Esercitazione 8
Radiazione (capitolo 11 Griffiths)
31 Maggio 2016

Dipole radiation (electric/magnetic dipoles and radiated power)

Problema 1

Problem 11.2 Equation 11.14 can be expressed in "coordinate-free" form by writing $p_0 \cos\theta = \mathbf{p}_0 \cdot \hat{\mathbf{r}}$. Do so, and likewise for Eqs. 11.17, 11.18, 11.19, and 11.21.

Problema 2

Problem 11.3 Find the **radiation resistance** of the wire joining the two ends of the dipole. (This is the resistance that would give the same average power loss—to heat—as the oscillating dipole in *fact* puts out in the form of radiation.) Show that $R = 790\,(d/\lambda)^2\,\Omega$, where λ is the wavelength of the radiation. For the wires in an ordinary radio (say, $d = 5$ cm), should you worry about the radiative contribution to the total resistance?

Problema 3

Problem 11.4 A *rotating* electric dipole can be thought of as the superposition of two *oscillating* dipoles, one along the x axis, and the other along the y axis (Fig. 11.7), with the latter out of phase by 90°:

$$\mathbf{p} = p_0[\cos(\omega t)\,\hat{\mathbf{x}} + \sin(\omega t)\,\hat{\mathbf{y}}].$$

Using the principle of superposition and Eqs. 11.18 and 11.19 (perhaps in the form suggested by Prob. 11.2), find the fields of a rotating dipole. Also find the Poynting vector and the intensity of the radiation. Sketch the intensity profile as a function of the polar angle θ, and calculate the total power radiated. Does the answer seem reasonable? (Note that power, being *quadratic* in the fields, does *not* satisfy the superposition principle. In this instance, however, it *seems* to. Can you account for this?)

Problema 4

Problem 11.5 Calculate the electric and magnetic fields of an oscillating magnetic dipole *without* using approximation 3. [Do they look familiar? Compare Prob. 9.33.] Find the Poynting vector, and show that the intensity of the radiation is exactly the same as we got using approximation 3.

Problema 5

Problem 11.6 Find the radiation resistance (Prob. 11.3) for the oscillating magnetic dipole in Fig. 11.8. Express your answer in terms of λ and b, and compare the radiation resistance of the *electric* dipole. [*Answer:* $3 \times 10^5\,(b/\lambda)^4\,\Omega$]

Problema 6

Problem 11.9 An insulating circular ring (radius b) lies in the xy plane, centered at the origin. It carries a linear charge density $\lambda = \lambda_0 \sin\phi$, where λ_0 is constant and ϕ is the usual azimuthal angle. The ring is now set spinning at a constant angular velocity ω about the z axis. Calculate the power radiated.

Problema 7

Problem 11.12 A current $I(t)$ flows around the circular ring in Fig. 11.8. Derive the general formula for the power radiated (analogous to Eq. 11.60), expressing your answer in terms of the magnetic dipole moment ($m(t)$) of the loop. [*Answer:* $P = \mu_0 \ddot{m}^2/6\pi c^3$]

Point charges

Problema 8

Problem 11.10 An electron is released from rest and falls under the influence of gravity. In the first centimeter, what fraction of the potential energy lost is radiated away?

Problema 9

Problem 11.16 In Ex. 11.3 we assumed the velocity and acceleration were (instantaneously, at least) *collinear*. Carry out the same analysis for the case where they are *perpendicular*. Choose your axes so that **v** lies along the z axis and **a** along the x axis (Fig. 11.15), so that $\mathbf{v} = v\,\hat{\mathbf{z}}$, $\mathbf{a} = a\,\hat{\mathbf{x}}$, and $\hat{\boldsymbol{\mathscr{r}}} = \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}}$. Check that P is consistent with the Liénard formula. [*Answer:*

$$\frac{dP}{d\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c}\frac{[(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\phi]}{(1-\beta\cos\theta)^5}, \quad P = \frac{\mu_0 q^2 a^2 \gamma^4}{6\pi c}.$$

For relativistic velocities ($\beta \approx 1$) the radiation is again sharply peaked in the forward direction (Fig. 11.16). The most important application of these formulas is to *circular* motion—in this case the radiation is called **synchrotron radiation**. For a relativistic electron the radiation sweeps around like a locomotive's headlight as the particle moves.]

Problema 10

Problem 11.14 In Bohr's theory of hydrogen, the electron in its ground state was supposed to travel in a circle of radius 5×10^{-11}m, held in orbit by the Coulomb attraction of the proton.

According to classical electrodynamics, this electron should radiate, and hence spiral in to the nucleus. Show that $v \ll c$ for most of the trip (so you can use the Larmor formula), and calculate the lifespan of Bohr's atom. (Assume each revolution is essentially circular.)

Problema 11

Problem 11.21 A particle of mass m and charge q is attached to a spring with force constant k, hanging from the ceiling (Fig. 11.19). Its equilibrium position is a distance h above the floor. It is pulled down a distance d below equilibrium and released, at time $t = 0$.

(a) Under the usual assumptions ($d \ll \lambda \ll h$), calculate the intensity of the radiation hitting the floor, as a function of the distance R from the point directly below q. [*Note:* The intensity here is the average power per unit area of *floor*.] At what R is the radiation most intense? Neglect the radiative damping of the oscillator. [*Answer:* $\mu_0 q^2 d^2 \omega^4 R^2 h/32\pi^2 c(R^2 + h^2)^{5/2}$]

(b) As a check on your formula, assume the floor is of infinite extent, and calculate the average energy per unit time striking the entire floor. Is it what you'd expect?

(c) Because it is losing energy in the form of radiation, the amplitude of the oscillation will gradually decrease. After what time τ has the amplitude been reduced to d/e? (Assume the fraction of the total energy lost in one cycle is very small.)