epl A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

March 2010

EPL, **89** (2010) 64005 www.epljournal.org
doi: 10.1209/0295-5075/89/64005

Tunability of a seeded free-electron laser through frequency pulling

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received 30 September 2009; accepted in final form 8 March 2010
published online 7 April 2010

PACS 41.60.Cr – Free-electron lasers
PACS 42.55.Ah – General laser theory
PACS 42.65.Ky – Frequency conversion; harmonic generation, including higher-order harmonic generation

Abstract – Frequency pulling is a well-known phenomenon leading to an output frequency shift in a conventional laser, when the cavity and the maximum gain frequencies are detuned. We demonstrate that a similar mechanism is at play in a seeded free-electron laser (FEL), when the seed frequency is out of resonance. Frequency pulling thus may give the possibility of fine-tuning the FEL frequency. On the basis of numerical simulations, we provide a general formula for the FEL output frequency. Such a formula generalizes the one normally used when treating the frequency pulling in conventional lasers.

Introduction. – Free-electron lasers (FELs) are widely recognized as one of new breed of the next-generation light sources. This is mainly due to the fact that FELs combine typical characteristics of synchrotron radiation, like short (X-ray) wavelengths and full tunability, with those usually pertaining to lasers, such as coherence, high brilliance and short pulses [1–8].

We concentrate our attention on single-pass FELs, for which two different schemes can be distinguished, depending on the origin of the optical wave that is used to initiate the process. In the self-amplified spontaneous emission (SASE) configuration [1,2,9], the initial seed is generated by the spontaneous emission of the electron beam. SASE-based devices produce tunable radiation at short (X-ray) wavelengths with several gigawatt peak power, excellent spatial mode, but rather poor temporal and spectral stability and coherence. Recently, new ideas have been proposed to produce fully coherent hard-X-ray SASE pulses, see, *e.g.*, [10]. A way to overcome the limits of the SASE configuration is to initialize the FEL process by means of an external coherent source [11–15]. This stabilizes FEL pulses and drastically improves their coherence.

However, the use of seeding has the intrinsic disadvantage that the tunability of the FEL is limited by that of the available seed source. When the seed wavelength is in the visible-UV range, such a problem may be easily overcome by using an Optical Parametric Amplifier [6]. On the contrary, when the seed is at shorter wavelengths, *e.g.* when it is produced by a process of high-harmonic generation in gases [11,16], the seed tunability is more difficult and, as a consequence, the same holds for the tunability of the FEL radiation. Recently, it has been suggested that some tunability may be recovered by using a short (*i.e.*, broad-band) seed pulse [17,18]. However, a precise estimate of the wavelength shift that such a method can provide has not been yet done, and the availability and utility of such a tuning is an argument of debate within the FEL community.

In this work we investigate in more detail the seeding process with short pulses, providing an explanation of the mechanism responsible for the FEL frequency shift with respect to both the resonant and the seed frequencies. Such a mechanism shares many similarities with the frequency pulling phenomenon, well known in the theory of conventional lasers [19]. In the following, after a brief reminder of the general method and the theory on which seeded FELs rely, we will focus on a case study by considering the parameters of the first stage of FERMI FEL [6]. FERMI is presently under construction at Sincrotrone Trieste and will initially produce coherent radiation in the 80–20 nm spectral range. The FEL process is studied by means of

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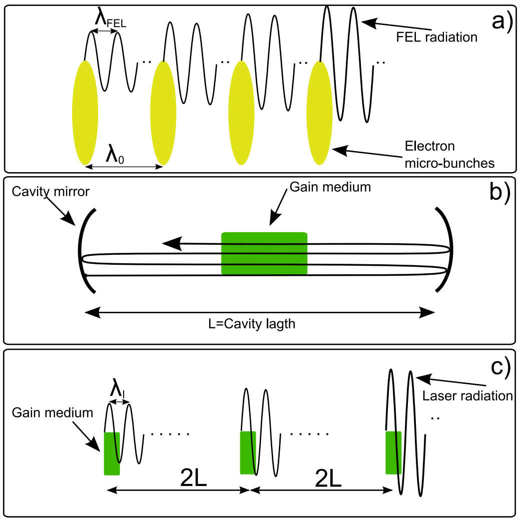

Fig. 1: (Colour on-line) Analogy between coherent FEL (a) and laser (b and c) emission processes. See text for an explanation of the symbols.

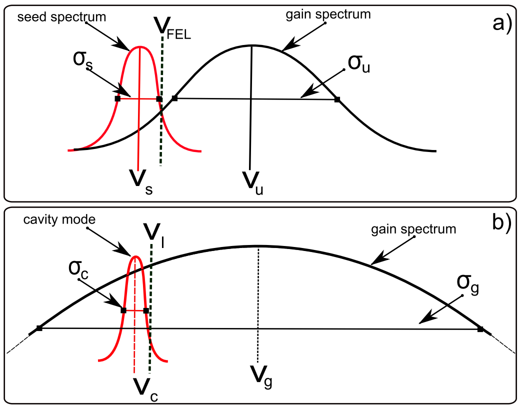

Fig. 2: (Colour on-line) a) Interplay between undulator (*i.e.*, gain) and seed spectra, leading to frequency pulling in a FEL. b) Interplay between active medium (*i.e.*, gain) and cavity mode spectra, leading to frequency pulling in a conventional laser. For symbols' explanation, see text.

numerical simulations with the FEL code GINGERH [20], that has been demonstrated to provide results in good agreement with experiments [21].

Detuning effect in seeded FELs and lasers. – The scheme we consider is called High-Gain Harmonic Generation (HGHG) and is based on the frequency up-conversion of a UV high power seed laser [13–15]. In such a configuration, electrons interact first with the seed laser within a short undulator, called the modulator. The result of this interaction is the creation of a coherent electron-density modulation, called micro-bunching, at the seed frequency ν_0. When passing through the second undulator, called the radiator, the bunched electron beam produces FEL radiation through coherent emission, see fig. 1a. Taking advantage of the fact that the electron-density modulation shows Fourier components also at the harmonics $\nu_s = N_h \cdot \nu_0$ (where the harmonic number N_h is an integer) it is possible to produce coherent FEL radiation not only at the original seed frequency, but also at its N_h-th harmonic [13]. When looking at the radiator, the coherent emission from these already micro-bunched electrons will occur only if their emission frequency, ν_{FEL}, is close to ν_s. In particular, in the simple view of having coherent emission from these bunches, one can expect that the larger the number of micro-bunches involved in the process, the closer ν_{FEL} will be to ν_s. This means that the frequency of the FEL emission depends on the number of micro-bunches, that is on the length of the seed pulse. On the other hand, the frequency of the emitted radiation also depends on the gain curve of the radiator: if ν_s is detuned with respect to the center of the gain curve of the radiator, ν_u, and positioned on a steep slope of the gain curve, see fig. 2a, ν_{FEL} will tend to be shifted towards the higher gain. The resulting FEL wavelength will be determined by the interplay of the two above mechanisms.

A similar behavior can be found in the operation of conventional laser oscillators, and is usually termed frequency pulling. In a conventional laser, the parameters of the laser emission are defined by the laser cavity (resonator) and the gain medium. The role of the resonator is to provide an optical feedback, by forcing multiple roundtrips of the radiation through the gain medium, see fig. 1b. As a result of these multiple passages, the radiation interacts several times with the gain medium. As shown in fig. 1c, this can be also viewed as a sequence of interactions with a number of consecutive gain media. Note the analogy with the FEL process illustrated in fig. 1a. A steady-state laser operation is reached only for wavelengths at which the phase-shift in a roundtrip is a multiple of 2π, which, in the case of a "cold" cavity (*i.e.*, in the absence of active medium), leads to the well-known relation between cavity length and frequency of allowed laser modes. Similarly to the FEL case, the larger the number of roundtrips in a "cold" cavity (high cavity finesse), the closer the laser frequency will tend to be to that of a cavity mode. On the other hand, the presence of an active medium induces a gain mechanism which tends to select the "cold" cavity modes that can reach threshold and laser action. Also in this case, if the cavity and the maximum gain frequencies are detuned, the two phenomena are in competition, see fig. 2b, resulting in a shift of the final laser wavelength [19,22].

An approximate formula which is normally used in literature for frequency pulling in conventional lasers is the following [19]:

$$\nu_{las} = \nu_c - (\nu_c - \nu_g)\frac{\sigma_c}{\sigma_g}. \tag{1}$$

Here ν_c and ν_g are, respectively, the cavity and the maximum gain freqsuencies, while σ_c and σ_g are their FWHM bandwidths.

This equation is valid with good accuracy for the case $\sigma_g \gg \sigma_c$, a situation often encountered in conventional lasers where high cavity finesse is used, and usually leads to a very small shift of the cavity mode frequencies towards the peak of the gain curve. Due to the similarity with the HGHG process, on may expect a similar expression to hold also in the latter case.

In the case of the FEL, however, the above mentioned approximation is not valid: the widths of the two curves are in general comparable. A more general formula exists for conventional lasers which could be used in such situation [22]:

$$\nu_{las} = \nu_c - (\nu_c - \nu_g)\frac{\sigma_c}{\sigma_c + \sigma_g}, \qquad (2)$$

which reduces to eq. (1) in the limit case $\sigma_c \ll \sigma_g$. It is important to point out that eq. (1) gives unphysical results for $\sigma_c \geqslant \sigma_g$ and that in this case eq. (2) must be used. Equation (2) shows that the laser oscillates at a frequency that lies between the peaks of the cavity resonance and gain curves, and is closer to the center of the narrower one.

Numerical results of frequency mismatch in seeded FEL. – Based on results of numerical simulations, in the following we attempt to determine a formula for frequency pulling in the case of an FEL. Predictions obtained using such a formula will be compared with those provided by eq. (2). As a paradigmatic example, we will study the case of the first stage of the FERMI FEL [6].

In the case of a tuned HGHG FEL, the harmonic of the seed frequency, ν_s, coincides with the peak of the gain curve, ν_u. This occurs when

$$\nu_s = \nu_u \equiv \left[\frac{\lambda_w}{2c\gamma^2}\left(1 + aw^2\right)\right]^{-1}, \qquad (3)$$

where γ is the electron-beam relativistic Lorentz factor, λ_w is the undulator period, aw is the undulator parameter and c the speed of light. The relative bandwidth of the gain medium σ_u is instead approximately equal to the Pierce parameter ρ [23], which is in the range 1–3 $\cdot 10^{-3}$ for the case study considered here.

In order to study the role of the seed bandwidth, σ_s, in the simulations we considered seed pulses with different durations. In this work, we concentrated on Fourier transform limited seed pulses, the effects of the nonlinear phase evolution along the seed pulse that may be present in very short seed pulses are not considered here and will be the subject of future studies.

The detuning on the radiator with respect to the resonant condition has been instead induced by varying the energy of the electron beam, *i.e.* γ in eq. (3). Although the electron beam energy is entering through the resonance condition into the equations that rule the process responsible for creating in the modulator the micro-bunched structure at ν_0, its effect is marginal as the modulator is usually characterized by a very large bandwidth. The same results could be obtained if the detuning were obtained by changing the aw of the radiator. We also considered cases with different bandwidths, σ_u, of the gain medium, by considering FEL setups with different ρ. We focus on the specific configuration with $\lambda_0 = c/\nu_0 = 240$ nm and $\lambda_s = c/\nu_s = 60$ nm.

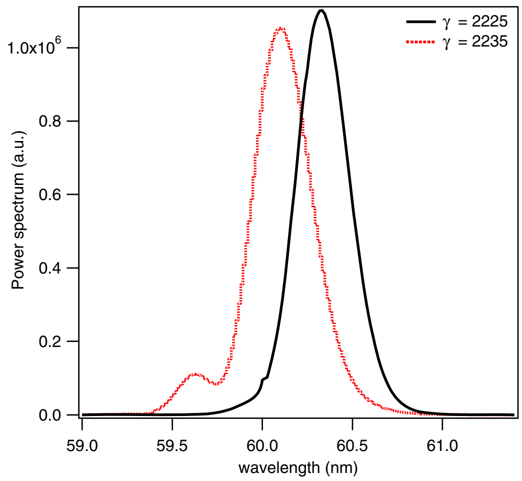

Fig. 3: (Colour on-line) FEL Spectrum for two different electron-beam energies (continuous curve: $\gamma = 2225$, dotted curve: $\gamma = 2235$), when the radiator is tuned at 60 nm (corresponding to a resonant energy $\gamma = 2231$). The seed pulse has a duration of 18 fs (FWHM), a wavelength of 240 nm and a power of about 100 MW. The other relevant electron-beam parameters used in the simulations are the following: peak current: 750 A; emittance: 1.5 mm mrad; energy spread: 150 keV. The radiator period is 55 mm, the radiator length is about 14.5 m. Additional information about the FERMI setup may be found in [6].

An example of the effect of a frequency detuning by changing the electron-beam energy (while keeping the seed frequency fixed) is shown in fig. 3. In this case the seed pulse has a duration of 18 fs (FWHM).

A more detailed analysis can be done by determining the FEL frequency as a function of the resonant frequency ν_u, for various seed pulse durations. The results of this series of simulations are reported in fig. 4. As expected, data show a linear dependence of the FEL frequency on detuning. For longer seed pulses (*i.e.*, narrower bandwidth), the FEL frequency displays a weak dependence on the resonant frequency, keeping close to the seed harmonic frequency. Instead, for shorter FEL pulses (*i.e.*, larger seed bandwidths) the dependence becomes stronger. The results of our simulations indicate that the produced FEL pulses preserve their properties in terms of pulse length and bandwidth also when the FEL is strongly detuned and the wavelength shifted. In order to derive an equation providing ν_{FEL} as a function of the ν_u and ν_s, as well as of their bandwidths, we need to determine the dependence of the slopes of the curves reported in fig. 4 on the latter parameters.

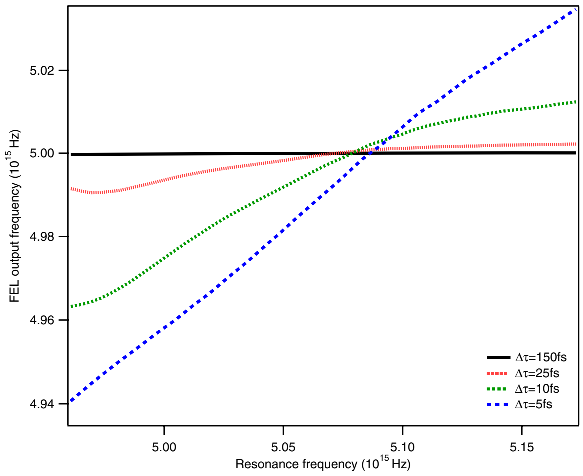

Fig. 4: (Colour on-line) FEL frequency *vs.* resonant frequency ν_u (varied by changing the electron-beam energy), for different bandwidths (varied by changing the seed pulse duration, Δt). The harmonic seed frequency is fixed at $\nu_s = c/\lambda_s$ with $\lambda_s = 60\,\text{nm}$.

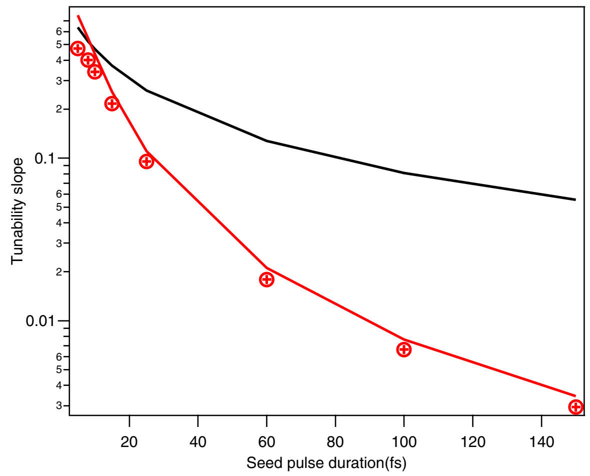

Fig. 5: (Colour on-line) Tunability slope as a function of the seed pulse duration: numerical results (crossed dots), data obtained from the standard frequency pulling equation (5) and from the modified equation (6).

According to eq. (2), the expected dependence of the laser frequency on the gain-medium frequency is given by

$$\frac{\mathrm{d}\nu_{las}}{\mathrm{d}\nu_g} = \frac{\sigma_c}{\sigma_c + \sigma_g}. \tag{4}$$

This equation can be easily adapted to the case of a FEL:

$$\frac{\mathrm{d}\nu_{FEL}}{\mathrm{d}\nu_u} = \frac{0.44/(\Delta T)_s}{0.44/(\Delta T)_s + 2.35\rho\nu_u}, \tag{5}$$

where $0.44/(\Delta T)_s$ is the FWHM bandwidth associated to a transform limited Gaussian pulse, whose duration is $(\Delta T)_s$; $2.35\rho\nu_u$ is the FWHM bandwidth associated to the FEL pulse.

By performing a linear fit of the data reported in fig. 4 one gets the tunability slope $\mathrm{d}\nu_{FEL}/\mathrm{d}\nu_u$ for different seed pulse durations. Results are shown in fig. 5 (crossed dots), together with the prediction obtained from the eq. (5) (black continuous line). As can be seen, the theoretical prediction does not fit with numerical results. The disagreement is particularly evident in the case of longer seed pulses.

Instead, data fit quite well with a different frequency pulling equation, whose prediction is shown by the continuous red line in fig. 5. Such an equation reads

$$\nu_{FEL} = \nu_s - (\nu_s - \nu_u)\frac{\sigma_s^2}{\sigma_s^2 + \sigma_u^2}, \tag{6}$$

and has a straightforward physical motivation. In fact, according to the standard formula, eq. (2), the laser frequency is given by the weighted average of the two frequencies, ν_c and ν_g, with the weighting factors proportional to the inverse of the corresponding linewidths [22]. The formula we propose, eq. (6), is the result of a more careful weighting, which takes into account the Gaussian profiles of both the gain and the loss curves. The output frequency, which is here the weighted average using the squares of the bandwidth, corresponds to the maximum of the function given by the product between the two Gaussian curves representing the gain and the losses for the laser process. It is also important to note that the standard formula for frequency pulling can be obtained from eq. (6) in the limit case in which $\sigma_u \gg \sigma_s$ ($\sigma_g \gg \sigma_c$ in the case of conventional lasers) and $(\nu_u - \nu_s) \ll \sigma_u$ ($\nu_g - \nu_c \ll \sigma_g$ in the case of conventional lasers), that is normally the case for conventional lasers.

Results of the numerical simulations differ slightly from the prediction given by eq. (6) only in the case of very short pulses, see fig. 5. This can be explained by taking into account the effect of light-electron slippage, that slightly increases the length of seeded part of the electron bunch.

The presented results refer to the case of HGHG. However, additional studies (not reported here) show that eq. (6) can be successfully applied also to the case of direct seeding. An additional confirmation for the validity of eq. (6) dsespite eq. (2) for describing the frequency pulling comes from an experiment [24] that has been recently performed in the Elettra storage ring FEL [15].

Conclusions. – In conclusion, on the basis of the results of numerical simulations, we have demonstrated and characterized the frequency pulling effect for seeded FELs. With these results we have fixed that the tuning of a FEL in the case of a UV seed laser is very limited and has not practical utility for standard seed pulse lengths. The only possibility of using the frequency pulling for a useful tuning of a FEL is limited to the case of very short pulses (few fs) and may be useful in the case of seeding with the short pulses of an HHG source. We also found a general formula providing the output FEL frequency as a function of the system parameters. Such a formula generalizes the one normally used when considering the frequency pulling in conventional lasers.

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The authors are indebted to W. Fawley, S. Milton and J. Wu for useful discussions. The work is partially supported by the Italian Ministry of University and Research under grants FIRB-RBAP045JF2 and FIRB-RBAP06AWK3.

REFERENCES

[1] Bonifacio R., Pellegrini C. and Narducci L., *Opt. Commun.*, **50** (1984) 373.
[2] Saldin E. L. *et al.*, *Opt. Commun.*, **148** (1998) 383.
[3] Ackermann W. *et al.*, *Nat. Photon.*, **1** (2007) 336.
[4] Shintake T., *Proceedings EPAC06*, **1** (2006) THOPA02.
[5] Galayda J. N., *Proceedings EPAC08*, **1** (2008) WEXM02.
[6] See, *e.g.*, `http://www.elettra.trieste.it/FERMI/`.
[7] See, *e.g.*, `http://www.xfel.eu/`.
[8] Vaccarezza C. *et al.*, *Proceedings EPAC06*, **1** (2006) MOPCH028.
[9] Milton S. V. *et al.*, *Phys. Rev. Lett.*, **85** (2000) 988; Andruszhow J., *Phys. Rev. Lett.*, **85** (2000) 3825.
[10] Rosenzweig J. B. *et al.*, *Nucl. Instrum. Methods A*, **593** (2008) 39; Thompson N. R. and McNeil B. W., *Phys. Rev. Lett.*, **100** (2008) 203901.
[11] Lambert G. *et al.*, *Nat. Phys.*, **4** (2008) 296.
[12] Gover A., *Phys. Rev. ST Accel. Beams*, **8** (2005) 030701.
[13] Coisson R. and De Martini F., *Phys. Quantum Electron.*, **9** (1982) 939; Bonifacio R., De Salvo L. and Pierini P., *Nucl. Instrum. Methods A*, **293** (1990) 627; Yu L. H., *Phys. Rev. A*, **44** (1991) 5178; Allaria E. and De Ninno G., *Phys. Rev. Lett.*, **99** (2007) 014801.
[14] Yu L. H. *et al.*, *Science*, **289** (2000) 932.
[15] De Ninno G. *et al.*, *Phys. Rev. Lett.*, **101** (2008) 053902.
[16] Salieres P. *et al.*, *Phys. Rev. Lett.*, **74** (1995) 3776.
[17] Wu J. *et al.*, *Opt. Express*, **16** (2008) 3255.
[18] Haixiao D., Xingtao W. and Zhimin D., *Phys. Rev. ST Accel. Beams*, **11** (2008) 040703.
[19] Yaryv A., *Quantum Electronics* (John Wiley & Sons Inc., NY) 1989.
[20] Fawley W., *Lawrence Berkley National Laboratory internal note* (2004) LBNL-49625.
[21] De Ninno G. *et al.*, *Phys. Rev. Lett.*, **100** (2008) 104801.
[22] Svelto O., *Principles of Lasers* (Plenum Press) 1989, Chapt. 4; Siegman A. E., *Lasers* (University Science Books) 1986, Chapt. 12.
[23] Colson W. B., *Phys. Lett. A*, **59** (1976) 187; Bonifacio R. *et al.*, *Opt. Commun.*, **50** (1984) 373.
[24] Spezzani C. *et al.*, in preparation.