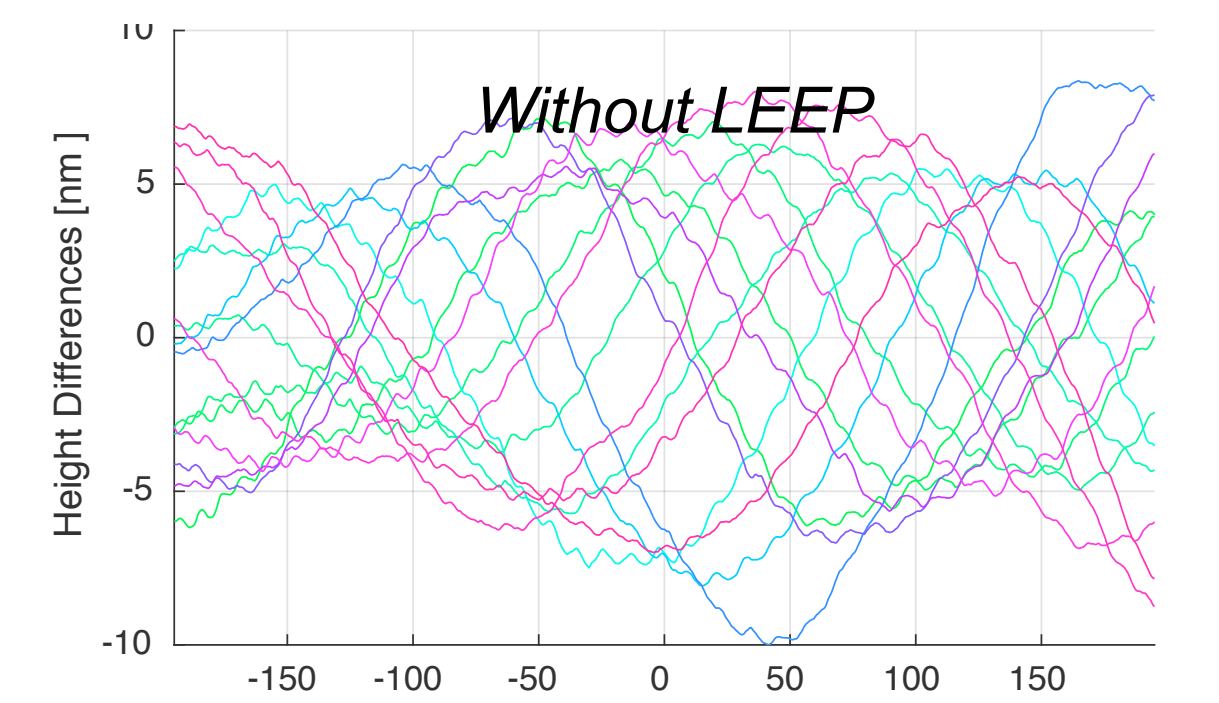
Given its high measurement speed, low noise and accuracy the ALBA-NOM is our main instrument for active optics characterization.



Acquisition	250 Hz
Noise	50 nrad / trace
Scan speed	20 mm/s
Estim. accuracy*	down to 0.04 nm
Temp. stability	<0.01 °C
Spatial resolution	~3.5 mm
Mirror Orientation	Up/Down/Side
Features	Continuous scan LEEP/QLEEP



Noise of measurements as a function of the measurement time (number of cycles)



AC error of individual scans.

Some facts about active optics

The polynomial expansion of an ellipse has an **infinite** number of terms.

The deformation of the substrate is given by a **cubic spline**.
(if section is constant, control points at force application points)

A two end-torque bender provides the **optimal** approximation up to cubic order. (better than many point actuators)

Point actuators cannot compensate **gravity sag** perfectly, since weight is continuously distributed.

The **sharpest** feature one can correct is $1.44\Delta x$ FWHM
(Δx being the minimum distance between actuators)

For every pair of large enough forces, there is a **width profile** for which the substrate bends to the exact ellipse.

