Benchmarking Coherent Synchrotron Radiation

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Prologue

- This is a review with an accent on **analytical modelling**, and **accuracy of predictions** relative to **experimental evidences**.
  - I will skip theoretical derivations and computational algorithms.

- I will report about a selection of case studies for **linac-driven FELs**.

- CSR will be analysed in terms of transverse ($\rightarrow$ **emittance**) and longitudinal instability ($\rightarrow$ **energy spread, microbunching**).
  - New paths of research on CSR instability could be identified.

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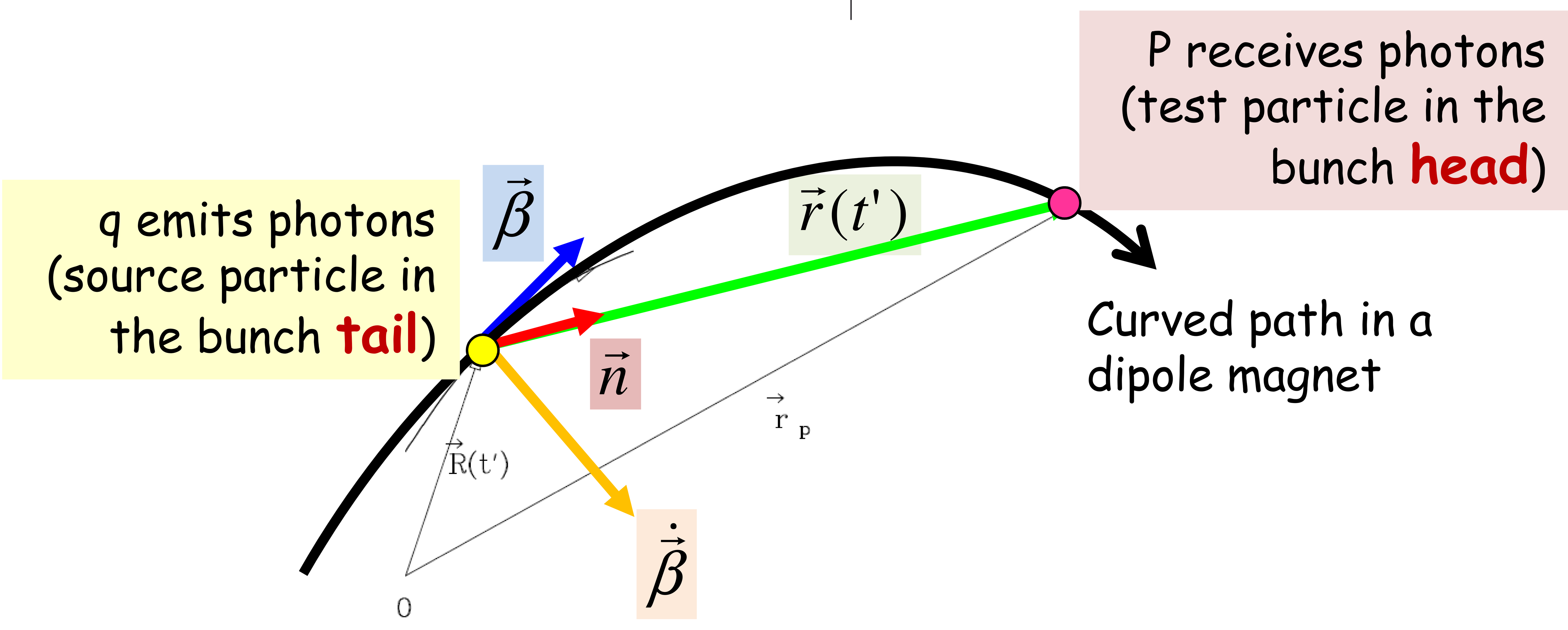
CSR Tail-Head Interaction



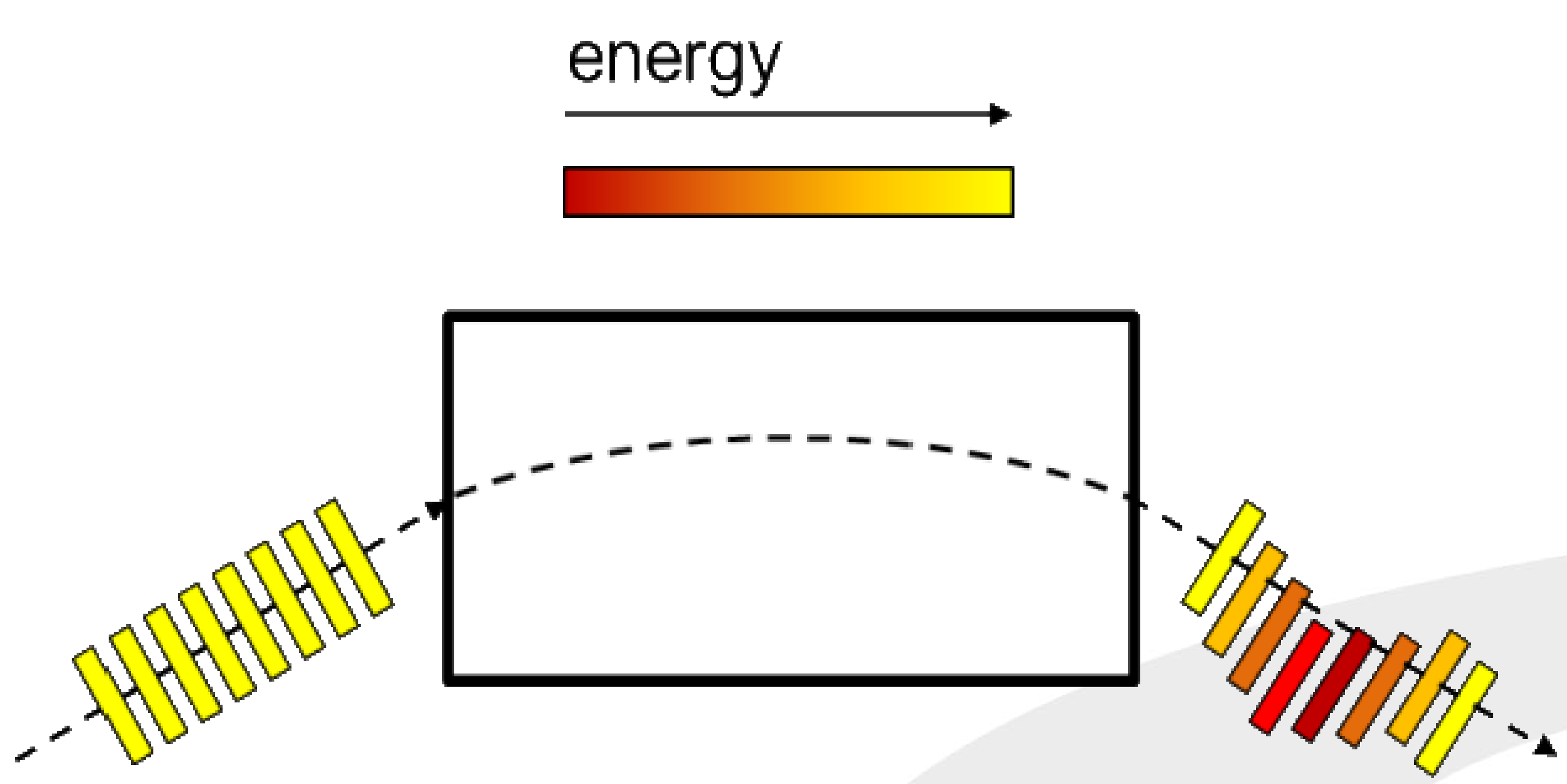
- Two particles on same circular path (**1-D model**),
- $\gamma^3 s/R >> 1$ (**steady-state approximation**),

ENERGY CHANGE ALONG BUNCH, per METER:

$$\frac{dU_e(z)}{dz} \cong -\frac{Ne^2}{3^{1/3}2\pi\varepsilon_0}\frac{1}{R^{2/3}}\int_z^{\infty}\frac{dz'}{(z'-z)^{1/3}}\frac{d\lambda(z')}{dz'}$$

Current spikes or fast rises enhance the z-CSR field.

- ❑ The longitudinal E-field comes from **geometrical considerations.**
- ❑ CSR instability is primarily a **chromatic effect.**
- ❑ Effect on transverse emittance can be described through **linear optics.**



C. Hall et al., PRST-AB 18, 030706 (2015)

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Energy Loss

- Example: Gaussian bunch in steady-state emission:

$$U_{tot} = -0.028 \times e^2 Z_0 c N \frac{\theta R^{1/3}}{\sigma_z^{4/3}} = -0.16\ \mathrm{MeV}$$ Total energy loss

$$\langle\delta\rangle_{csr} \simeq -0.35 \times r_e \frac{N}{\gamma} \frac{\theta R^{1/3}}{\sigma_z^{4/3}} = 0.20\%$$ Relative mean energy loss

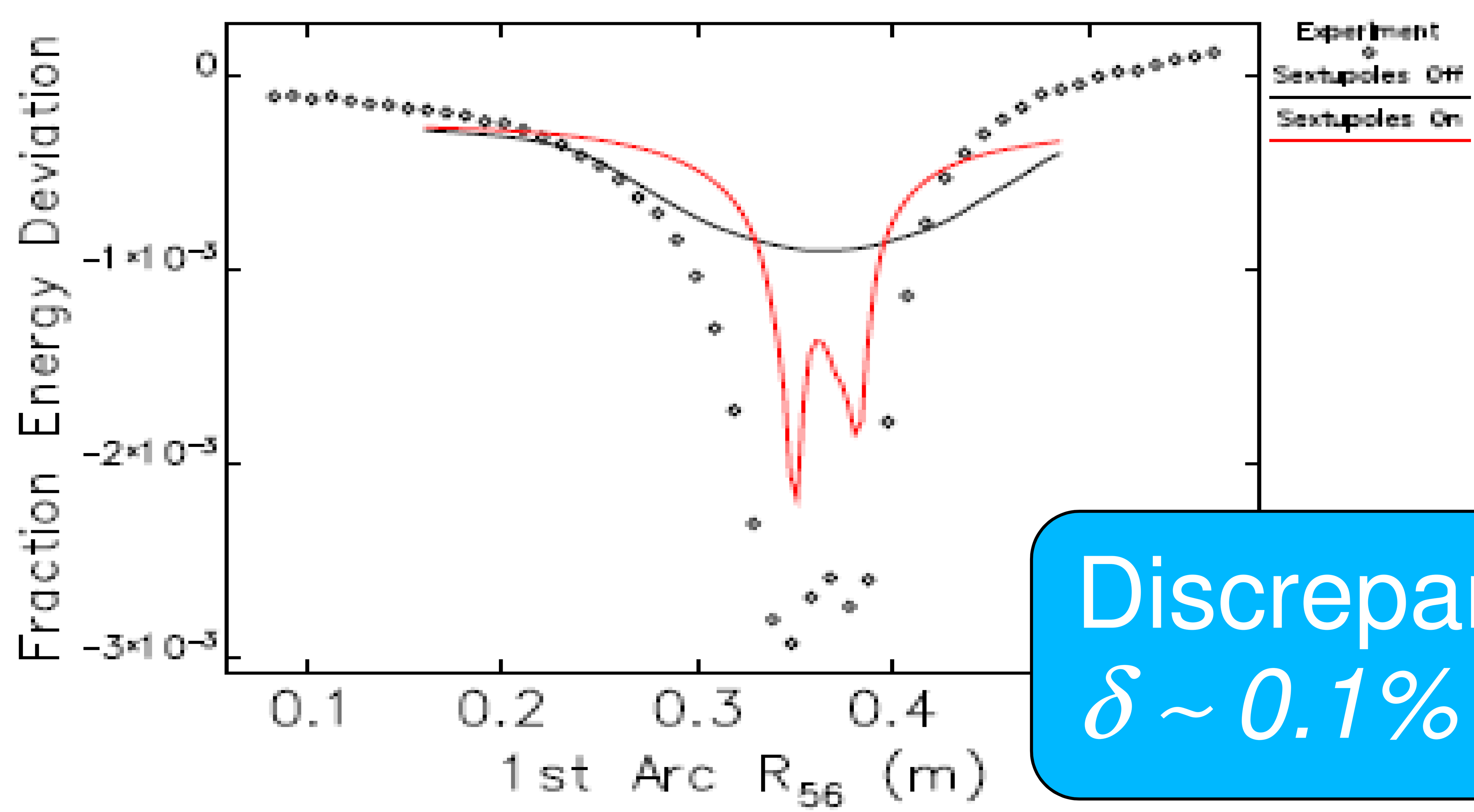
$$\sigma_{\delta,\,csr} \simeq 0.7 \times |\langle\delta\rangle_{csr}| = 0.14\%$$ Relative rms energy spread

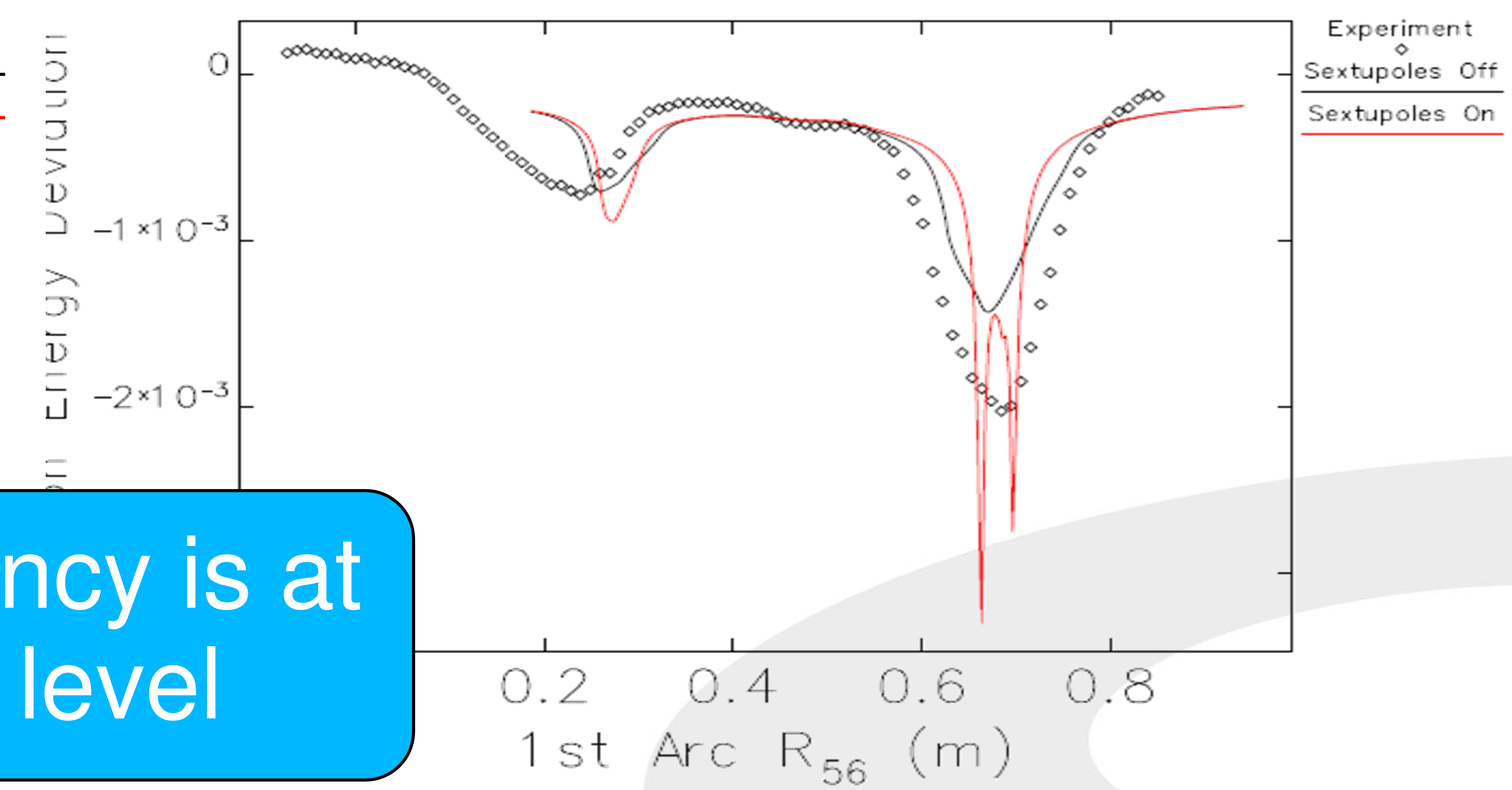
$$\begin{aligned} \sigma_z &= 50\mu m \\ Q &= 300pC \\ L_B &= 1m \\ \theta_B &= 10^o \\ R &= 5.7m \\ I_{pk} &= 715\ A \\ E &= 700\ MeV \end{aligned}$$

We should be able to resolve energy loss at this level

Arc
(TBA or Bates cell)

0.1 GeV, 0.1 nC, < 1 kA





Discrepancy is at $\delta \sim 0.1\%$ level

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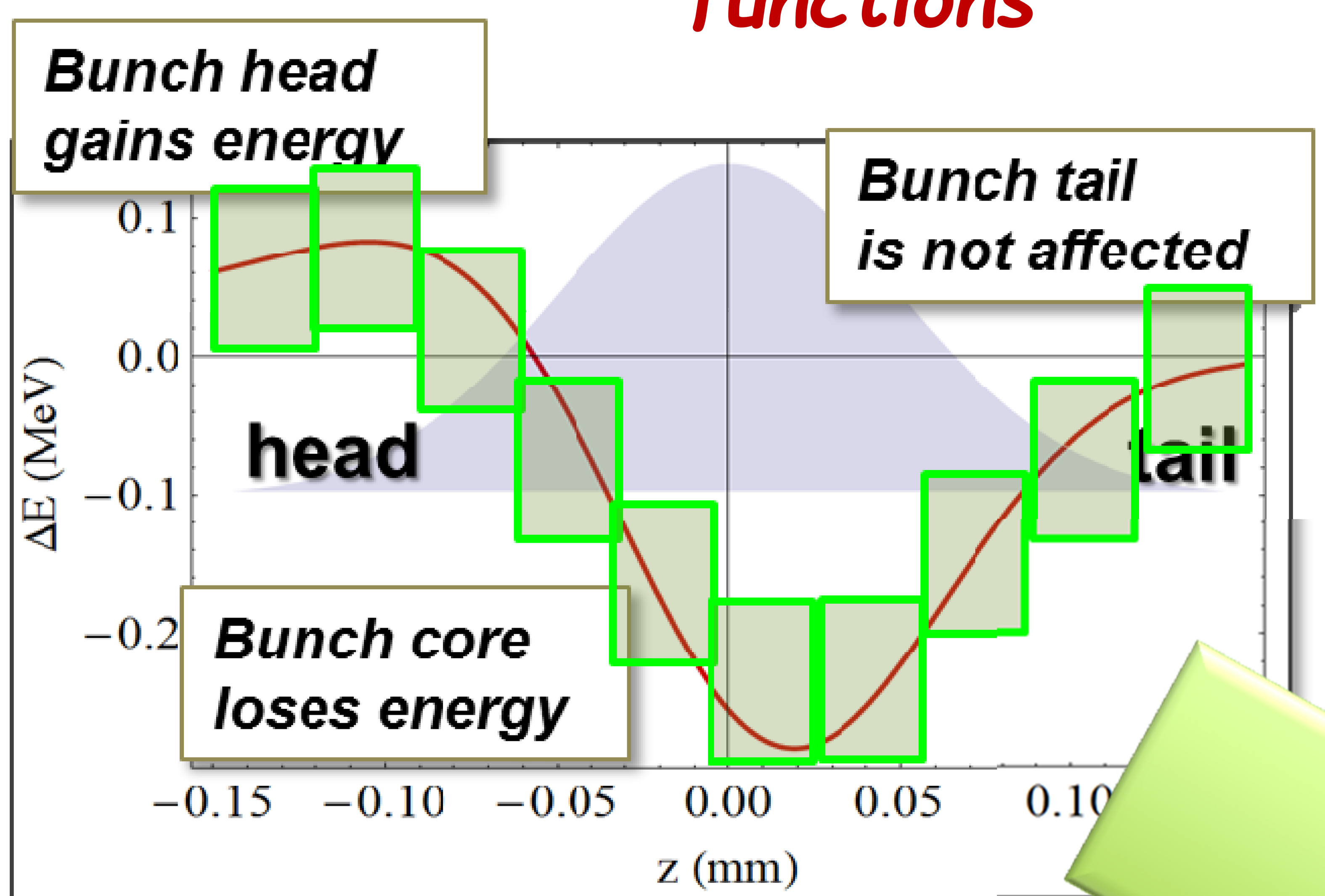
Collapsing the Physics onto RMS Values

- Particle coordinates transform according to:

$$x(s) = x_\beta(s) + R_{16}(s_0 \to s)\,\delta(s) \equiv x_\beta + \Delta x$$
$$x'(s) = x'_\beta(s) + R_{26}(s_0 \to s)\,\delta(s) \equiv x'_\beta + \Delta x'$$

~ Energy dispersion functions

Change of longitudinal momentum by absorption of radiation



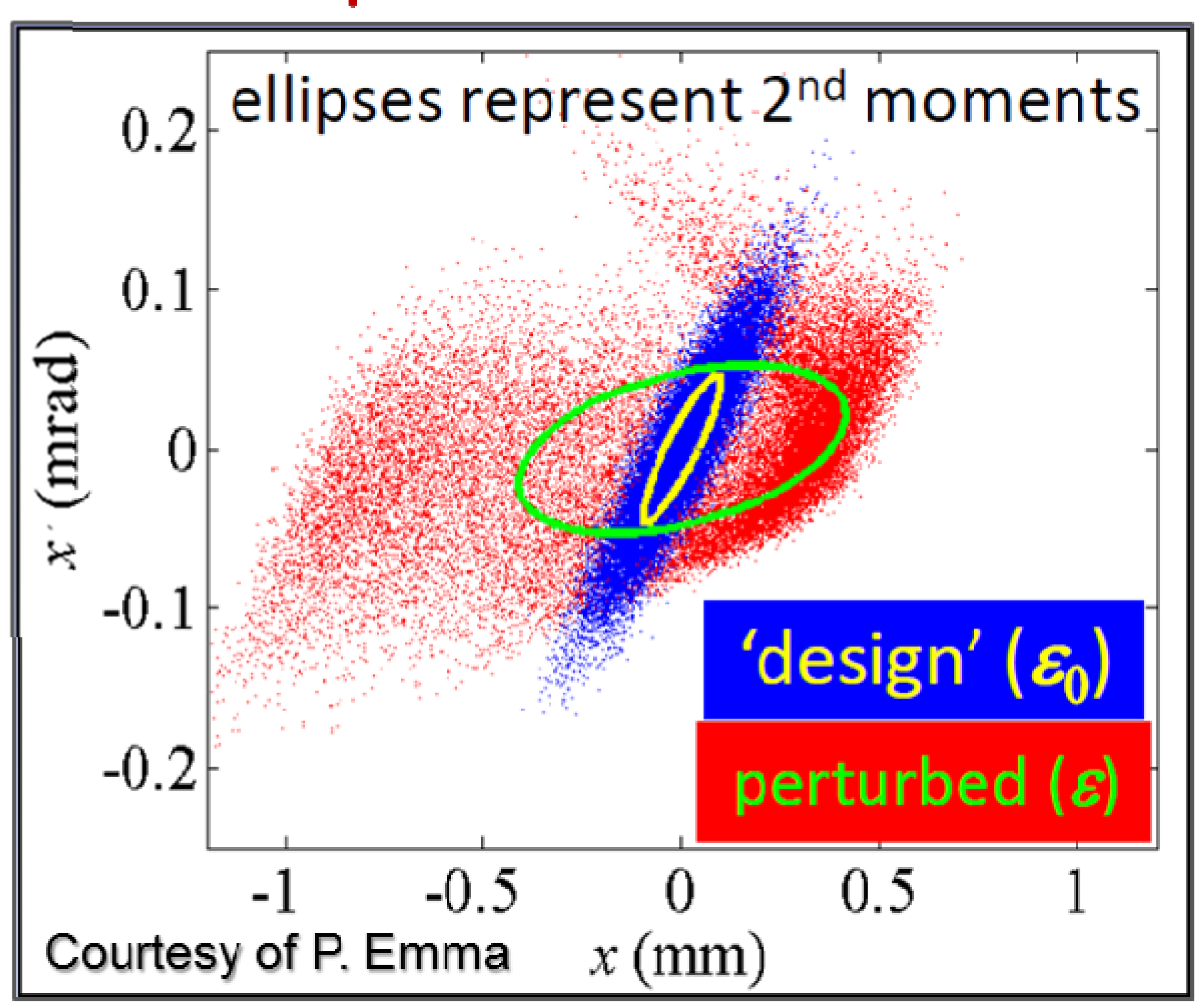
RMS EMITTANCE

$$\varepsilon_x = \sqrt{\langle x_\beta^2\rangle\langle x'^2_\beta\rangle - \langle x_\beta x'_\beta\rangle^2}$$

$$\varepsilon_x^2 = \varepsilon_{x,0}^2 + \varepsilon_{x,0}\left(\beta_x\langle\Delta x'^2\rangle + 2\alpha_x\langle\Delta x\Delta x'\rangle + \gamma_x\langle\Delta x^2\rangle\right) +$$
$$+\left(\langle\Delta x^2\rangle\langle\Delta x'^2\rangle - \langle\Delta x\Delta x'\rangle^2\right)$$

Since Δx and Δx' from CSR field are correlated, this goes to 0.

Phase space at chicane's exit



Is this simplified picture good enough? At which level of accuracy?

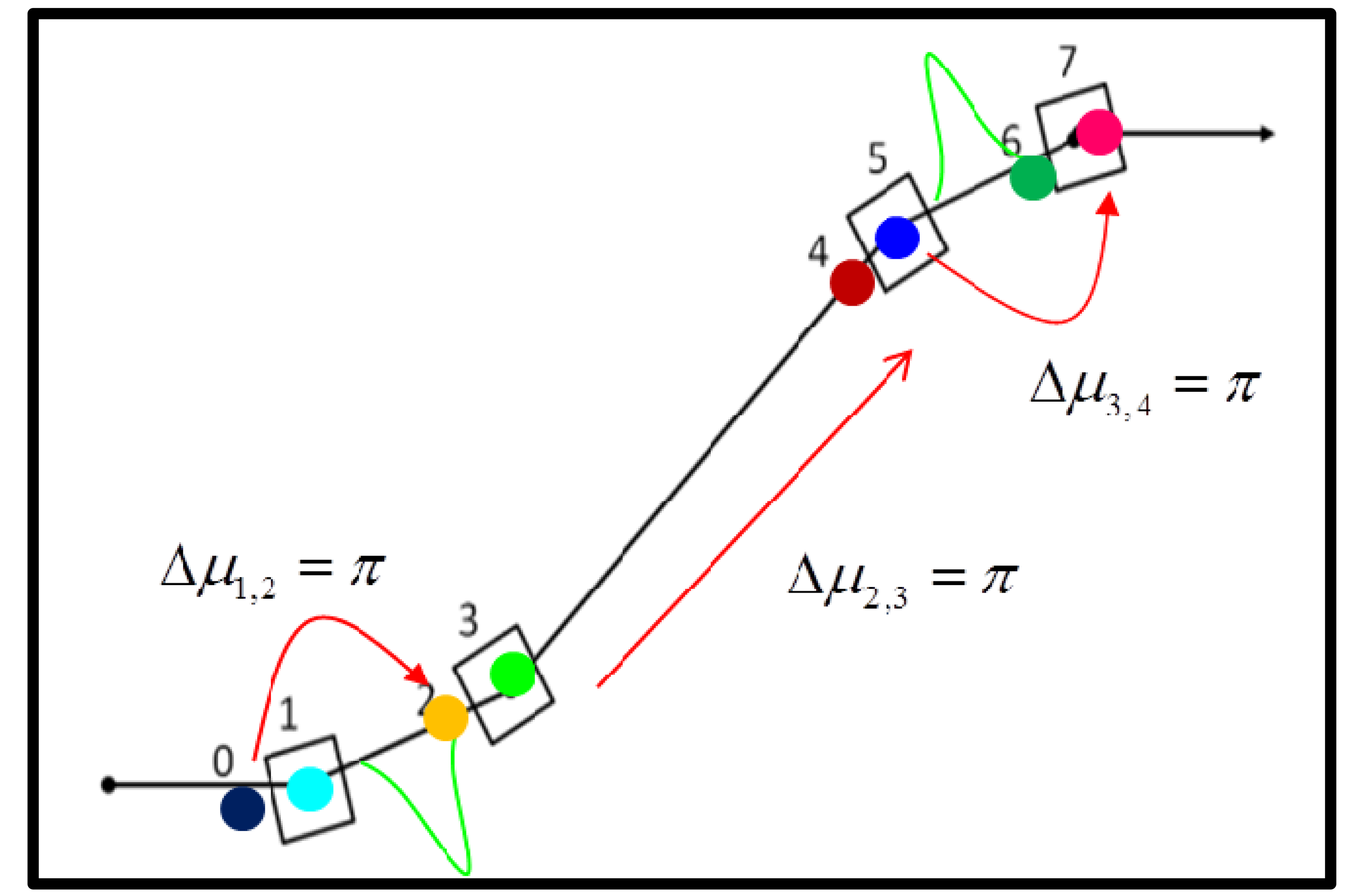
S. Di Mitri et al., PRL 110, 014801 (2013)

T. Hara @ FEL'17 and NOCE'17

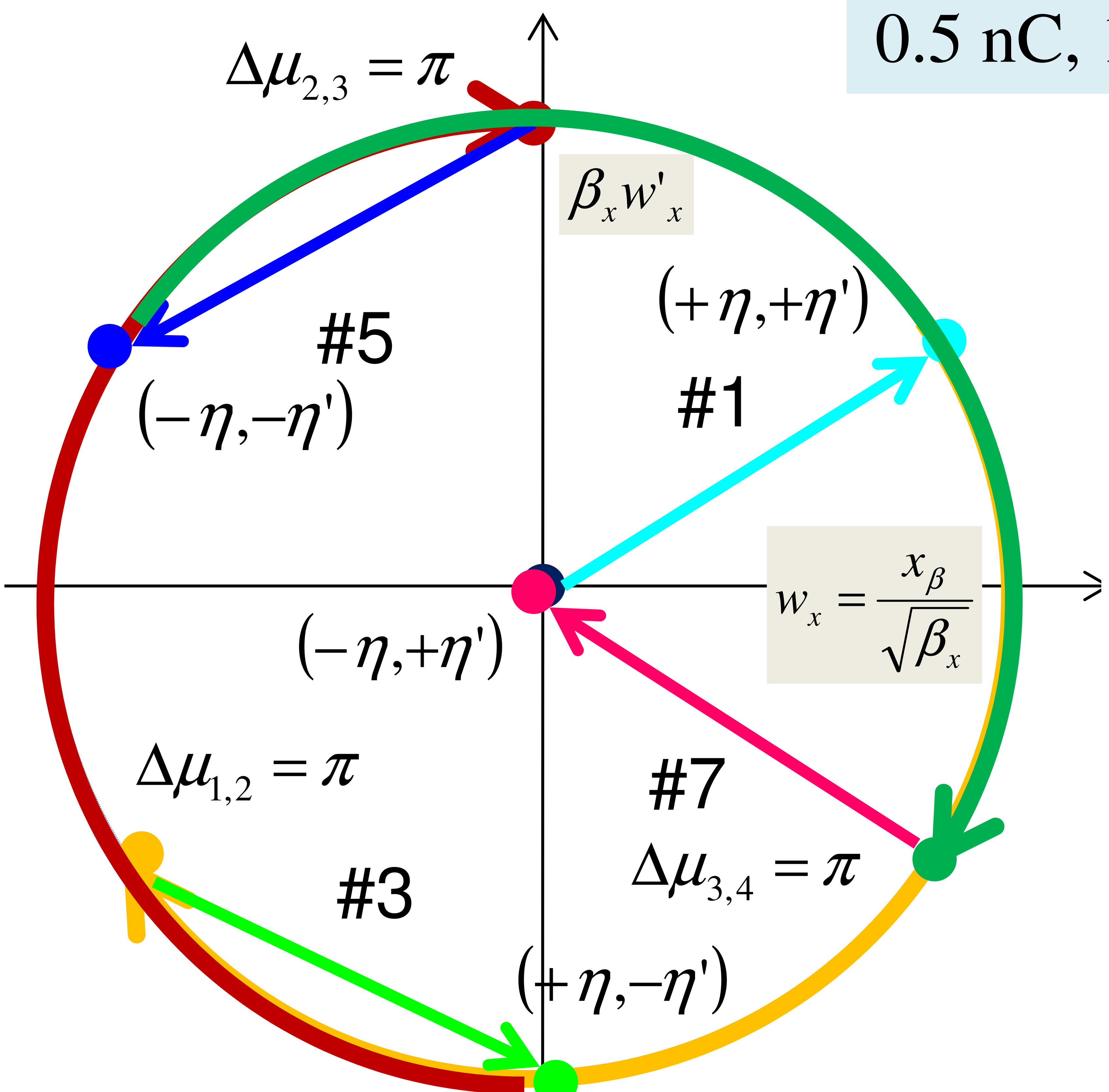
Horizontal Emittance

Dogleg
(almost isochronous)

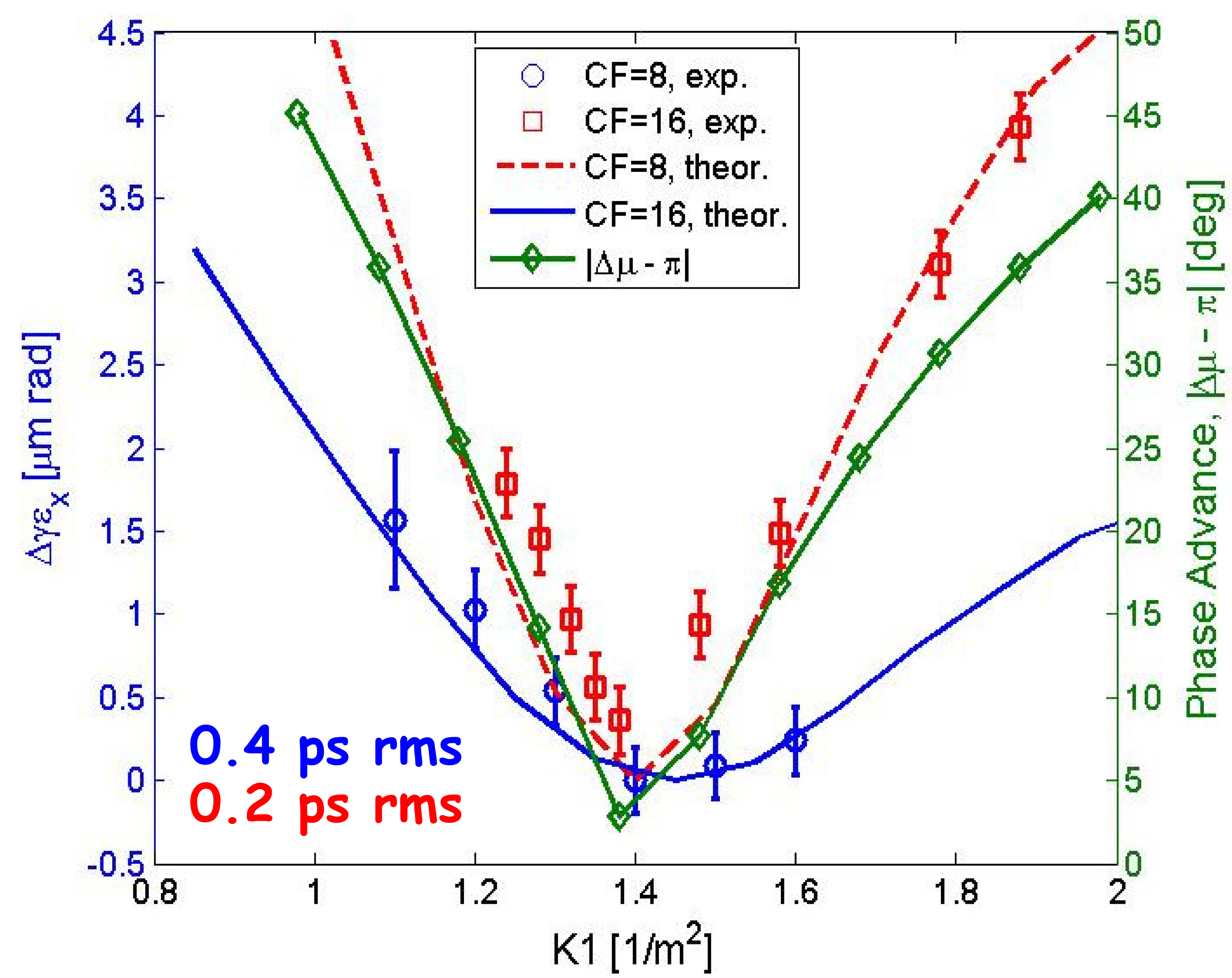
$$\varepsilon_x^2 = \varepsilon_{x,0}^2 + \varepsilon_{x,0} H_x(s_1)\sigma_{\delta,CSR}^2 X(\alpha_x, \beta_x, \Delta\mu_x)_{s_f}$$



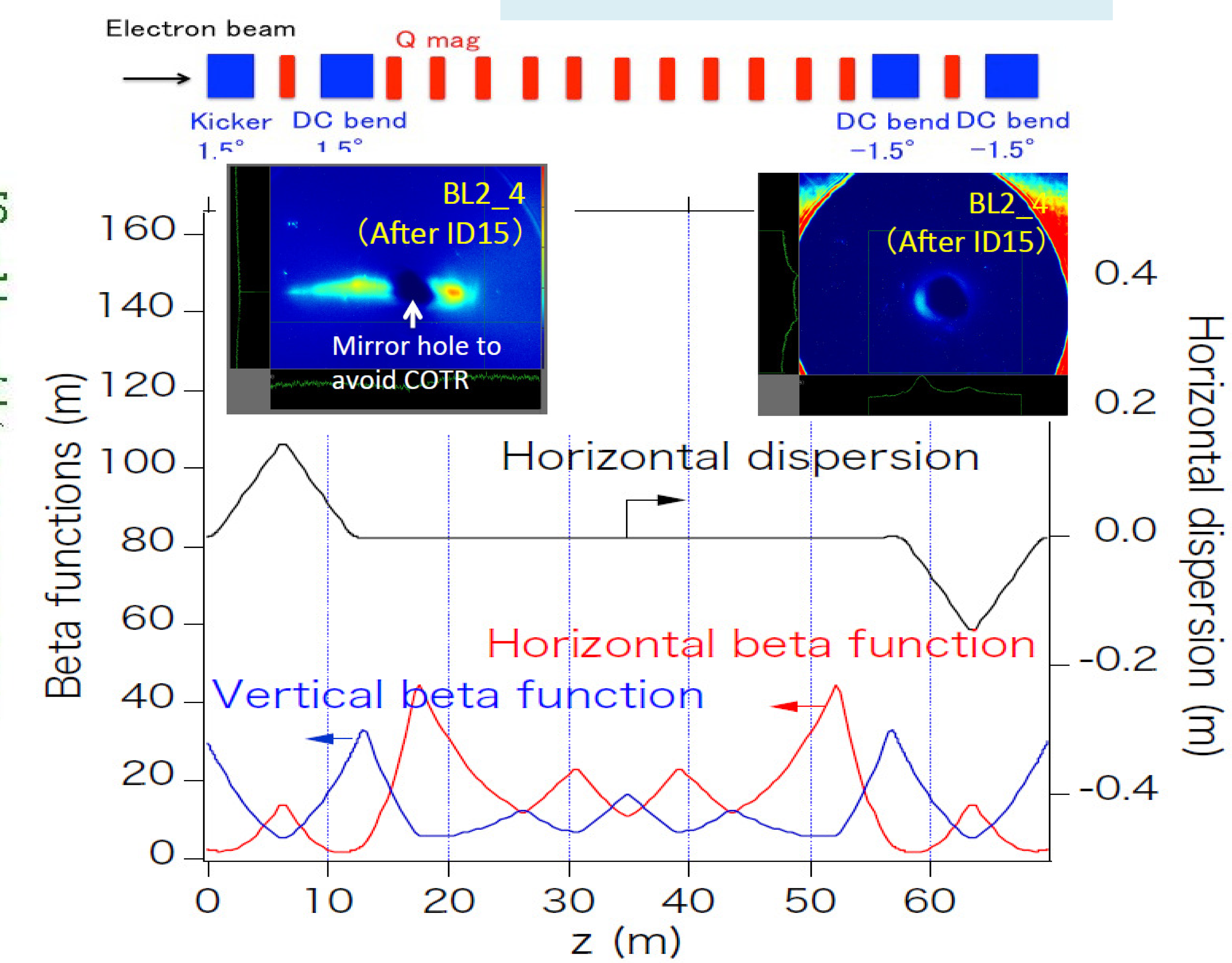
Accuracy of prediction is at $\Delta\varepsilon_{n,x} \sim 0.1 \mu m$ level



0.5 nC, 1 GeV, 0.5 kA



0.2 nC, 7 GeV, 3 kA



Y. Jiao et al., PRST-AB 17, 060701 (2014)
S. Di Mitri, NIM A 806 (2016)

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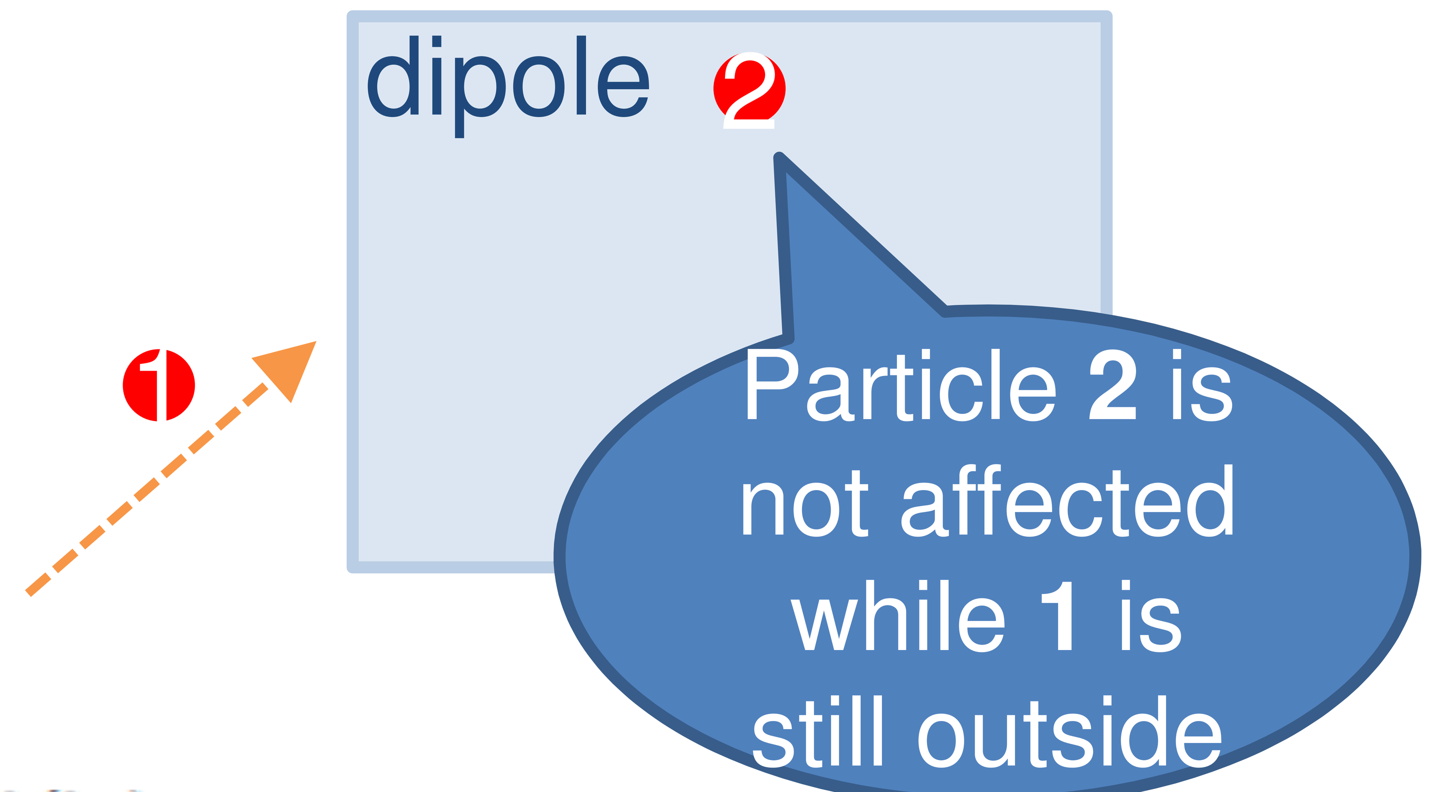
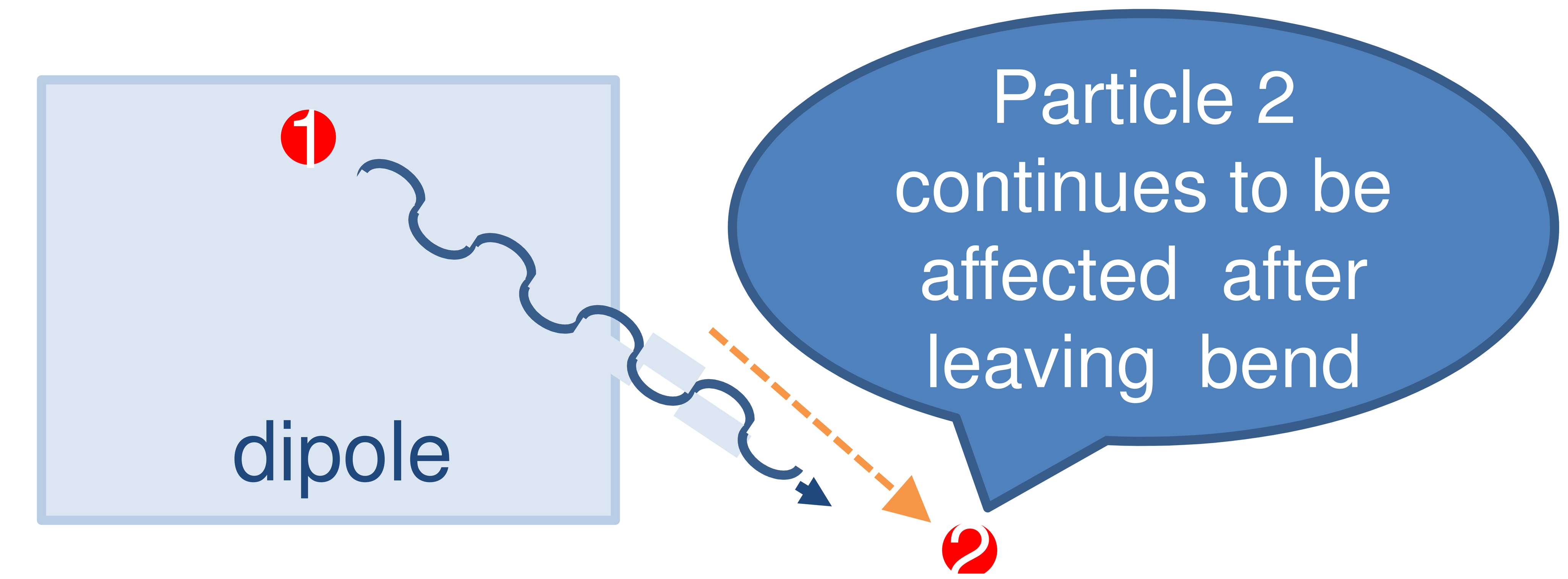
Transient Effects

- Steady-state model doesn't account for **transient effects**, which are relevant over lengths $L_t \simeq (24R^2\sigma_z)^{1/3} \simeq 0.1 - 1$ m

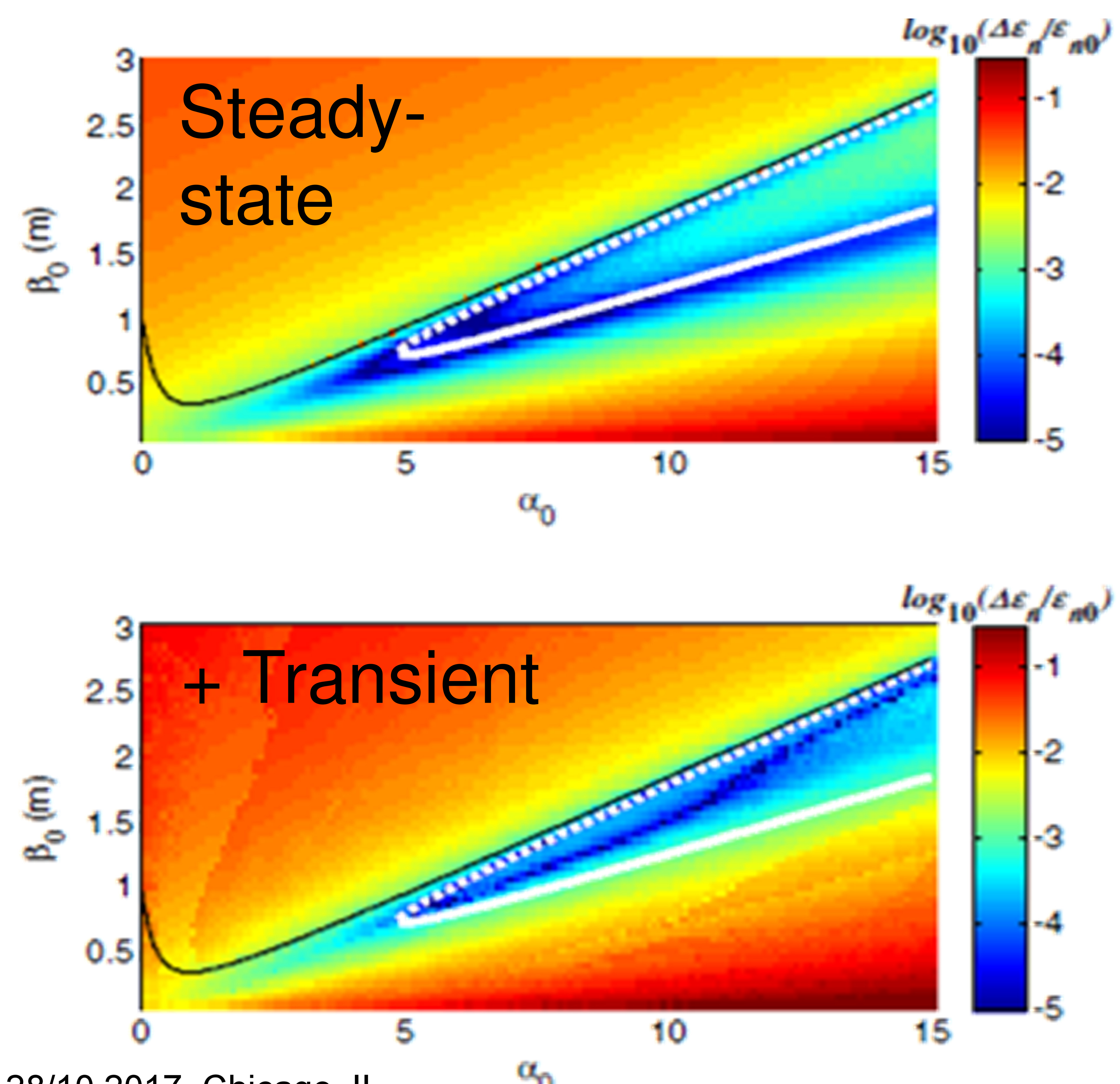
Are these contributions noticeable in the overall CSR energy budget?

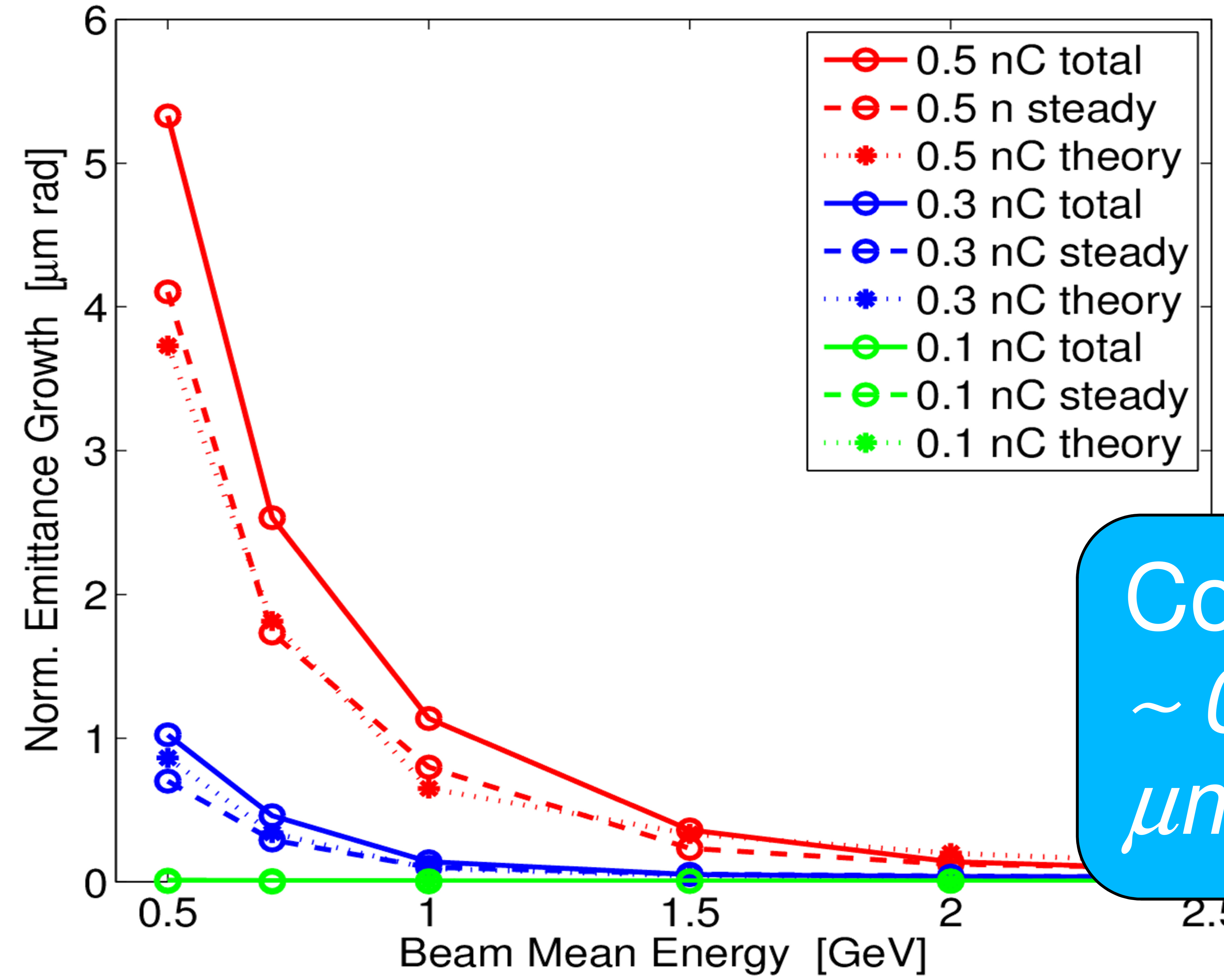




Chicane

0.5 nC,
1 GeV,
~ 1 kA





Arc (6 DBA)

Contribution is at $\Delta\varepsilon_{n,x}$ ~ *0.1 (chicane) – 1.0 µm* level (long arc).

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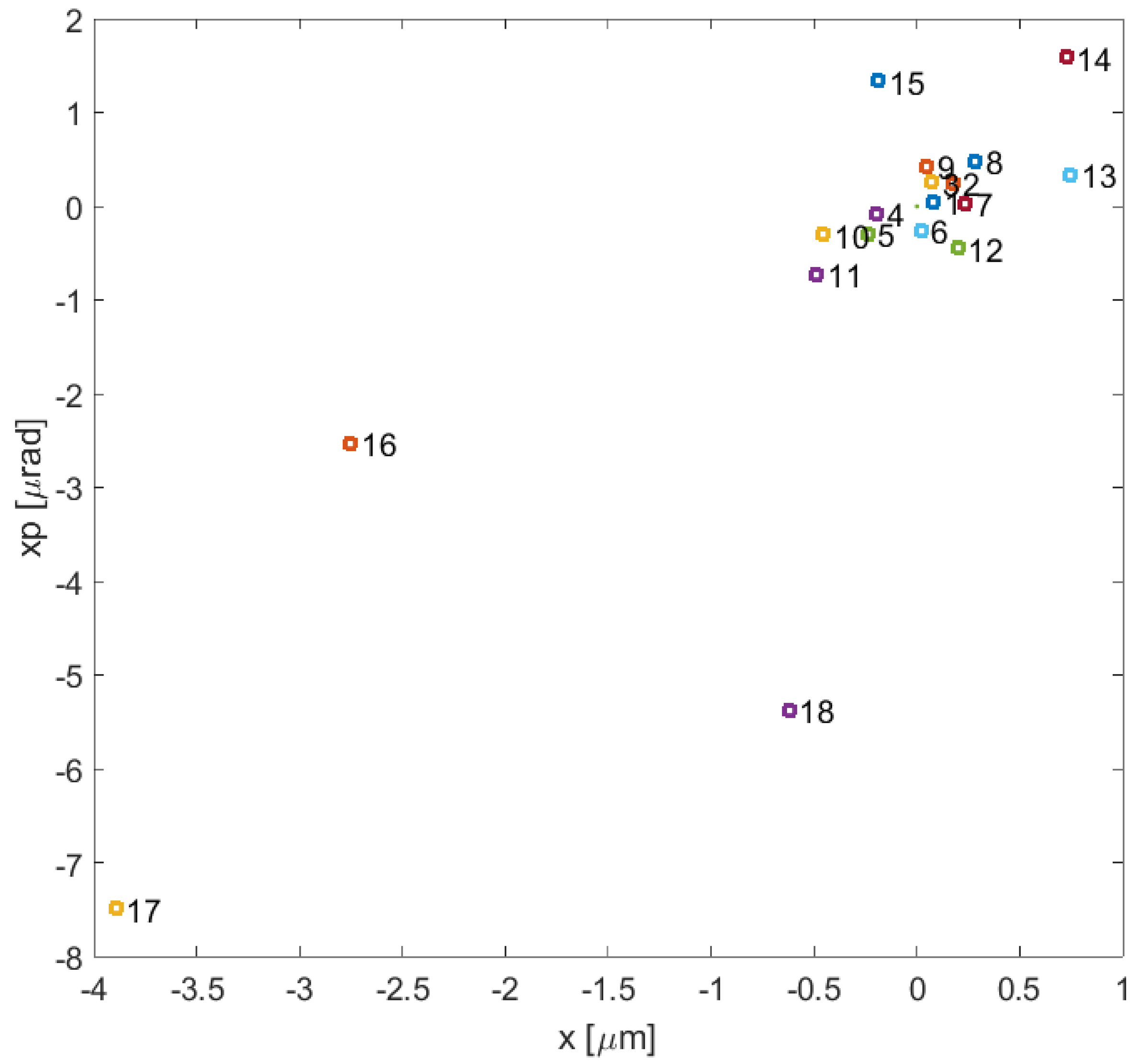
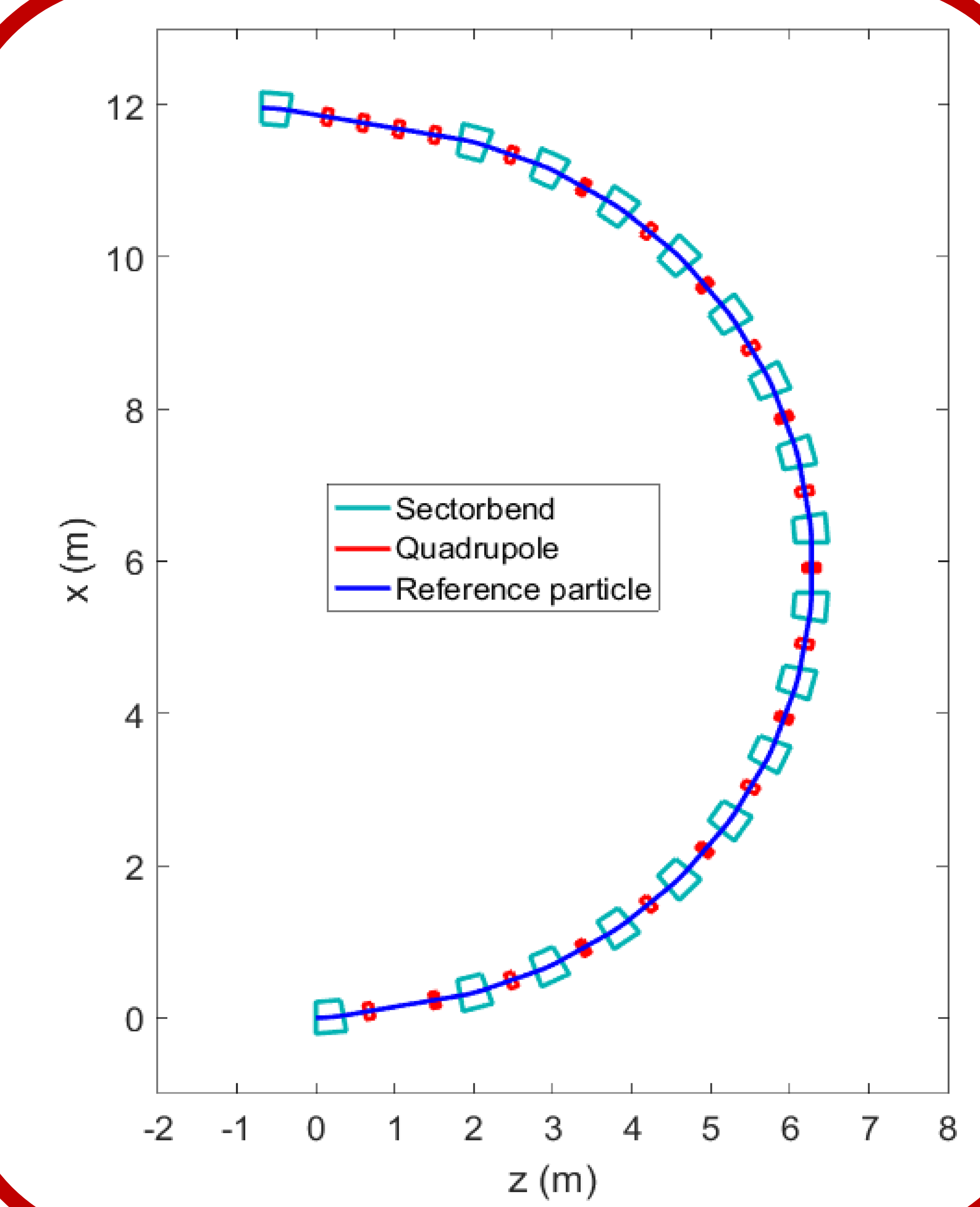
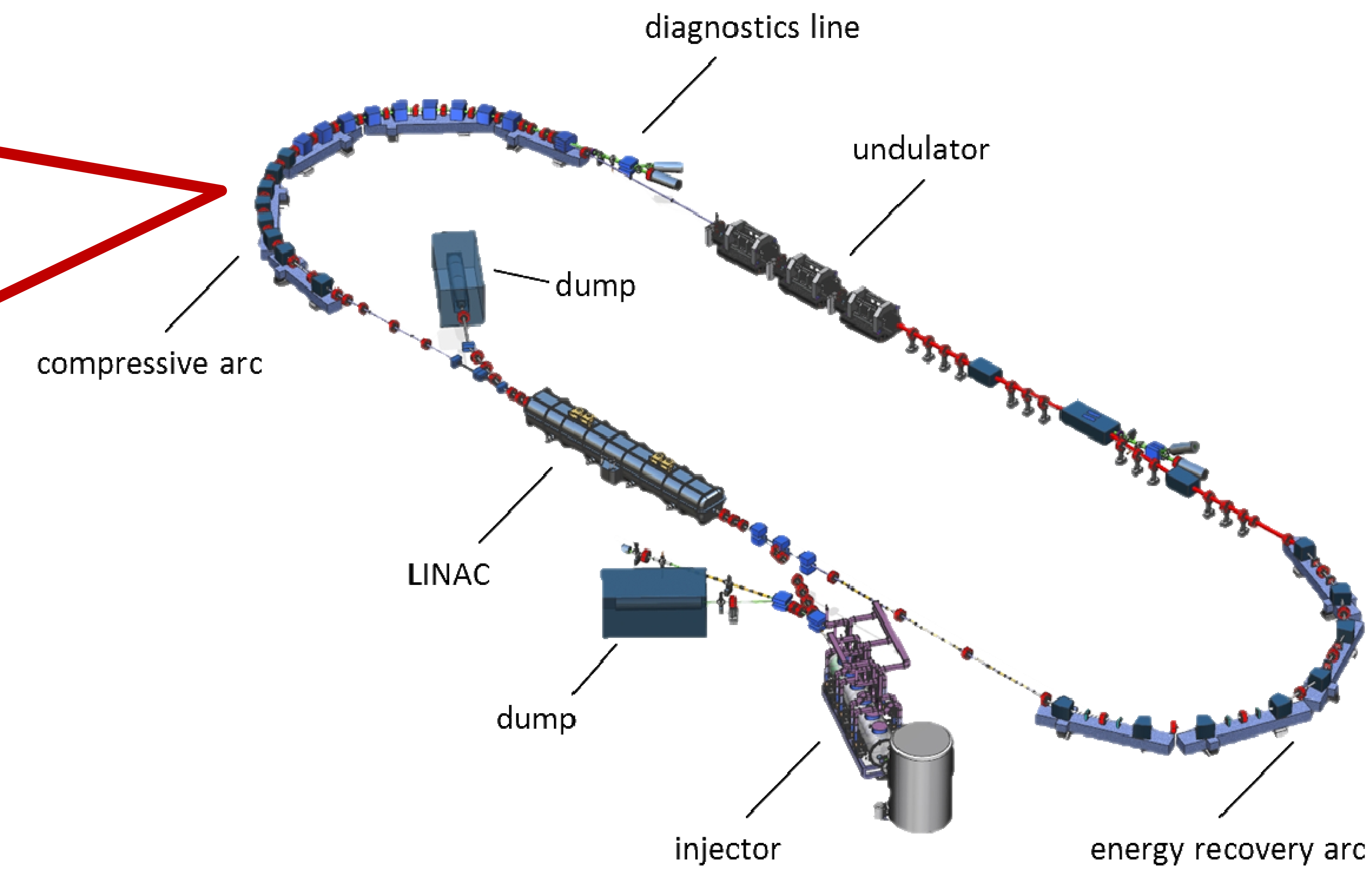
Example: Compact ERL – UV FEL

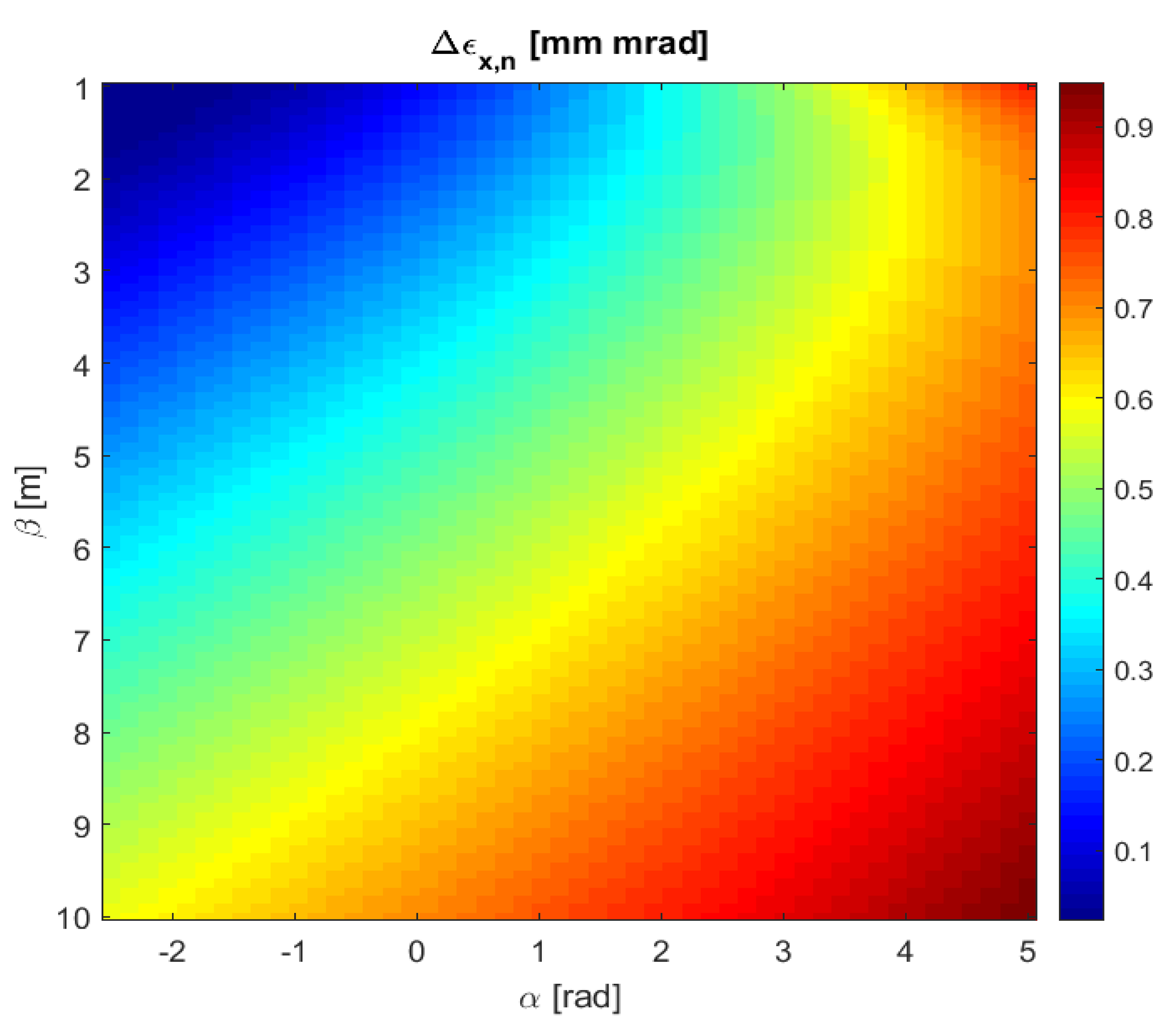
CSR kicks are calculated analytically at each dipole:

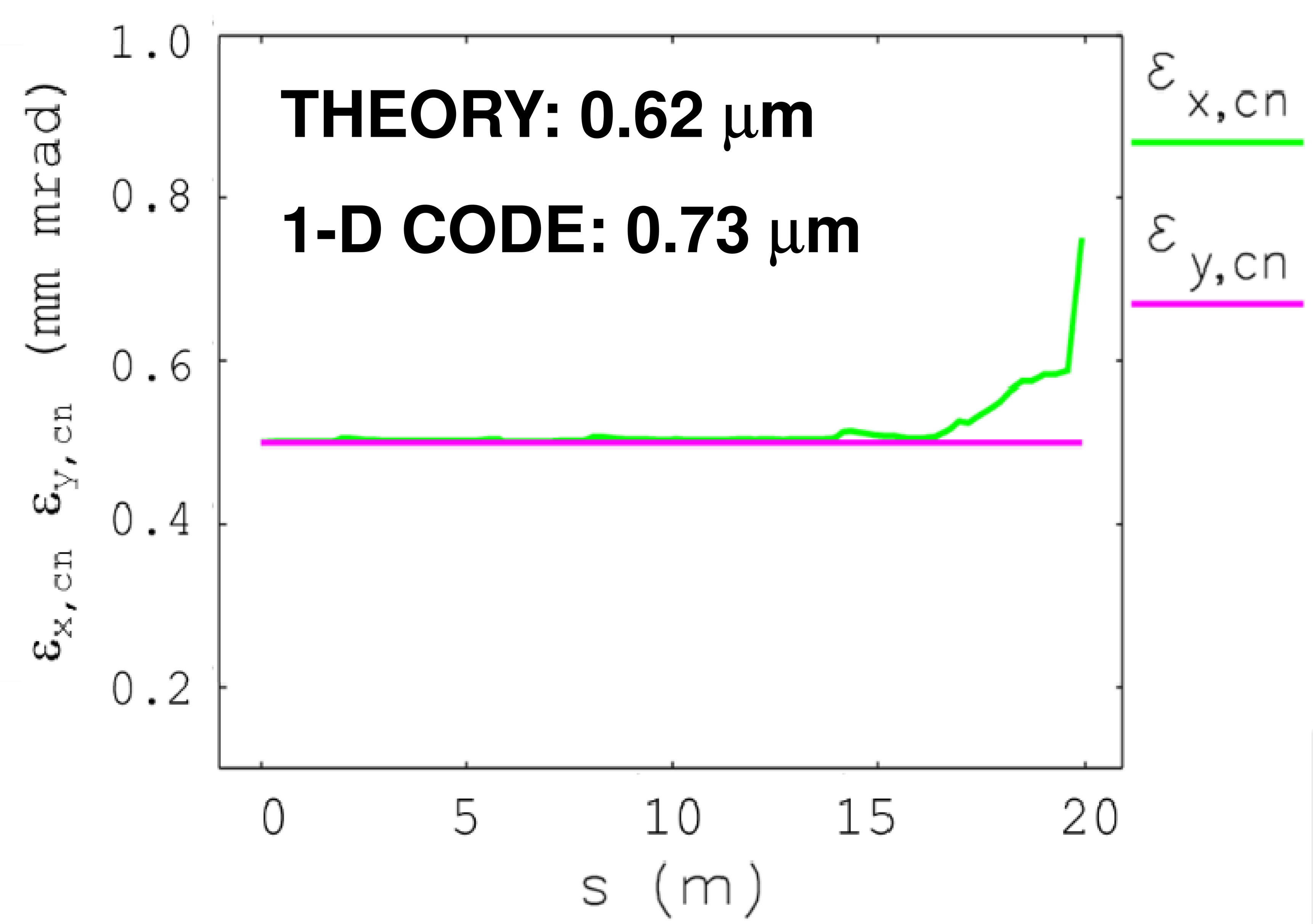






Analytical scan of final emittance vs. initial Twiss parameters:





elegant simulation, including transient effects

S. Di Mitri et al., NIM A 608 (2009)

K. Bane et al., PRST-AB 12, 030704 (2009)

FERMI Team, P. Williams,
A. Brynes & ASML, in progress

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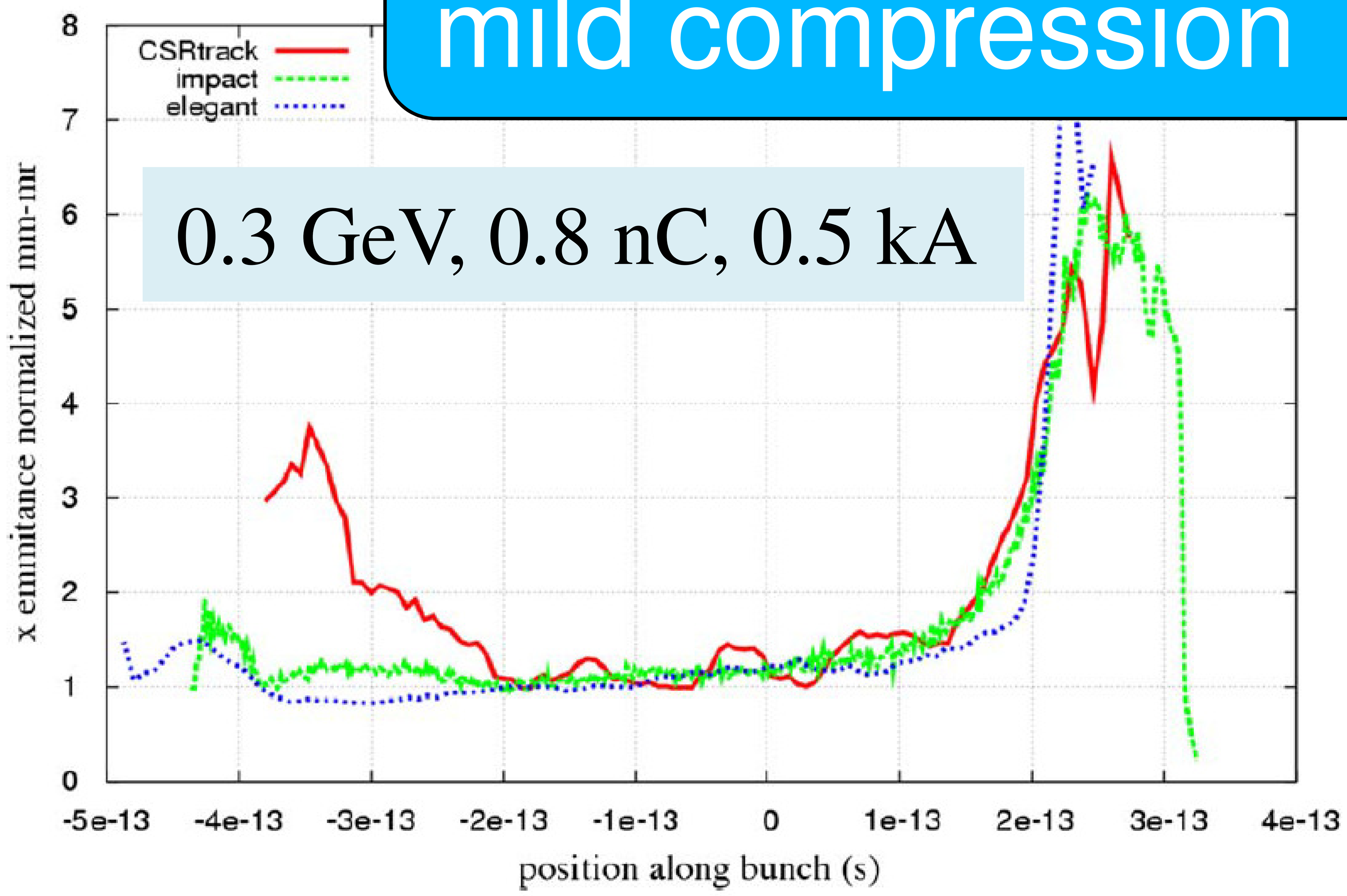
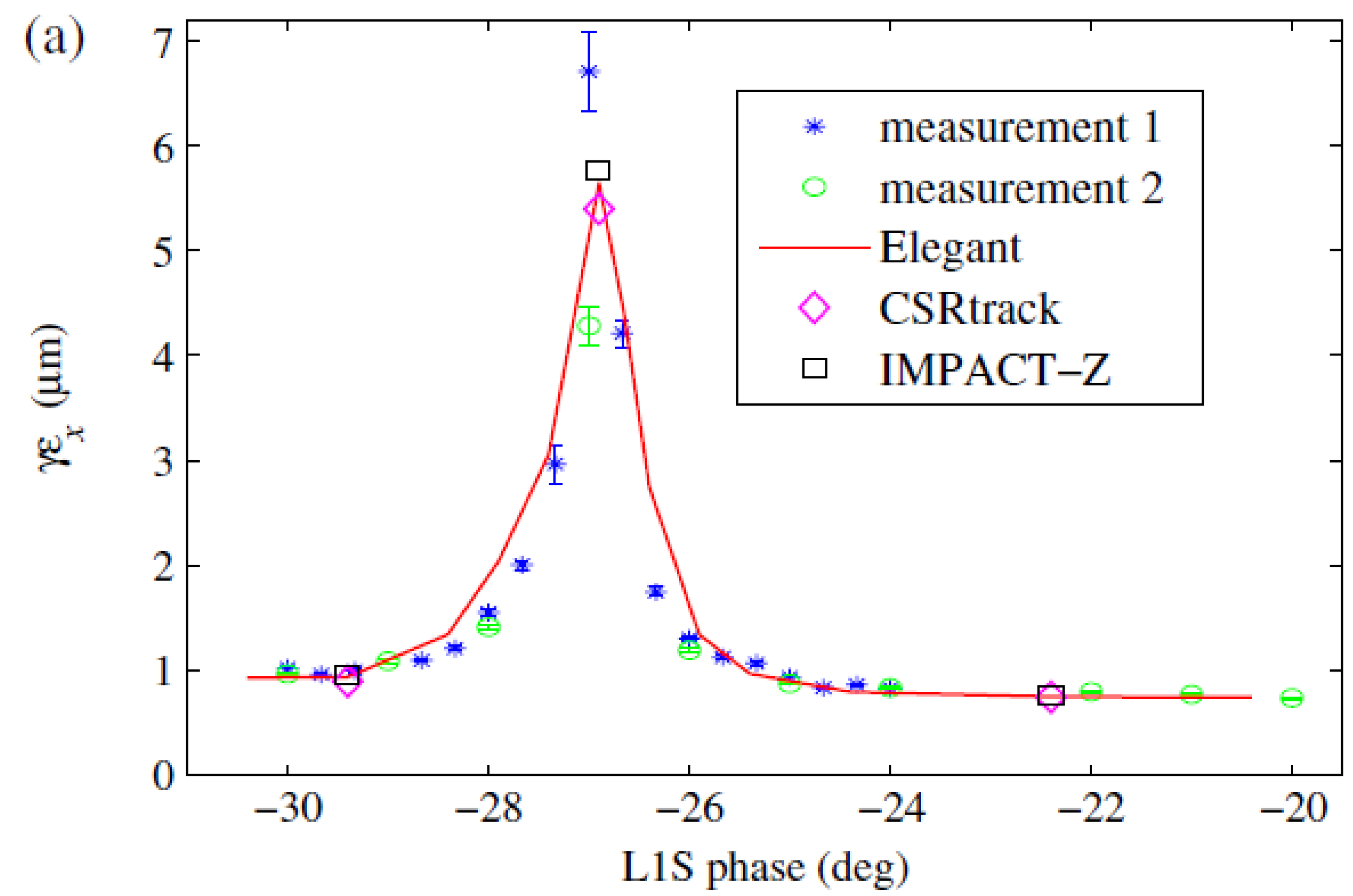
1-D Codes

1-D codes agree for mild compression

0.3 GeV, 0.8 nC, 0.5 kA

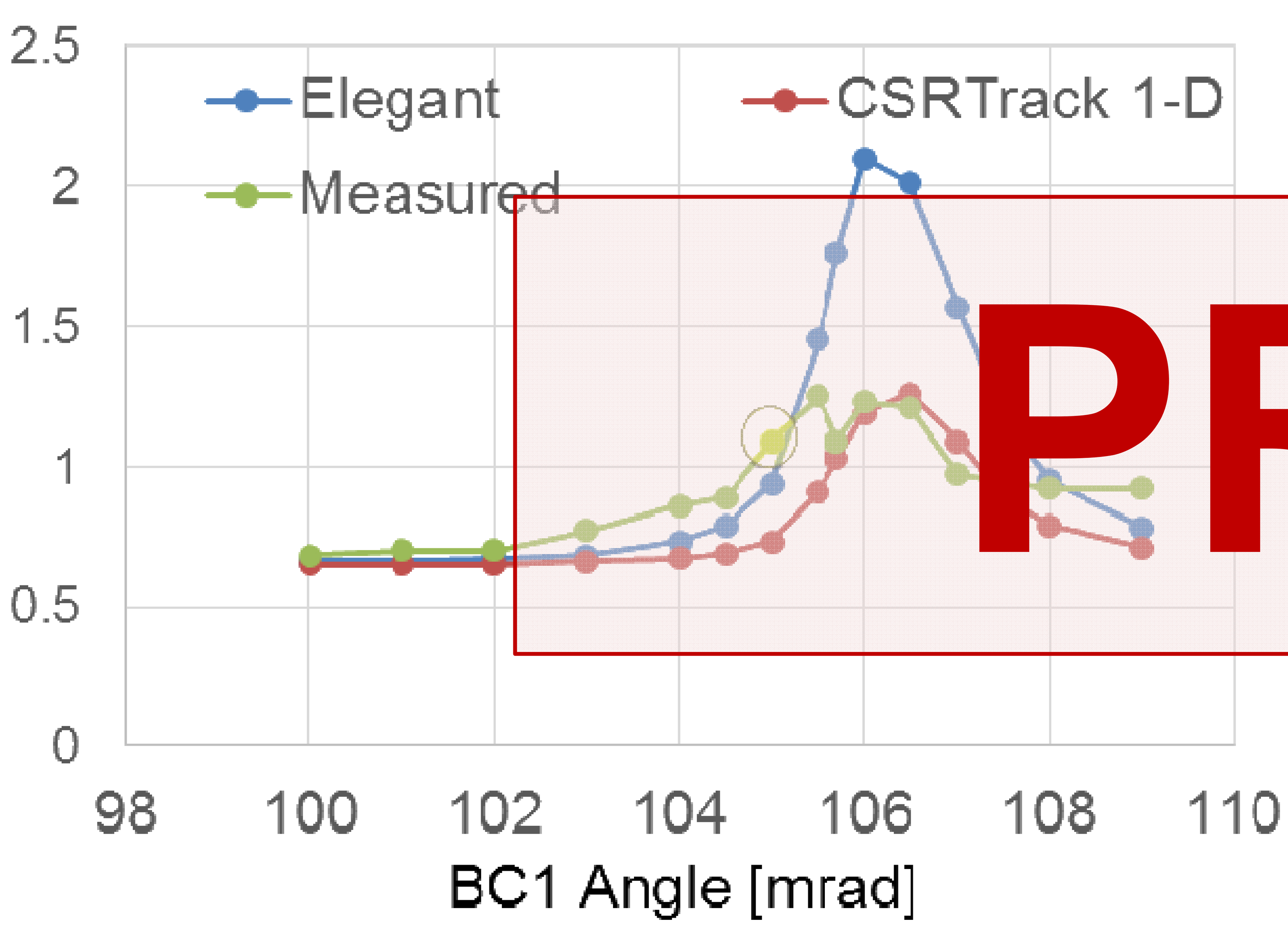


0.3 GeV, 0.3 nC, < 1 kA

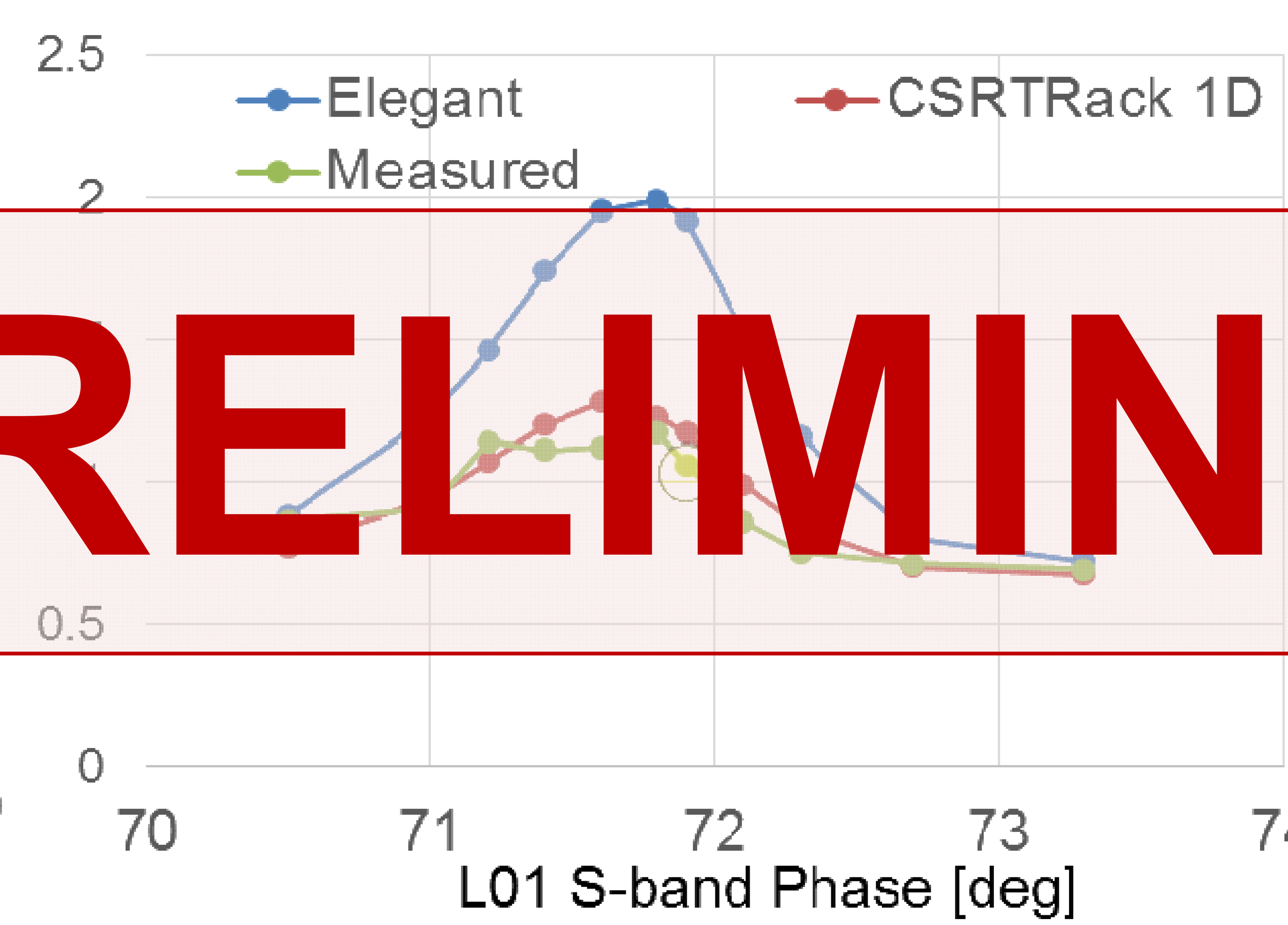


1-D codes agree at full compression even (??). 1-D approx. still seems to be good for $\sigma_\perp \approx (R\sigma_z{}^2)^{\frac{1}{3}}$ $(see\ next\ slide)$

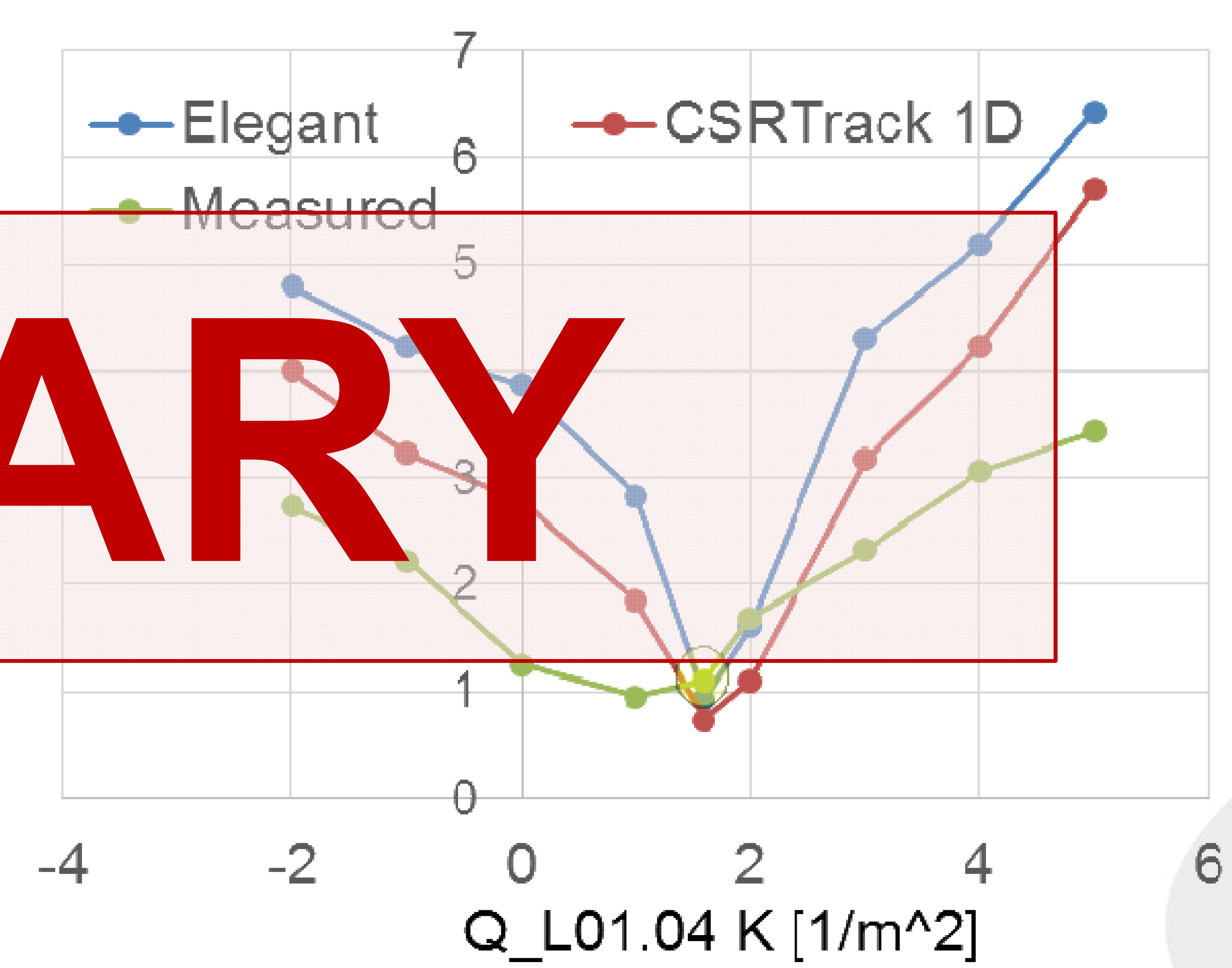
BC1 Angle Scan



Linac Phase Scan



Quadrupole Strength Scan



PRELIMINARY

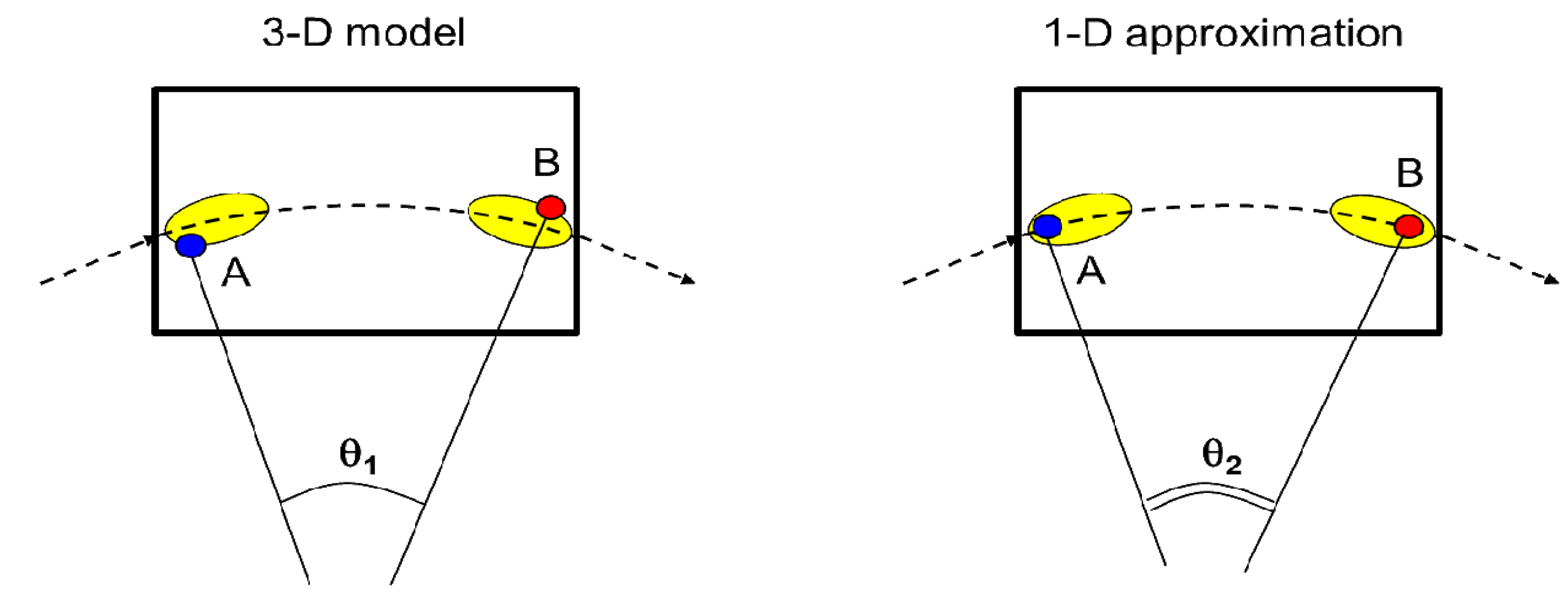
0.3 GeV, 0.7 nC, < 3 kA

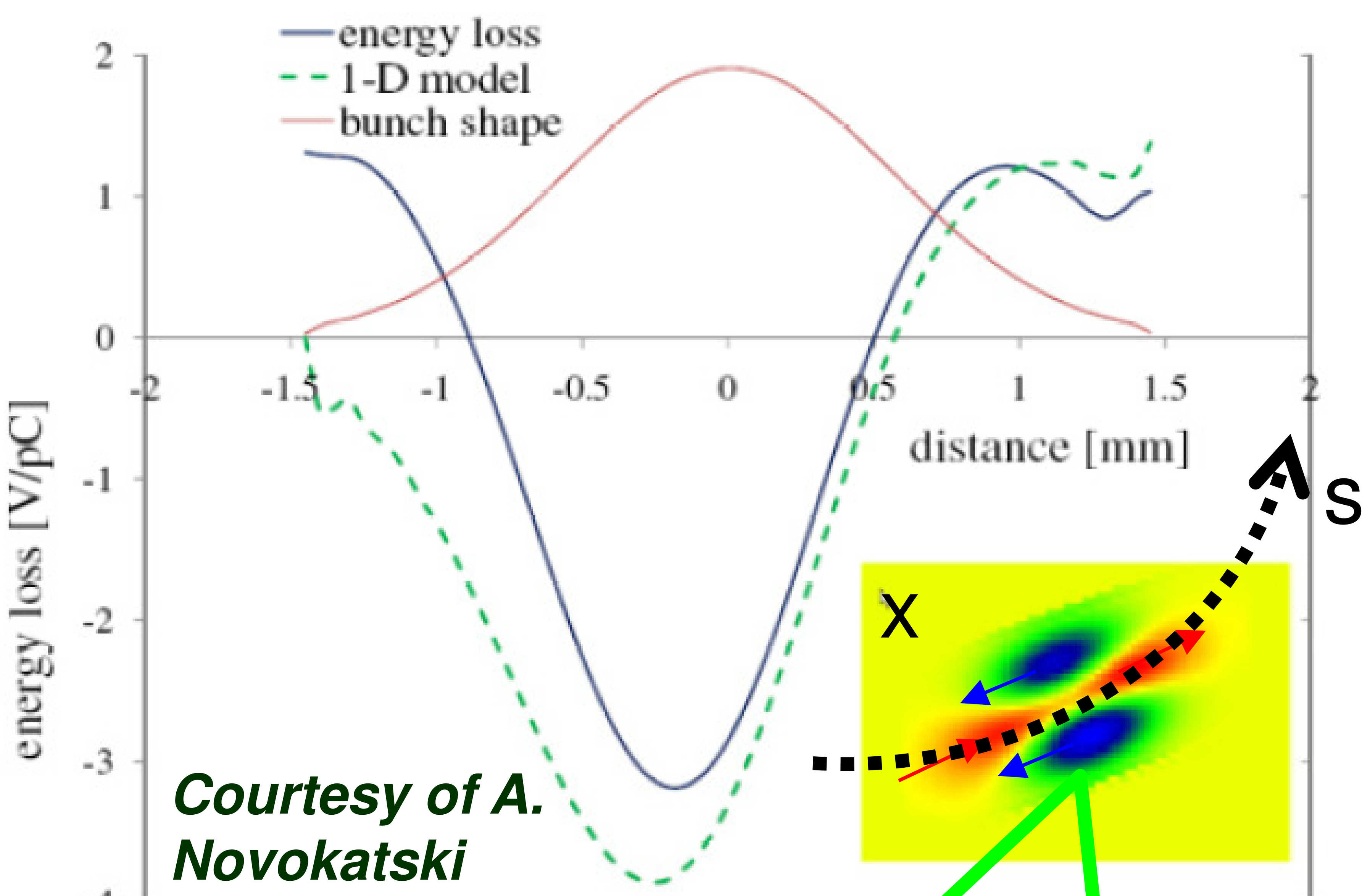
2,3-D Codes

H. Owen et al. for NLS (2008)

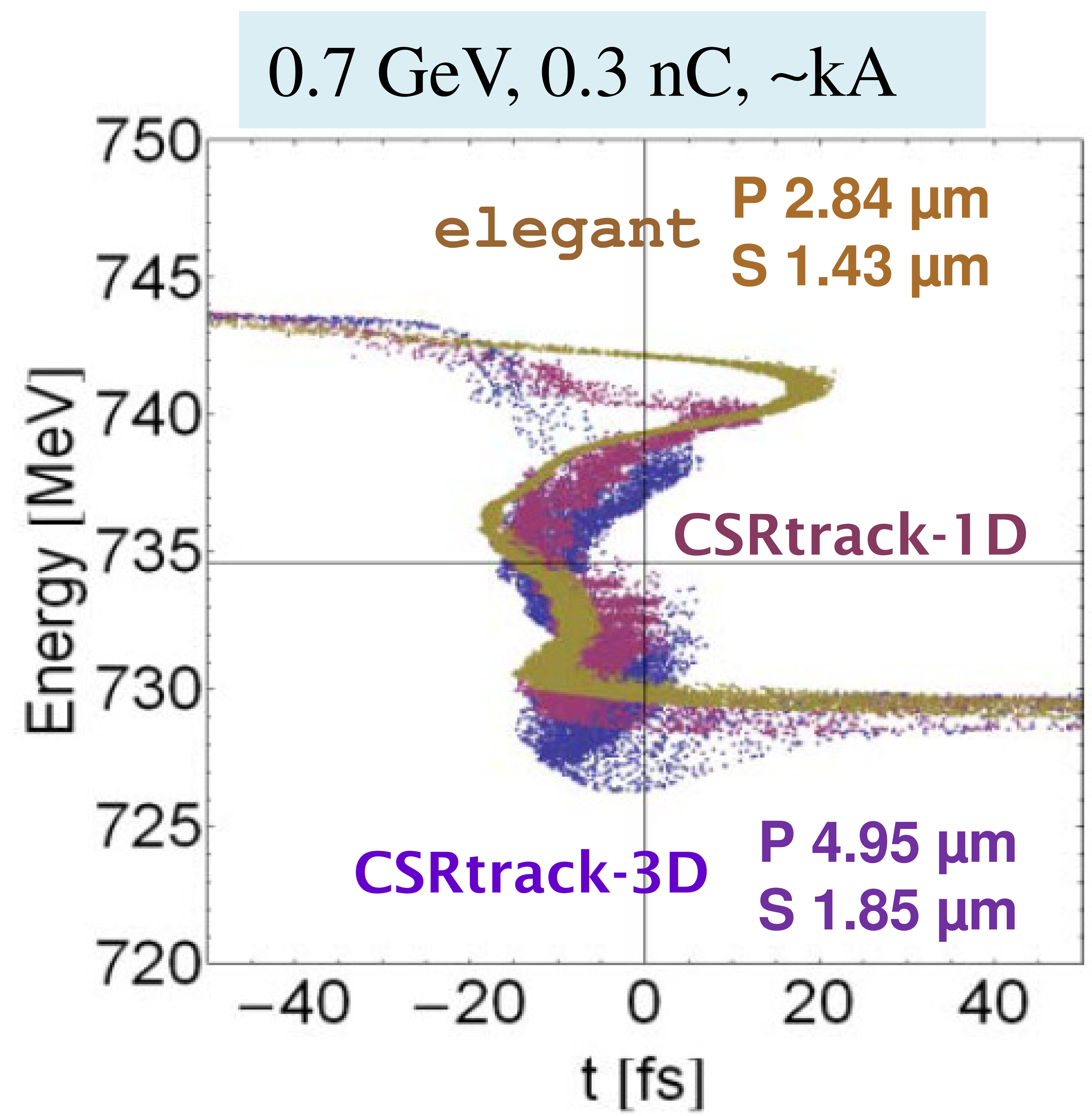
C. Prokop et al., NIM A 719 (2013)

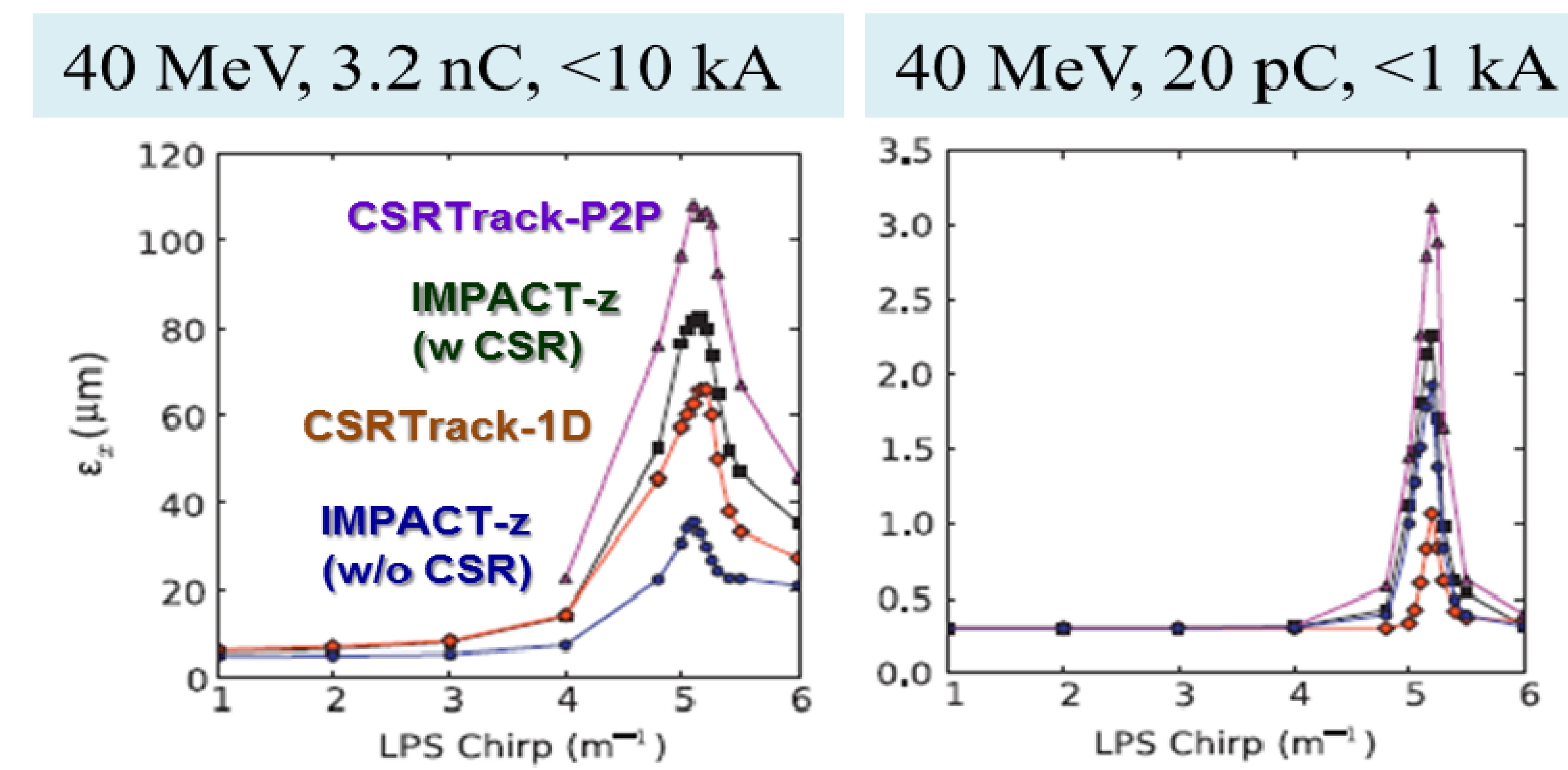
- 1-D model doesn't account for CSR field **radial dependence**:
  - Forcing $\theta_1 \cong \theta_2$ leads to the so-called *Derbenev criterion*: transverse effects become important when $\sigma_\perp \gg (R\sigma_z^2)^{1/3}$

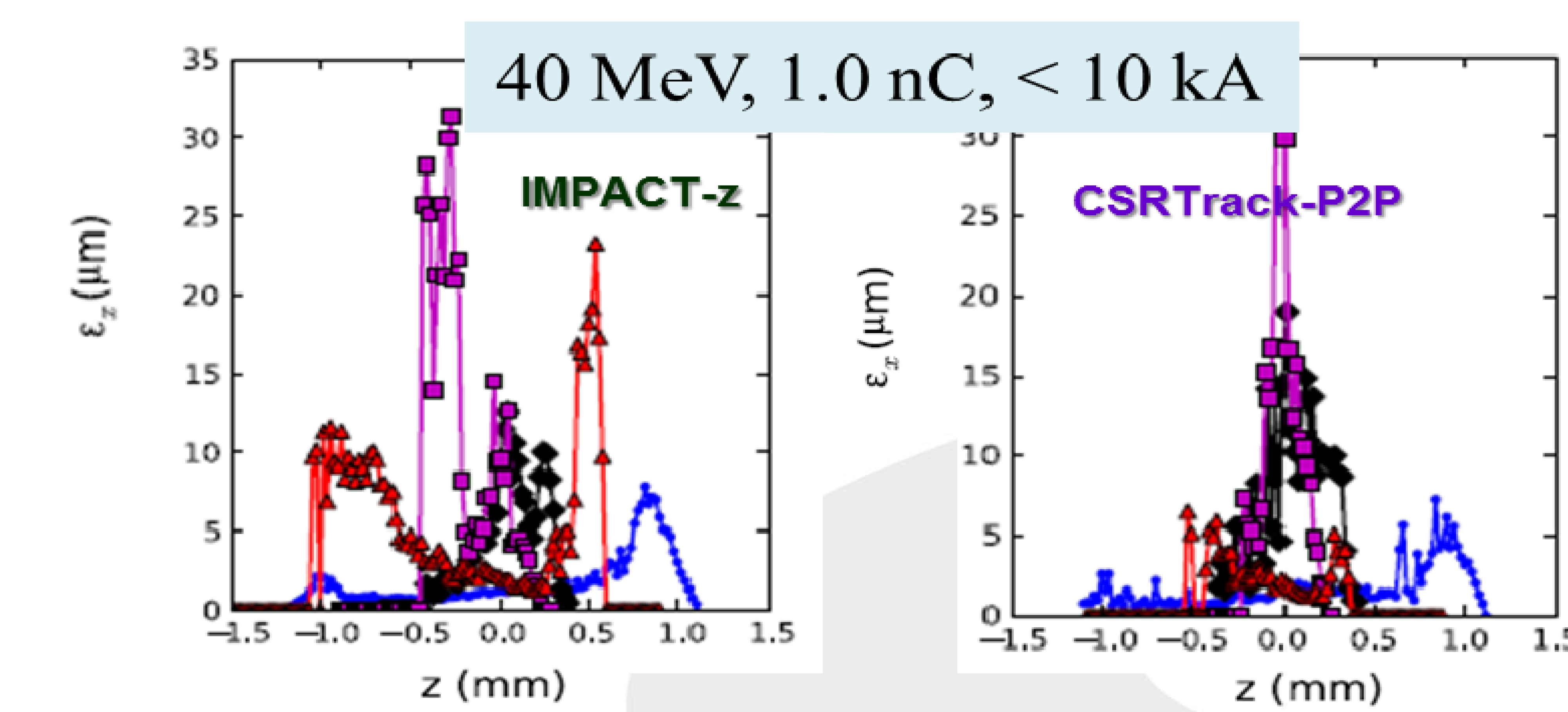




2-D CSR field modifies the beam energy distribution. Energy spread is correlated both along z and x.







S. Bettoni et al., PRAB 19, 034402 (2016)

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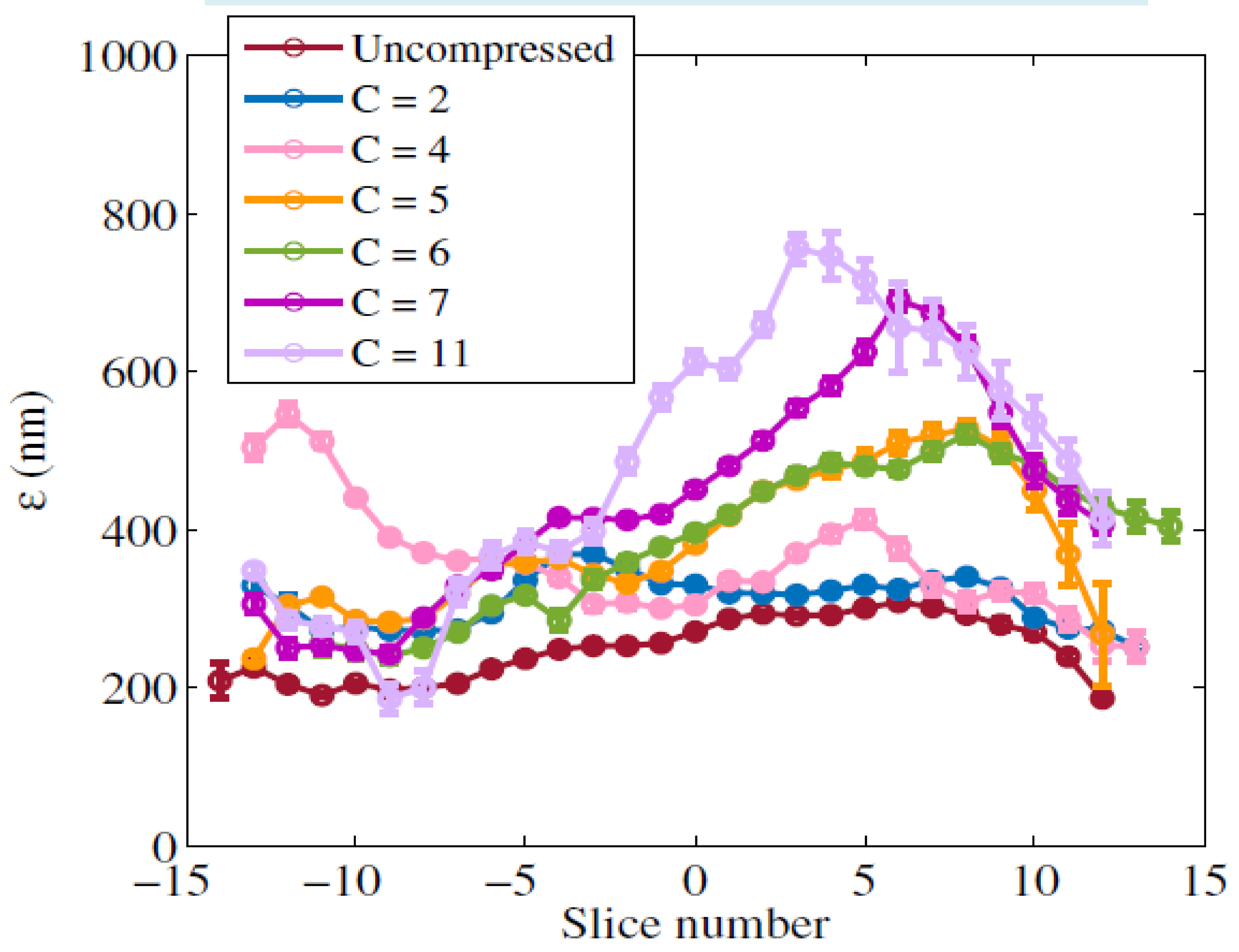
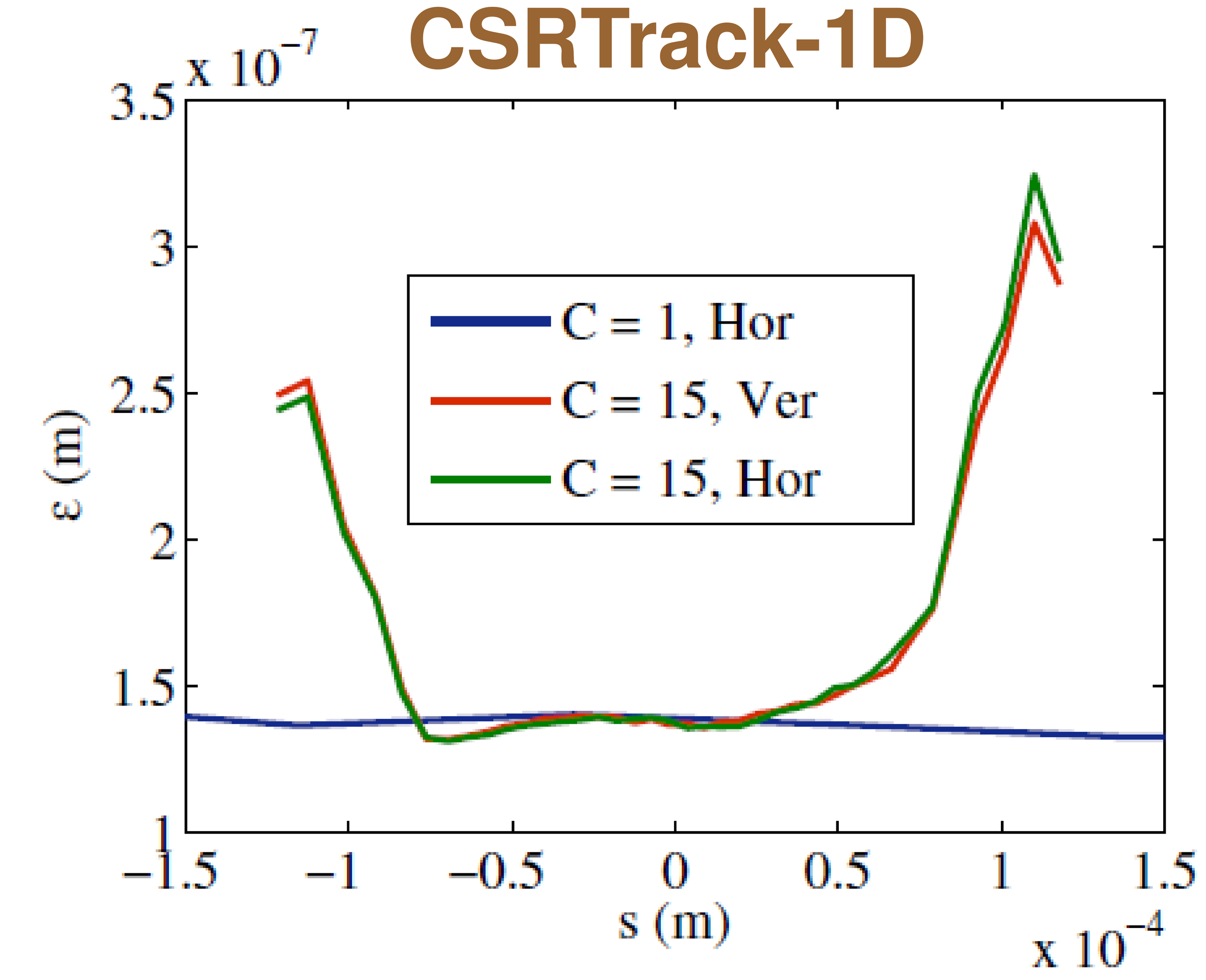
Slice Emittance

- Slice emittance growth becomes noticeable in ultra-low emittance beams.
  - 3-D effects are weakened in “pencil” beams…
  - …but charge density is higher (space charge forces ?)
  - Slice linear optics mismatch
  - Chromatic aberrations

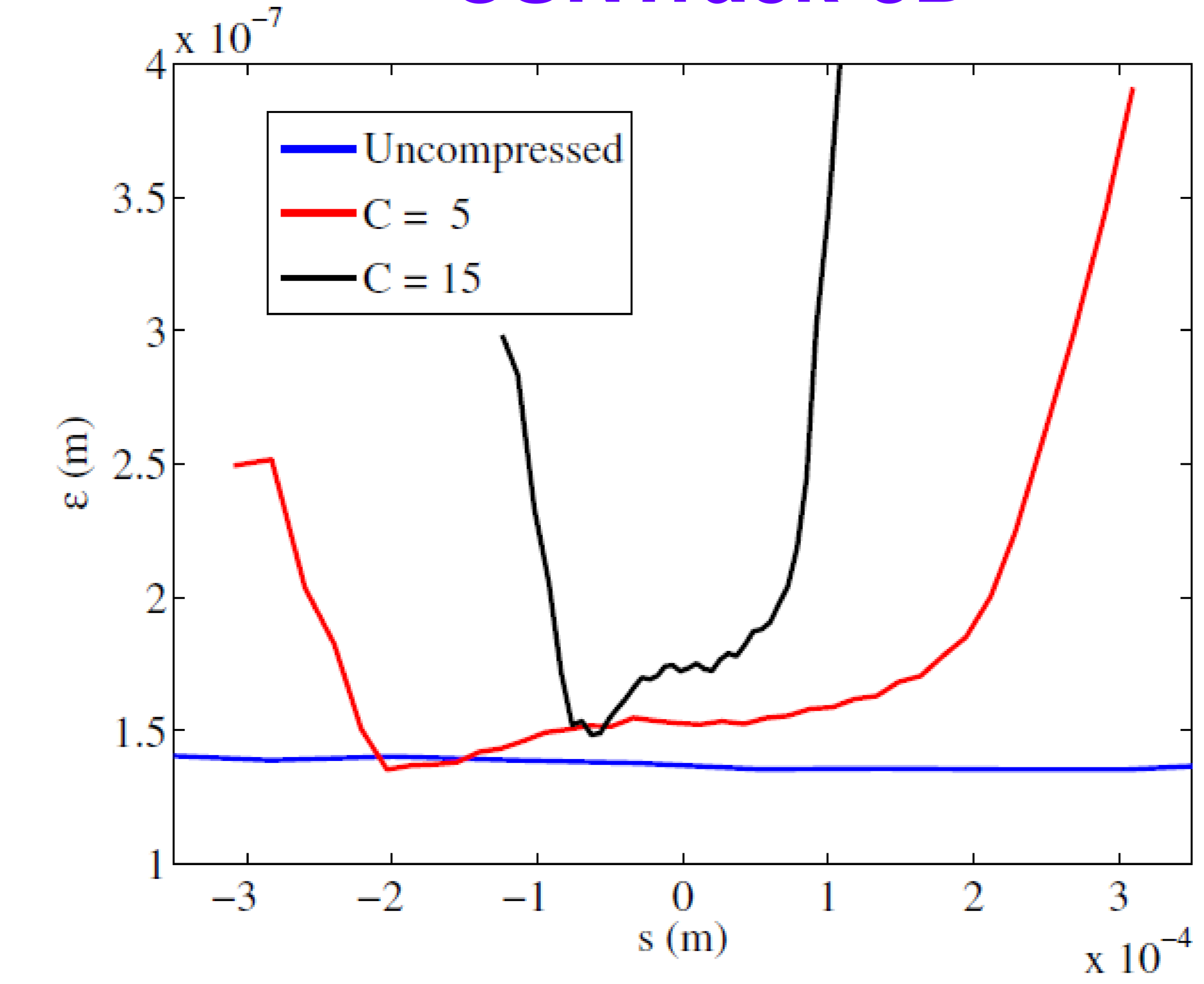
0.2 GeV, 0.2 nC, < 0.2 kA

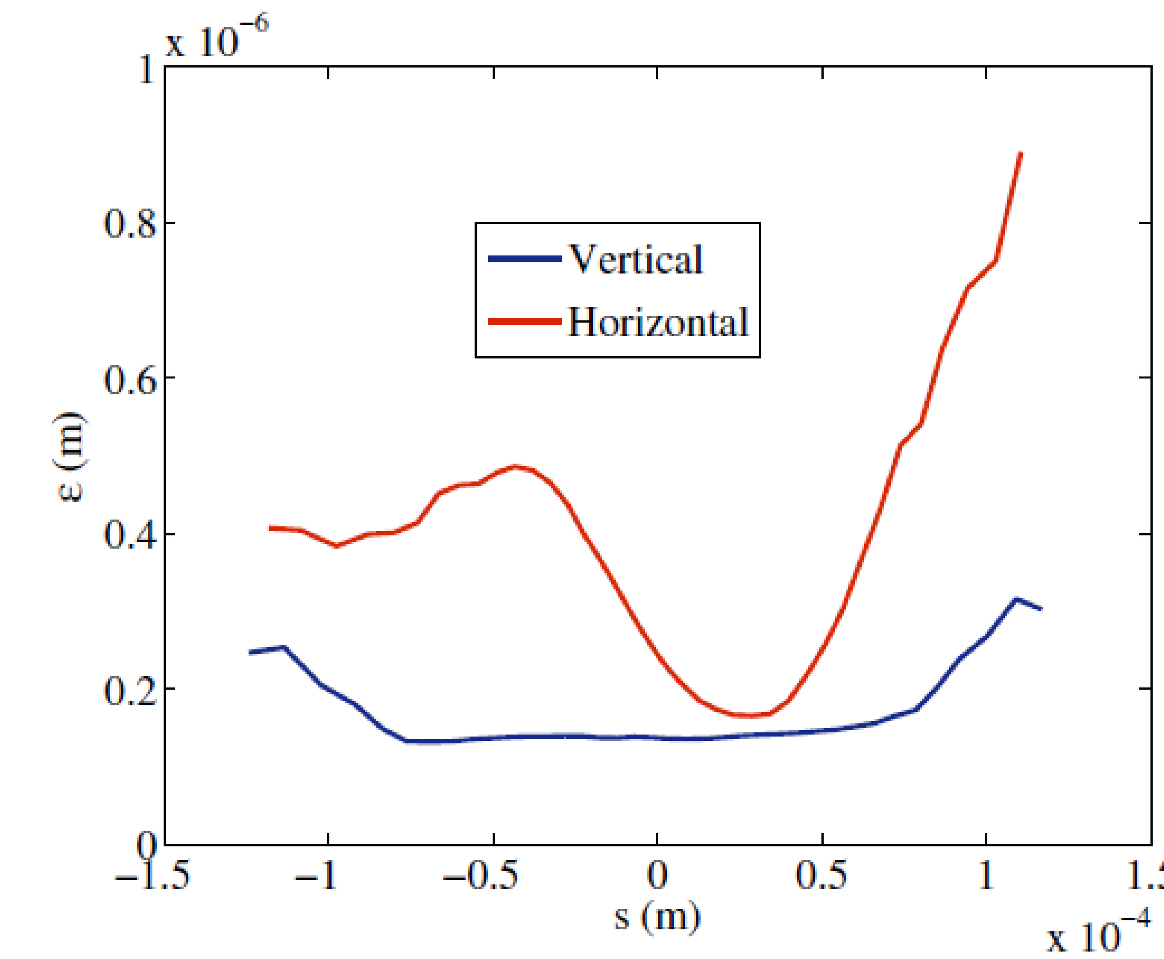


CSRTrack-1D



CSRTrack-3D





3D + Beam Optics Mismatch

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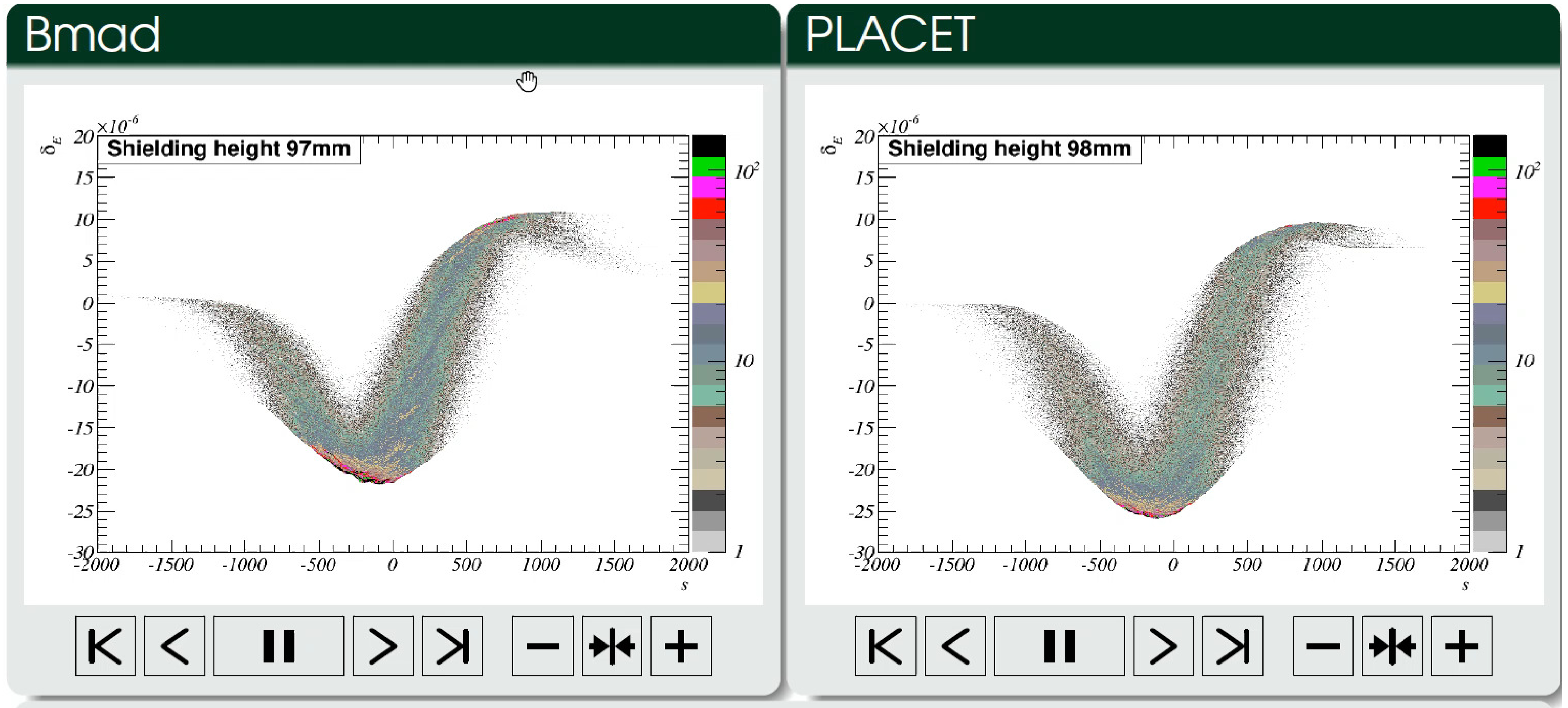
Shielding

- Shielding of CSR field would require pipe gap as small as < 2 mm or so.

J. Esberg et al. for CERN (2015)



- Without shielding, there is some discrepancy between Bmad and PLACET.
- PLACET with no shielding shows perfect agreement with ELEGANT (E. Adli).
- When decreasing the parallel plate distance, the shielding wake can start to interact with the tail of the bunch.
- Large difference between Bmad and new PLACET implementation for small plate separations.

D. Sagan et al. (2009)

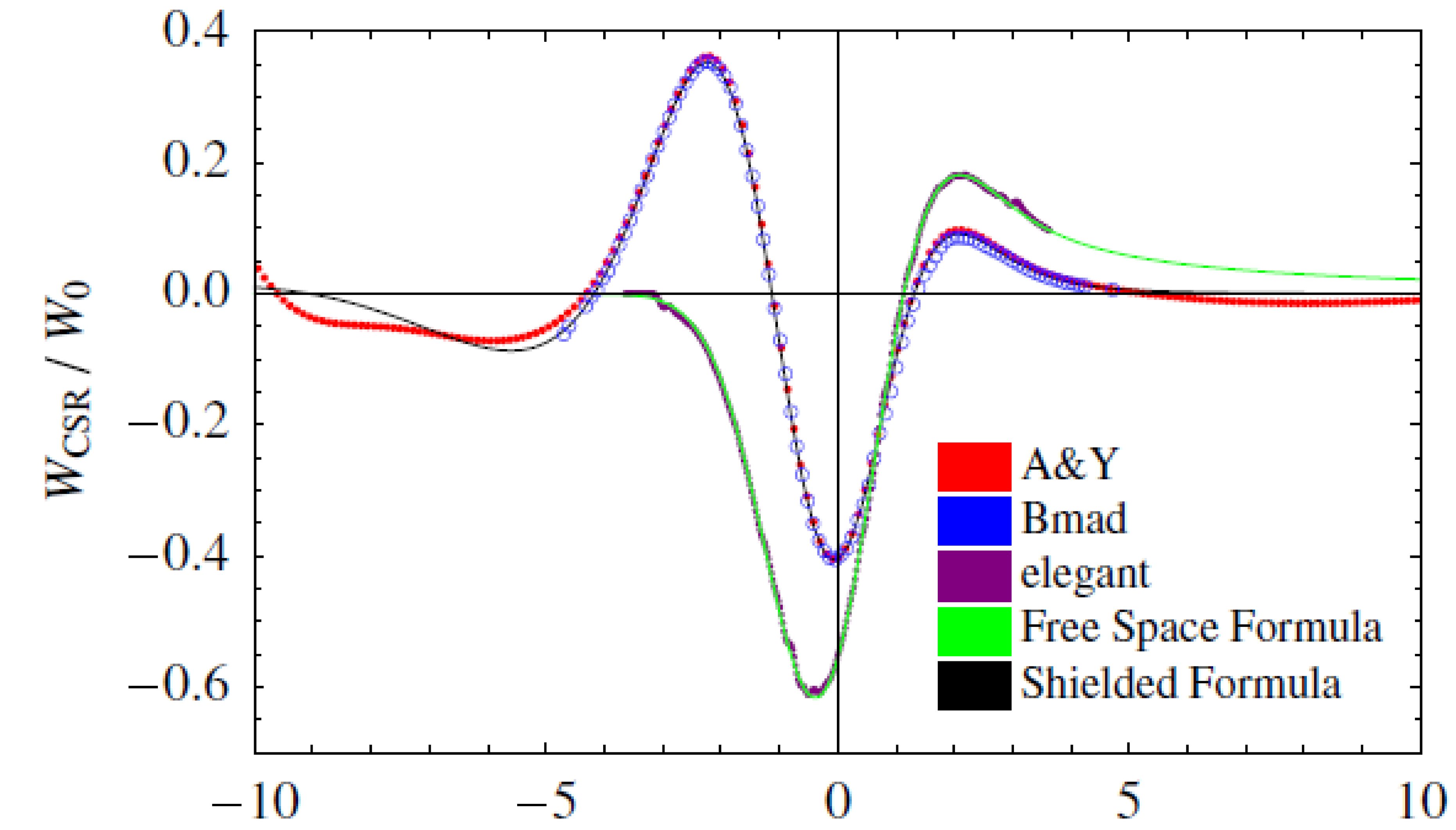
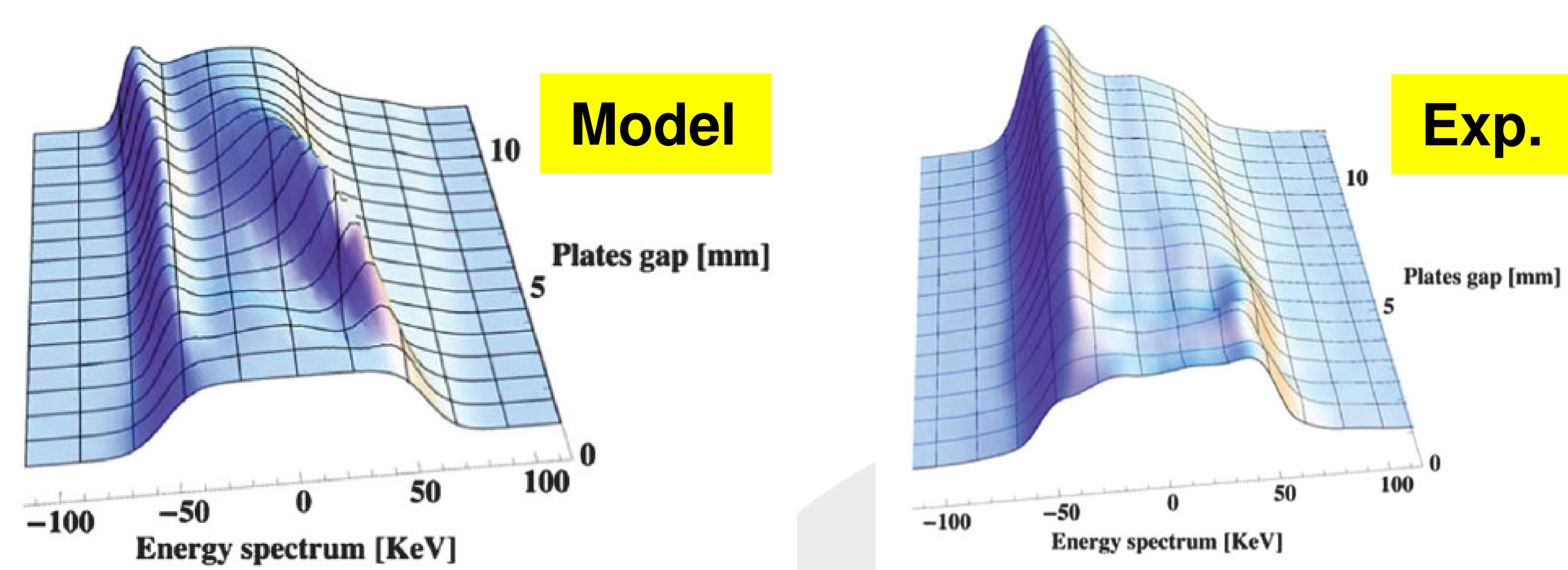


FIG. 14. (Color) Realistic magnets: Parameter set E (JLab TH2 magnet) line (top), set F (CESR analyzer magnet) (bottom). Bmad agrees with the CSR-wake formula Eq. (53) better than the other codes at the bunch tail.

V. Yakimenko et al. @ ATF (2012)



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CSR-Microbunching: Theory

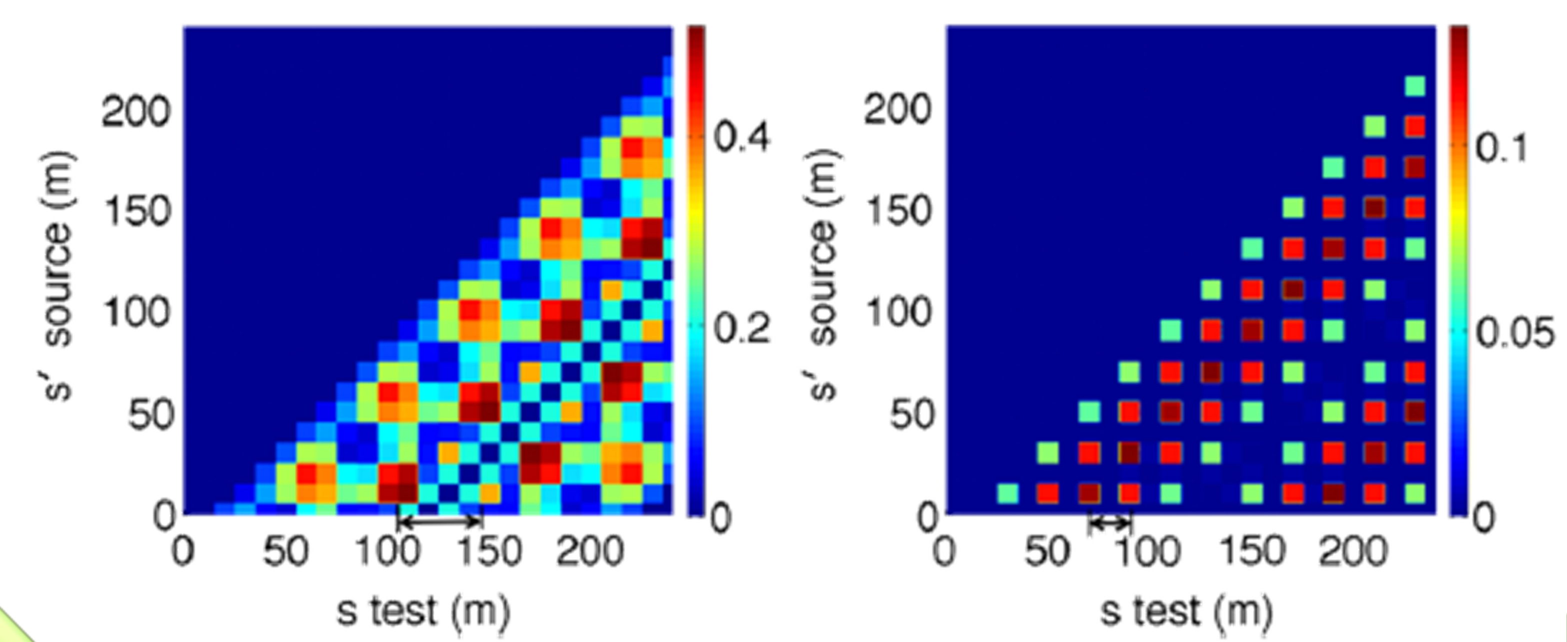
- CSR amplifies microbunching gain typically driven by longitudinal space charge force. In a multi-dipole line, CSR can even dominate the instability gain.
- Theoretical prescriptions on beam optics were found for minimizing both the CSR-induced **emittance growth** and **microbunching gain** (local isochronicity, π-phase advance, small betas, ecc.).

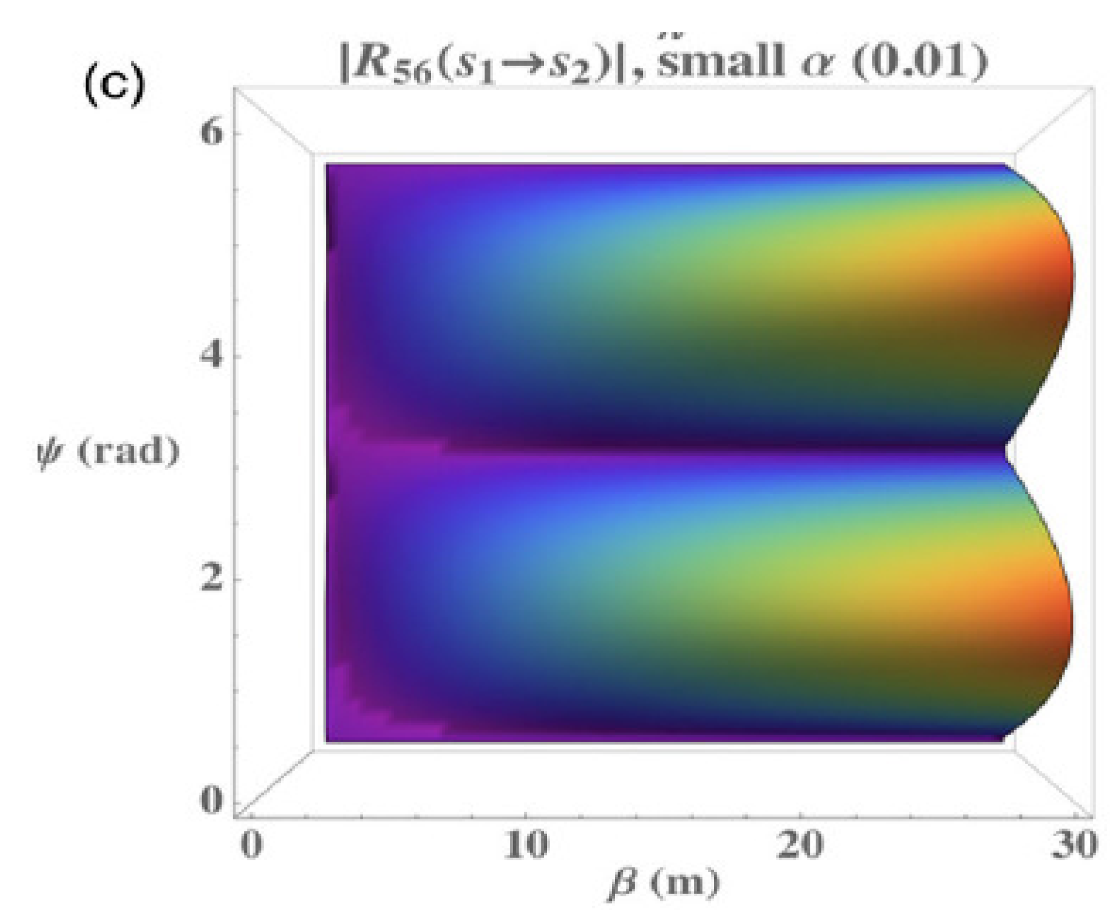
MERIT FUNCTION

$$\xi = \left| \max\{R_{56}^{s'\to s}\} \frac{k_0^{1/3}}{\rho^{2/3}} \Delta L \right|$$

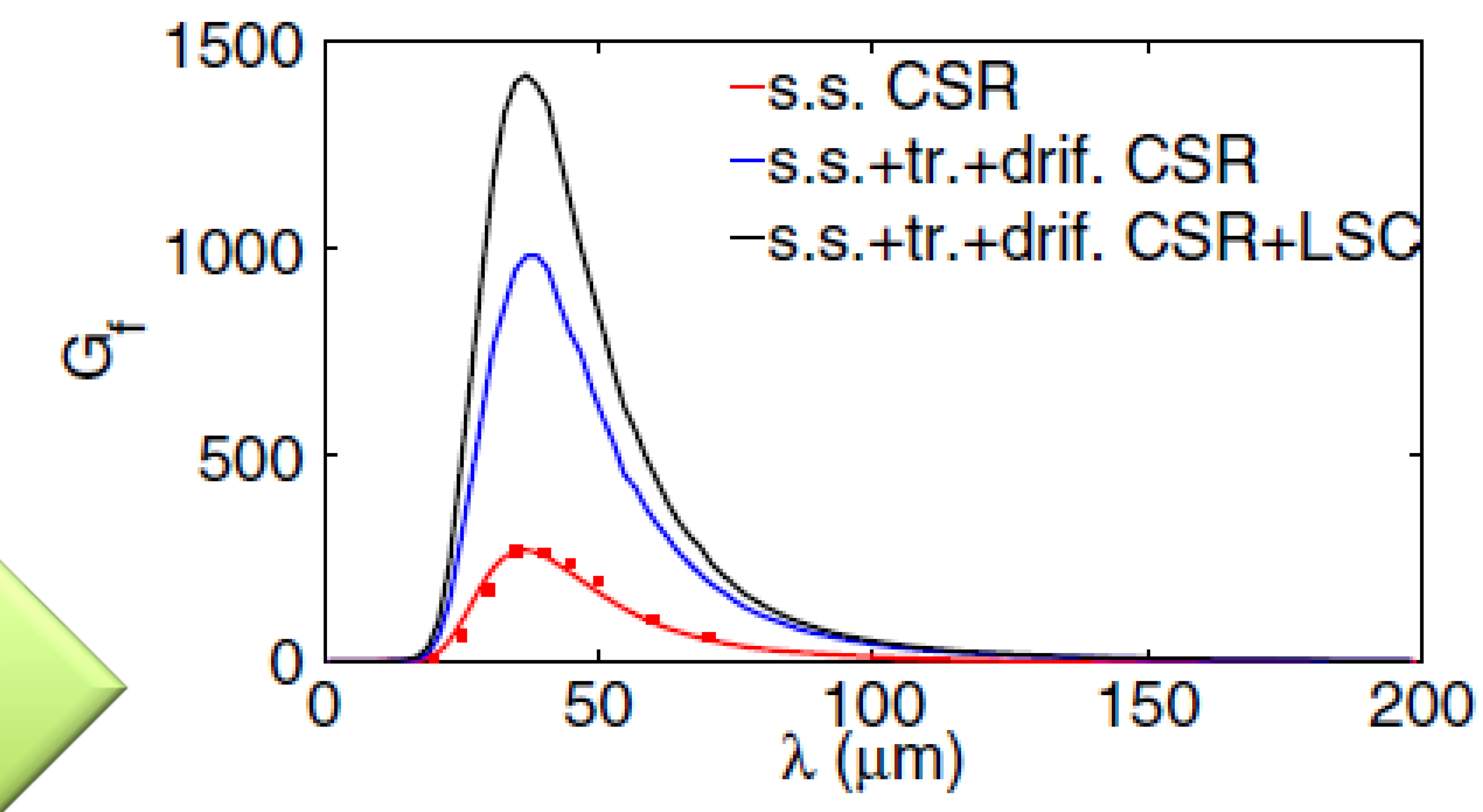
Bunching is proportional to this parameter when CSR only is considered.

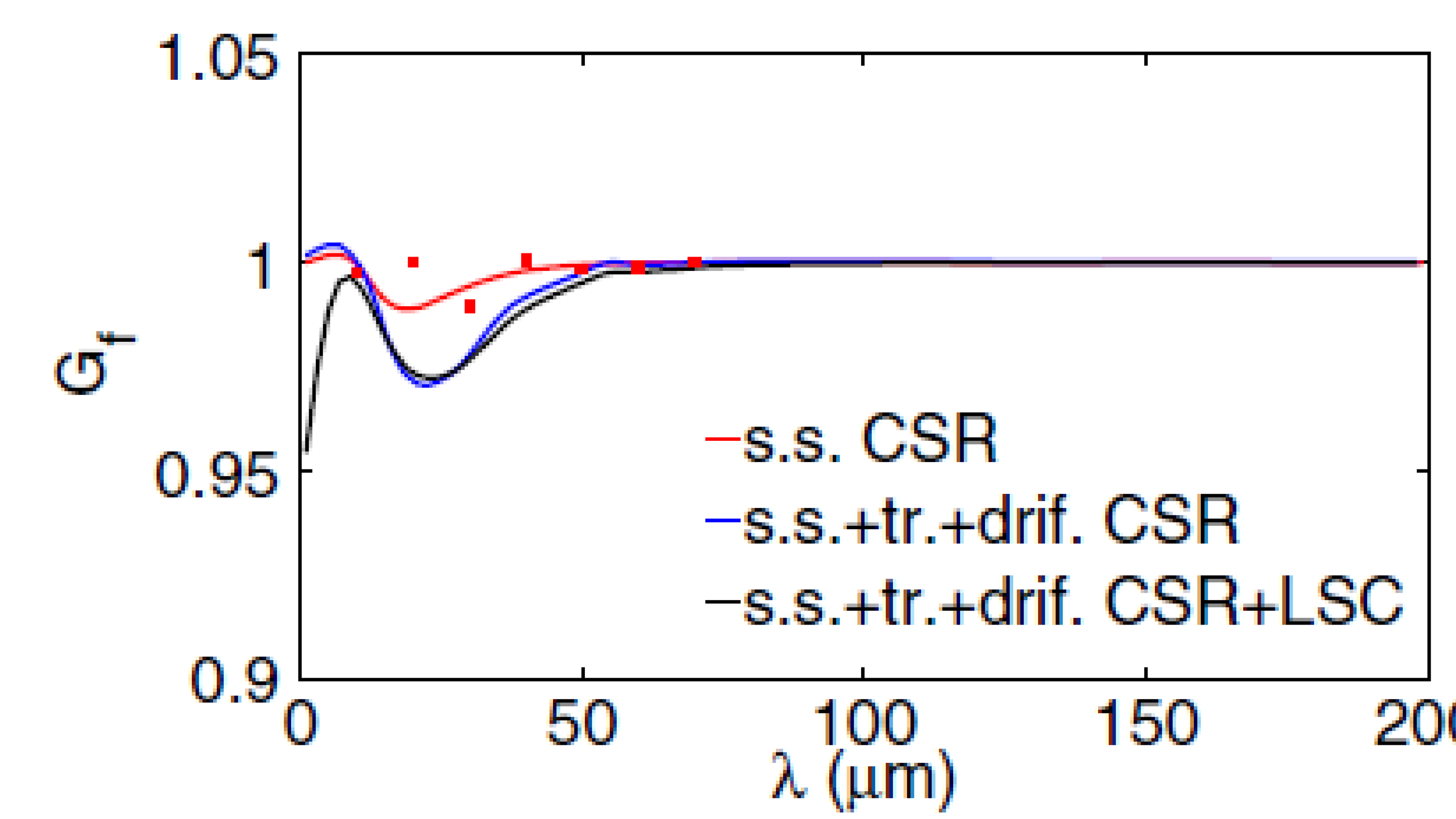
CALCULATE





DESIGN & SIMULATE





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Final Remarks

- **1-D steady-state analytical formulas** guide to the design of e-beam lines (chicanes, transfer lines, arcs).
  - $\sigma_\delta \sim 0.1\%$, $\Delta\varepsilon_{n,x} \sim 0.1$ μm accuracy of prediction for E > 300 MeV, σ_z > 10s μm, Q < 0.5 nC (I < kAs)
  - Control of CSR-induced emittance growth through beam optics is well-established
  - Predictions get worse at lower beam energies, and with multiple bends in long beam lines

- 1-D approximation seems to be ok for a relaxed **Derbenev criterion** even
  - 3-D effects are expected to raise at full compression
  - *Picture is unclear to me yet*

- Proposal of **research lines:**
  - Systematic investigation of 1-D vs. 3-D effects (codes benchmarking, experimental accuracy)
  - Validation of the CSR-driven microbunching gain vs. beam optics
  - Direct characterization of CSR-induced distortions in (z,E), (z,x) and (z,x') phase space.

Thank you for Your attention

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