beneath the glass will seem decidedly grayer than the rest of the paper, because the slide will reflect at both its interfaces, and the light reaching and returning from the paper will be diminished appreciably. Now hold the slide near your eye and again view the page through it as you tilt it, increasing θ_i. The amount of light reflected will increase, and it will become more difficult to see the page through the glass. When $\theta_i \approx 90°$ the slide will look like a perfect mirror as the reflection coefficients (Fig. 4.41) go to -1.0. Even a rather poor surface (see photo), such as the cover of this book, will be mirrorlike at glancing incidence. Hold the book horizontally at the level of the middle of your eye and face a bright light; you will see the source reflected rather nicely in the cover. This suggests that even X-rays could be mirror-reflected at glancing incidence (p. 242), and modern X-ray telescopes are based on that very fact.

At normal incidence Eqs. (4.35) and (4.41) lead rather straightforwardly to

$$[t_\parallel]_{\theta_i=0} = [t_\perp]_{\theta_i=0} = \frac{2n_i}{n_i + n_t} \tag{4.48}$$

It will be shown in Problem 4.50 that the expression

$$t_\perp + (-r_\perp) = 1 \tag{4.49}$$

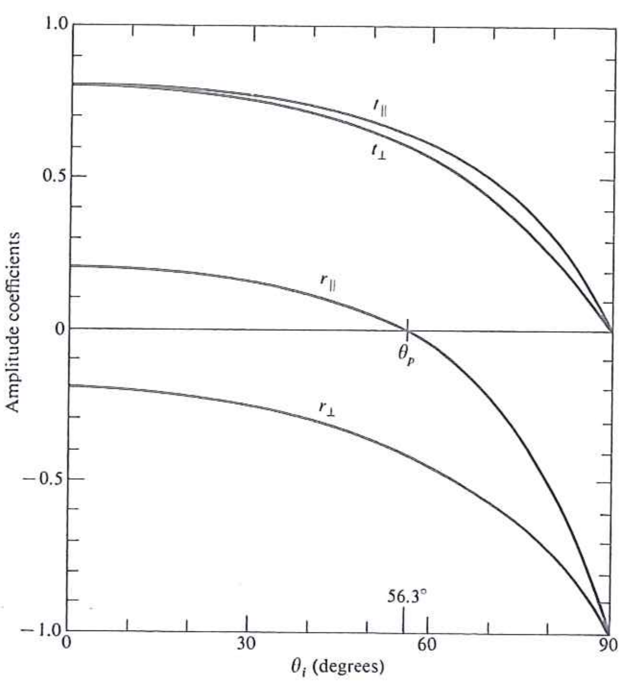

Figure 4.41 The amplitude coefficients of reflection and transmission as a function of incident angle. These correspond to external reflection $n_t > n_i$ at an air–glass interface ($n_{ti} = 1.5$).

At near-glancing incidence the walls and floor are mirrorlike—this despite the fact that the surfaces are rather poor reflec tors at $\theta_i = 0°$. (Photo by E.H.)

holds for all θ_i, whereas

$$t_\parallel + r_\parallel = 1 \tag{4.50}$$

is true only at normal incidence.

The foregoing discussion, for the most part, was restricte to the case of **external reflection** (i.e., $n_t > n_i$). The opposit situation of **internal reflection**, in which the incident mediun is the more dense ($n_i > n_t$), is of interest as well. In tha instance $\theta_t > \theta_i$, and $r_\perp$, as described by Eq. (4.42), wil always be positive. Figure 4.42 shows that $r_\perp$ increases fron its initial value [Eq. (4.47)] at $\theta_i = 0$, reaching $+1$ at what i called the **critical angle**, θ_c. Specifically, θ_c is the special val ue of the incident angle (p. 122) for which $\theta_t = \pi/2$. Like wise, $r_\parallel$ starts off negatively [Eq. (4.47)] at $\theta_i = 0$ anc thereafter increases, reaching $+1$ at $\theta_i = \theta_c$, as is evident fron the Fresnel Equation (4.40). Again, $r_\parallel$ passes through zero a the *polarization angle* θ'_p. It is left for Problem 4.66 to shov that the polarization angles θ'_p and θ_p for internal and externa reflection at the interface between the same media are simpl the complements of each other. We will return to interna reflection in Section 4.7, where it will be shown that $r_\perp$ and r are complex quantities for $\theta_i > \theta_c$.

Phase Shifts

It should be evident from Eq. (4.42) that $r_\perp$ is negative regard- less of θ_i when $n_t > n_i$. Yet we saw earlier that had we choser $[\vec{E}_r]_\perp$ in Fig. 4.37 to be in the opposite direction, the first Fres-

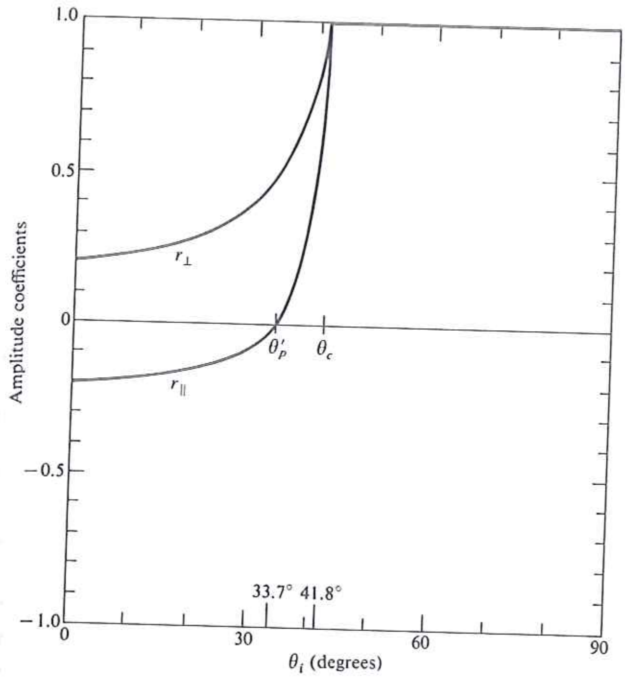

Figure 4.42 The amplitude coefficients of reflection as a function of incident angle. These correspond to internal reflection $n_t < n_i$ at an air–glass interface ($n_{ti} = 1/1.5$).

nel Equation (4.42) would have changed signs, causing $r_\perp$ to become a positive quantity. The sign of $r_\perp$ is associated with the relative directions of $[\vec{\mathbf{E}}_{0i}]_\perp$ and $[\vec{\mathbf{E}}_{0r}]_\perp$. Bear in mind that a reversal of $[\vec{\mathbf{E}}_{0r}]_\perp$ is tantamount to introducing a phase shift, $\Delta\varphi_\perp$, of π radians into $[\vec{\mathbf{E}}_r]_\perp$. Hence at the boundary $[\vec{\mathbf{E}}_i]_\perp$ and $[\vec{\mathbf{E}}_r]_\perp$ will be antiparallel and therefore π out-of-phase with each other, as indicated by the negative value of $r_\perp$. When we consider components normal to the plane-of-incidence, there is no confusion as to whether two fields are in-phase or π radians out-of-phase: if parallel, they're in-phase; if antiparallel, they're π out-of-phase. In summary, then, ***the component of the electric field normal to the plane-of-incidence undergoes a phase shift of π radians upon reflection when the incident medium has a lower index than the transmitting medium.*** Similarly, $t_\perp$ and $t_\parallel$ are always positive and $\Delta\varphi = 0$. *Furthermore, when $n_i > n_t$ no phase shift in the normal component results on reflection, that is, $\Delta\varphi_\perp = 0$ so long as $\theta_i < \theta_c$.*

Things are a bit less obvious when we deal with $[\vec{\mathbf{E}}_i]_\parallel$, $[\vec{\mathbf{E}}_r]_\parallel$, and $[\vec{\mathbf{E}}_t]_\parallel$. It now becomes necessary to define more explicitly what is meant by *in-phase*, since the field vectors are coplanar but generally not colinear. The field directions were chosen in Figs. 4.39 and 4.40 such that if you looked down any one of the propagation vectors toward the direction from which the light was coming, $\vec{\mathbf{E}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{k}}$ would appear to have the same relative orientation whether the ray was incident, reflected, or transmitted. We can use this as the required condition for two E-fields to be in-phase. Equivalently, but more simply, ***two fields in the incident plane are in-phase if their y-components are parallel and are out-of-phase if the components are antiparallel.*** Notice that when two $\vec{\mathbf{E}}$-fields are out-of-phase so too are their associated $\vec{\mathbf{B}}$-fields and vice versa. With this definition we need only look at the vectors normal to the plane-of-incidence, whether they be $\vec{\mathbf{E}}$ or $\vec{\mathbf{B}}$, to determine the relative phase of the accompanying fields in the incident plane. Thus in Fig. 4.43*a* $\vec{\mathbf{E}}_i$ and $\vec{\mathbf{E}}_t$ are in-phase, as are $\vec{\mathbf{B}}_i$ and $\vec{\mathbf{B}}_t$, whereas $\vec{\mathbf{E}}_i$ and $\vec{\mathbf{E}}_r$ are out-of-phase, along with $\vec{\mathbf{B}}_i$ and $\vec{\mathbf{B}}_r$. Similarly, in Fig. 4.43*b* $\vec{\mathbf{E}}_i$, $\vec{\mathbf{E}}_r$, and $\vec{\mathbf{E}}_t$ are in-phase, as are $\vec{\mathbf{B}}_i$, $\vec{\mathbf{B}}_r$, and $\vec{\mathbf{B}}_t$.

Now, the amplitude reflection coefficient for the parallel component is given by

$$r_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

which is positive ($\Delta\varphi_\parallel = 0$) as long as

$$n_t \cos\theta_i - n_i \cos\theta_t > 0$$

that is, if

$$\sin\theta_i \cos\theta_i - \cos\theta_t \sin\theta_t > 0$$

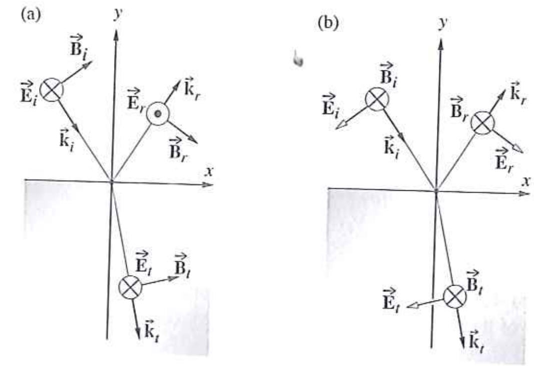

Figure 4.43 Field orientations and phase shifts.

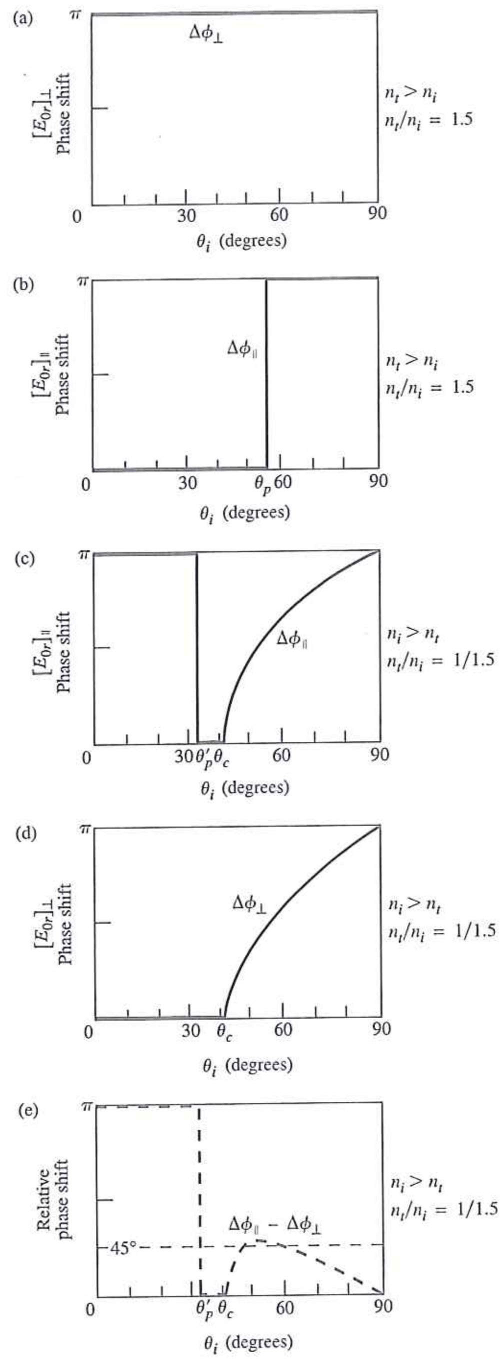

Figure 4.44 Phase shifts for the parallel and perpendicular components of the $\vec{\mathbf{E}}$-field corresponding to internal and external reflection.

or equivalently

$$\sin(\theta_i - \theta_t)\cos(\theta_i + \theta_t) > 0 \tag{4.51}$$

This will be the case for $n_i < n_t$ if

$$(\theta_i + \theta_t) < \pi/2 \tag{4.52}$$

and for $n_i > n_t$ when

$$(\theta_i + \theta_t) > \pi/2 \tag{4.53}$$

Thus when $n_i < n_t$, $[\vec{\mathbf{E}}_{0r}]_\parallel$ and $[\vec{\mathbf{E}}_{0i}]_\parallel$ will be in-phase ($\Delta\varphi_\parallel = 0$) until $\theta_i = \theta_p$ and out-of-phase by π radians thereafter. The transition is not actually discontinuous, since $[\vec{\mathbf{E}}_{0r}]_\parallel$ goes to zero at θ_p. In contrast, for internal reflection $r_\parallel$ is negative until θ'_p, which means that $\Delta\varphi_\parallel = \pi$. From θ'_p to θ_c, $r_\parallel$ is positive and $\Delta\varphi_\parallel = 0$. Beyond θ_c, $r_\parallel$ becomes complex, and $\Delta\varphi_\parallel$ gradually increases to π at $\theta_i = 90°$.

Figure 4.44, which summarizes these conclusions, will be of continued use to us. The actual functional form of $\Delta\varphi_\parallel$ and $\Delta\varphi_\perp$ for internal reflection in the region where $\theta_i > \theta_c$ can be found in the literature,* but the curves depicted here will suffice for our purposes. Figure 4.44*e* is a plot of the relative phase shift between the parallel and perpendicular components, that is, $\Delta\varphi_\parallel - \Delta\varphi_\perp$. It is included here because it will be useful later on (e.g., when we consider polarization effects). Finally, many of the essential features of this discussion are illustrated in Figs. 4.45 and 4.46. The amplitudes of the reflected vectors are in accord with those of Figs. 4.41 and 4.42 (for an air–glass interface), and the phase shifts agree with those of Fig. 4.44.

Many of these conclusions can be verified with the simplest experimental equipment, namely, two linear polarizers, a piece of glass, and a small source, such as a flashlight or high-intensity lamp. By placing one polarizer in front of the source (at 45° to the plane-of-incidence), you can easily duplicate the conditions of Fig. 4.45. For example, when $\theta_i = \theta_p$ (Fig. 4.45*b*) no light will pass through the second polarizer if its transmission axis is parallel to the plane-of-incidence. In comparison, at near-glancing incidence the reflected beam will vanish when the axes of the two polarizers are almost normal to each other.

*Born and Wolf, *Principles of Optics*, p. 49.

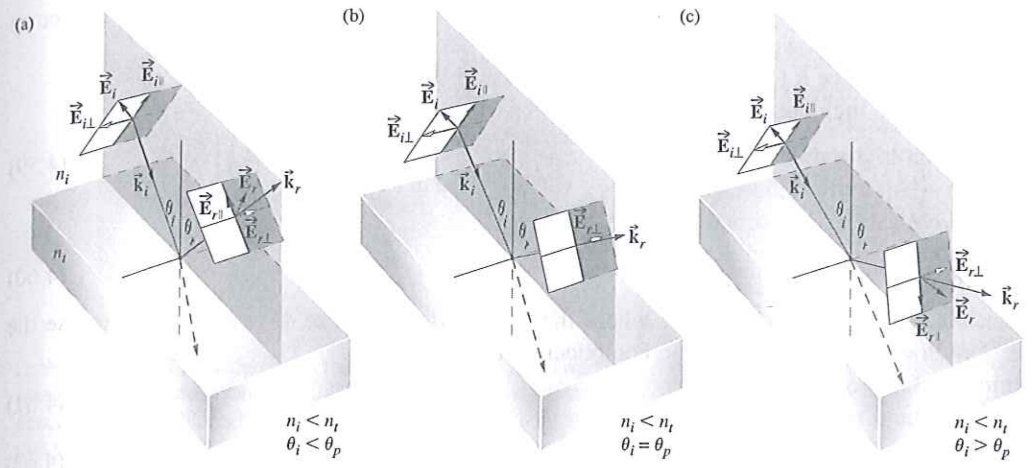

Figure 4.45 The reflected $\vec{E}$-field at various angles concomitant with external reflection.

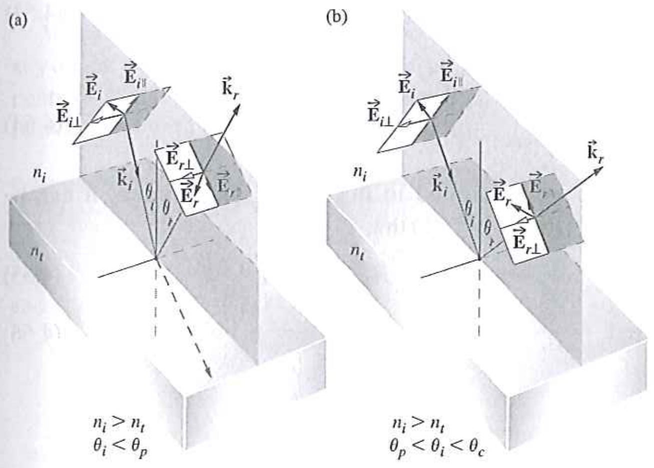

Figure 4.46 The reflected $\vec{E}$-field at various angles concomitant with internal reflection.

Reflectance and Transmittance

Consider a circular beam of light incident on a surface, as shown in Fig. 4.47, such that there is an illuminated spot of area A. Recall that the power per unit area crossing a surface in vacuum whose normal is parallel to $\vec{S}$, the Poynting vector, is given by

$$\vec{S} = c^2 \epsilon_0 \vec{E} \times \vec{B} \tag{3.40}$$

Furthermore, the radiant flux density (W/m^2) or irradiance is

$$I = \langle S \rangle_T = \frac{c\epsilon_0}{2} E_0^2 \tag{3.44}$$

This is the average energy per unit time crossing a unit area normal to $\vec{S}$ (in isotropic media $\vec{S}$ is parallel to $\vec{k}$). In the case at hand (Fig. 4.47), let I_i, I_r, and I_t be the incident, reflected, and transmitted flux densities, respectively. The cross-sectional areas of the incident, reflected, and transmitted beams are, respectively, $A \cos \theta_i$, $A \cos \theta_r$, and $A \cos \theta_t$. Accordingly, the incident power is $I_i A \cos \theta_i$; this is the energy per unit time flowing in the incident beam, and it's therefore the power

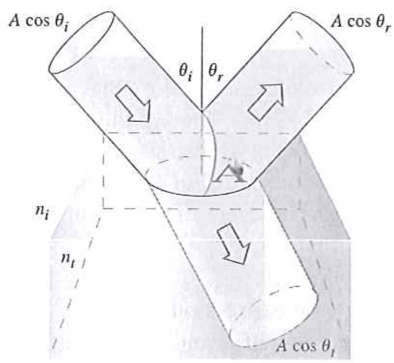

Figure 4.47 Reflection and transmission of an incident beam.