Simulation of the Trickle Heating Effect

LCLS – Trickle Heating, Measurement and Theory
(SLAC-PUB-13854 Z. Huang et. al.)

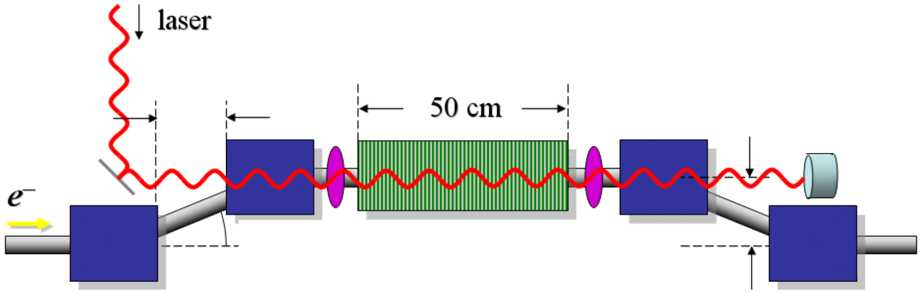
Poisson Solver for Periodic Micro Structures

LCLS – Trickle Heating, Simulation

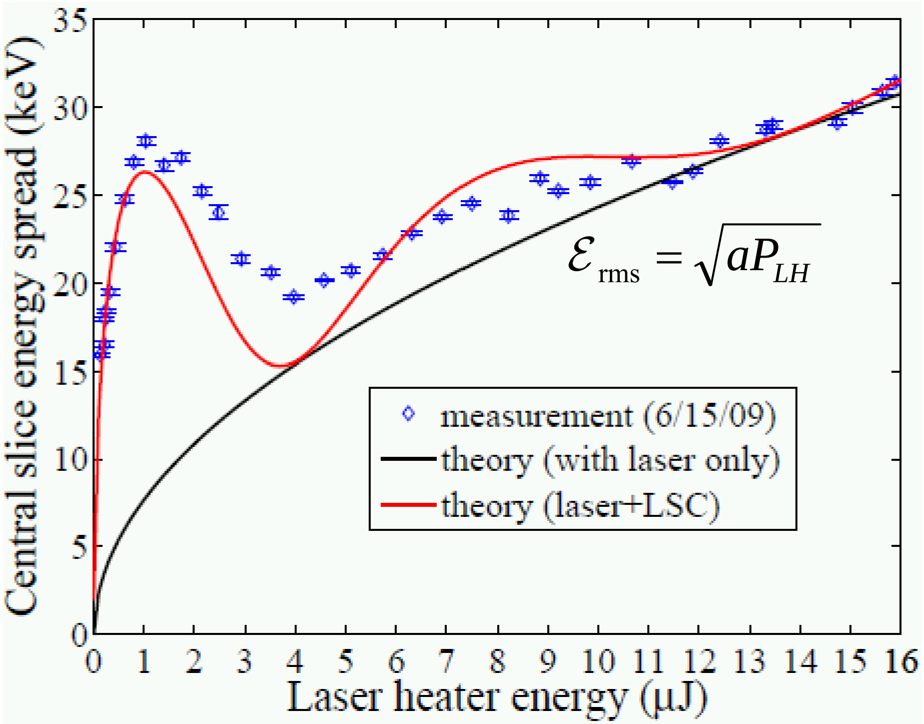
EuXFEL – Trickle Heating, Simulation

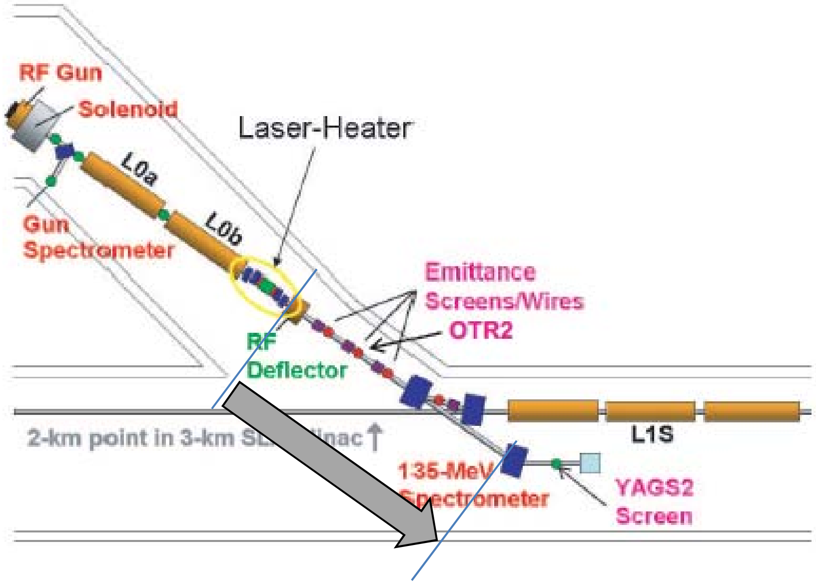
LCLS – Trickle Heating, Measurement and Theory

(SLAC-PUB-13854 Z. Huang et. al.)



it is induced energy modulation **after** the dogleg





(SLAC-PUB-13854 Z. Huang et. al.)

3D Impedance

$$E_z^{(3\mathrm{D})}(\mathbf{r}) = \frac{1}{4\pi\varepsilon}\int dV' \times \rho(\mathbf{r}')\frac{\gamma(z-z')}{\left((x-x')^2+(y-y')^2+\gamma^2(z-z')^2\right)^{3/2}}$$

Fourier transformation on axis:

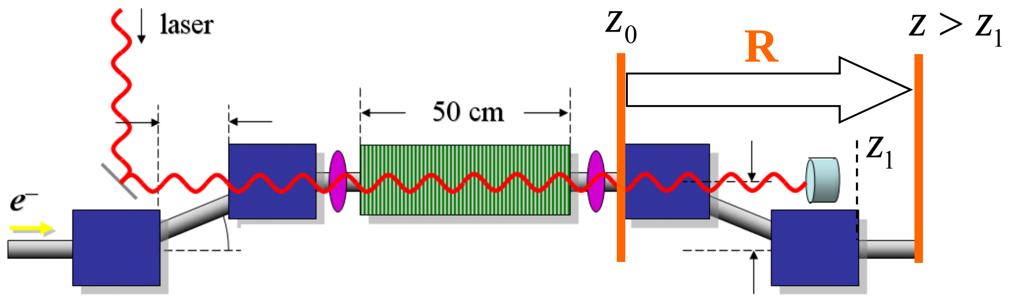
$$E_z(k) = \int dz \times E_z(z\mathbf{e}_z)\exp(-ikz)$$

$$E_z(k_0) = \frac{-ik_0}{2\pi\varepsilon_0\gamma^2\lambda_0}\int \underbrace{dxdydz \times \rho(x,y,z)}\, e^{-ik_0 z} K_0\left(\frac{k_0 r}{\gamma}\right)$$

with phase space:

$$dxdx'dydy'dzd\delta \times f(x,x',y,y',z,z')$$

(SLAC-PUB-13854 Z. Huang et. al.)



integration for (nominal) Gaussian transverse phase space (round beam):

$$E_z(k_0) \approx \frac{iI_0 Z_0}{2\pi k_0 \sigma_r^2} J_1(k_0 R_{56} \delta_L) \exp\left(-\tfrac{1}{2}(k_0 R_{56} \sigma_{\delta 0})^2\right) \exp\left(-\tfrac{\varepsilon}{2\beta}(k_0 R_{11} R_{52})^2\right) \frac{1}{1+\gamma^2 R^2}$$

for $k_0 \sigma_r / \gamma >> 1$ with $\varepsilon, \alpha, \beta$ initial Twiss parameters

$$R = \frac{R_{51} R_{11} \beta_{x0} + (R_{51} R_{12} + R_{52} R_{11}) \alpha_{x0} + R_{52} R_{12} \gamma_{x0}}{\beta_{x0} R_{11}^2 - 2\alpha_{x0} R_{11} R_{12} + \gamma_{x0} R_{12}^2}$$

induced energy modulation (round beam, on axis):

$$\mathcal{E}_{\mathrm{LSC}} = e \int E_z(k_0) dz$$

(SLAC-PUB-13854 Z. Huang et. al.)

$\delta_L = \mathcal{E}_L / \mathcal{E}$ amplitude of relative energy modulation in LH,
assumption: offset independent

$$\boxed{\mathcal{E}_{\mathrm{LSC}}} = J_1\left(k_0 R_{56} \boxed{\delta_L}\right) \times \boxed{\frac{2i\mathcal{E}_0}{k_0} \frac{I_0}{I_A} \int dz \times \frac{1}{\sigma_r^2} \exp\left(-\tfrac{\varepsilon}{2\beta}\left(k_0 R_{52} R_{11}\right)^2\right) \frac{1}{1+\gamma^2 R^2}}$$

optical function
assumptions: round beam, perturbation, $\sigma_{\delta 0} \to 0$, $\sigma_{\mathrm{L}} \to \mathrm{inf}$

$2\mathcal{E}_{\mathrm{LSC}}$ amplitude of induced energy modulation
on axis

total rms energy spread

$\mathcal{E}_{\mathrm{rms,before}}$ rms spread before LH

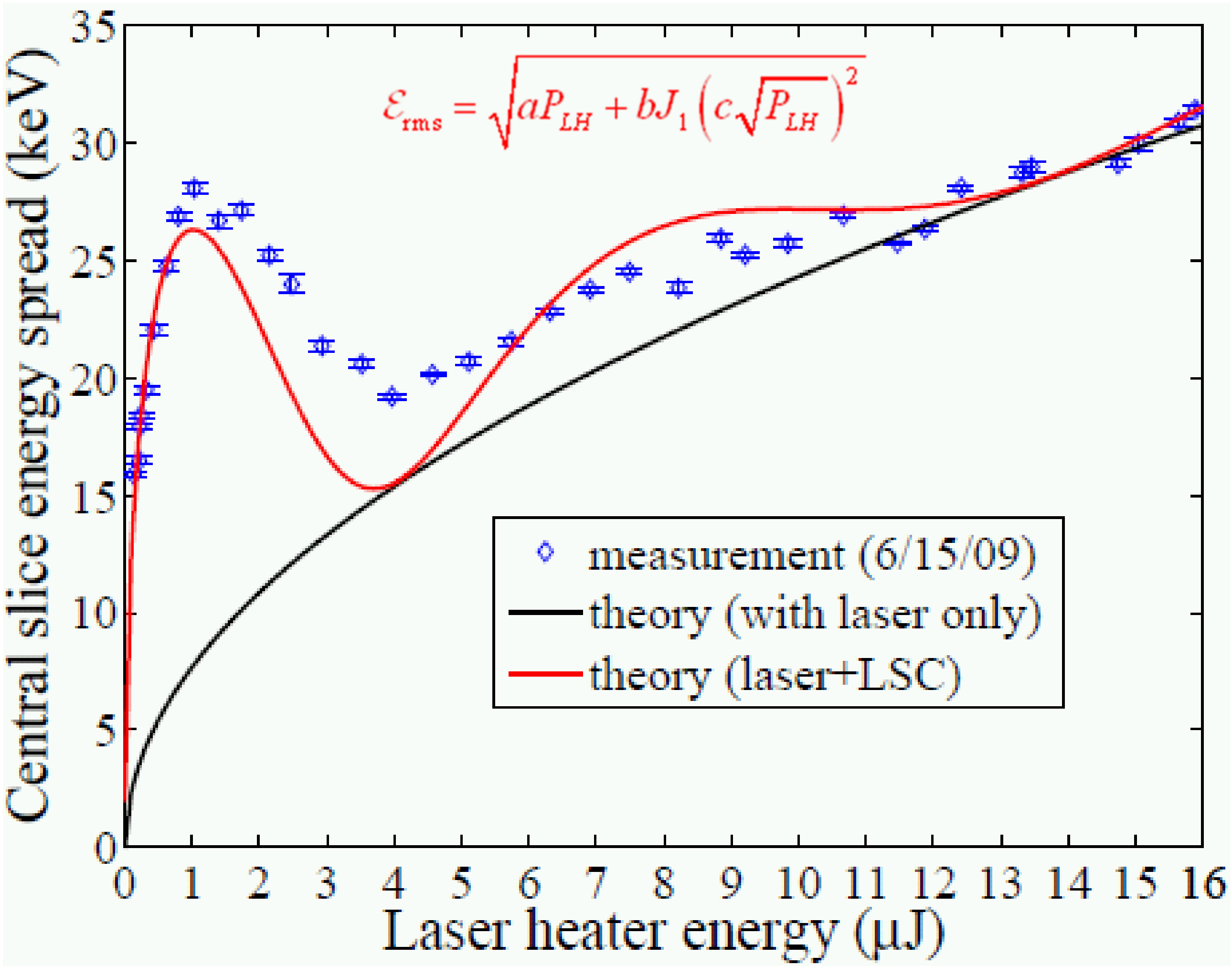
$\mathcal{E}_{\mathrm{rms,L}} = f_{\mathrm{L}} \mathcal{E}_{\mathrm{L}}$ rms spread induced by LH, with shape factor f_{L}

$\mathcal{E}_{\mathrm{rms,LSC}} = f_{\mathrm{LSC}} \mathcal{E}_{\mathrm{LSC}}$ rms spread induced by trickle heating, with shape factor f_{LSC}

$$\mathcal{E}_{\mathrm{rms}} = \sqrt{\mathcal{E}^2_{\mathrm{rms,before}} + \left(\mathcal{E}^2_{\mathrm{rms,L}} \boxed{+} \mathcal{E}^2_{\mathrm{rms,LSC}}\right)}$$

if these effects are uncorrelated!

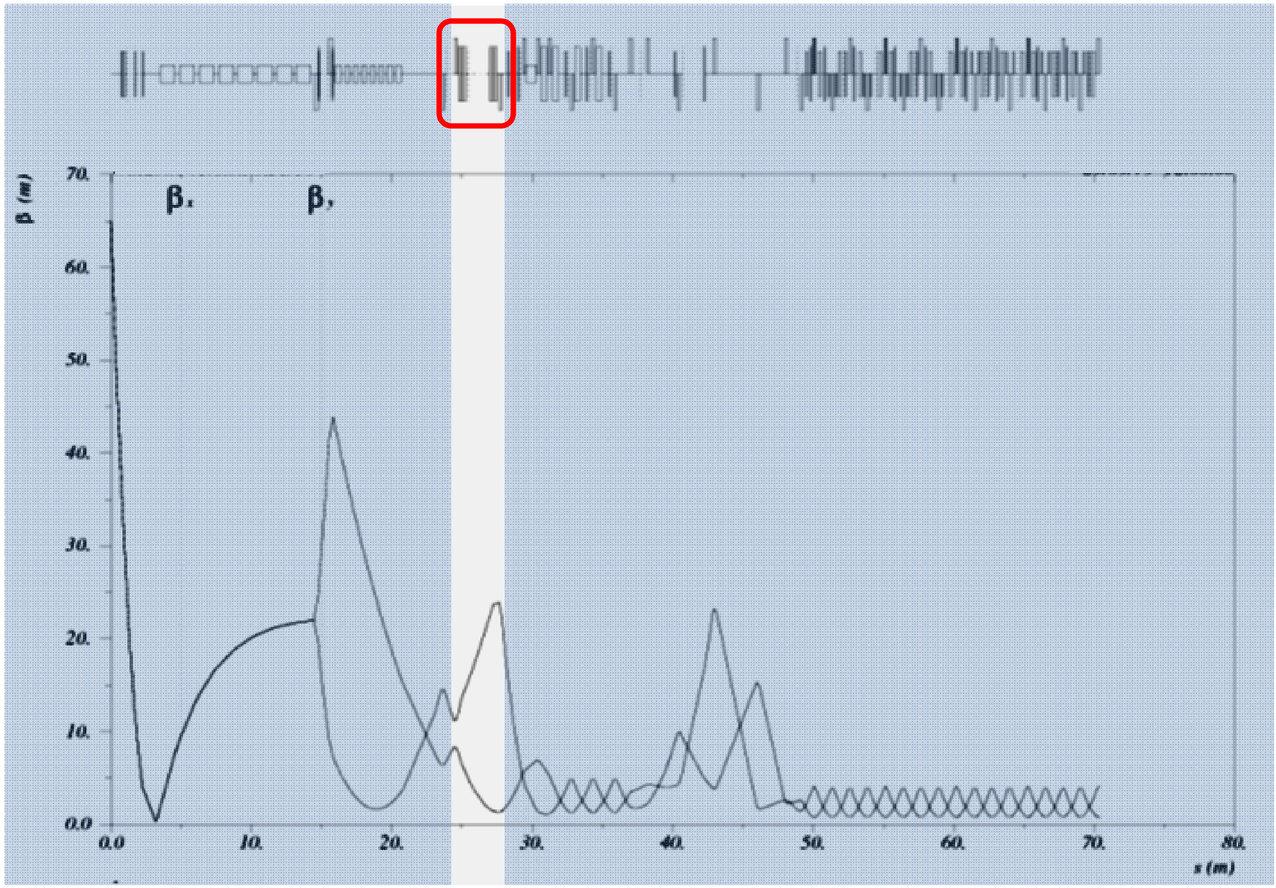
(SLAC-PUB-13854 Z. Huang et. al.)



(SLAC-PUB-13854 Z. Huang et. al.)

EuXFEL Laser Heater

beam optics



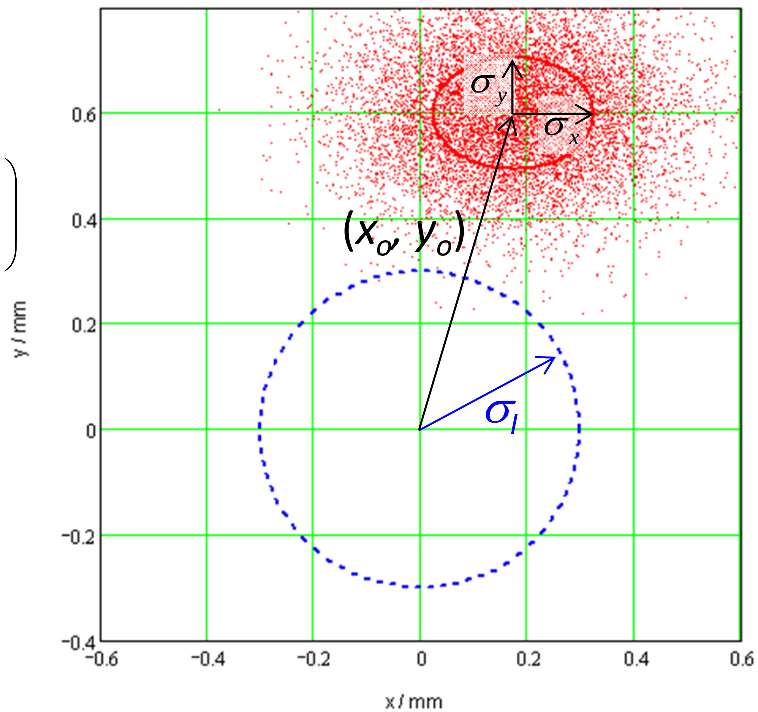
beam is **not** round

$\mathcal{E}_{LH} \approx$ 130 MeV, already with chirp $\delta\mathcal{E}_{rms}/\mathcal{E}_{LH} \approx 1.4\%$

non-axial overlap (photon-electron)

particle beam: $P_{xy}(x,y)=\frac{1}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{1}{2}\left(\frac{x-x_o}{\sigma_x}\right)^2-\frac{1}{2}\left(\frac{y-y_o}{\sigma_y}\right)^2\right)$

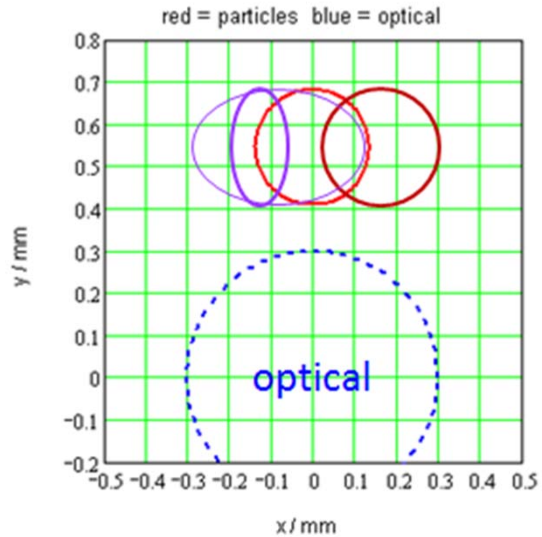
laser beam: $\|\mathbf{E}(r,0)\|^2=E_{x0}^2\exp\left(-\frac{1}{2}\left(\frac{r}{\sigma_l}\right)^2\right)$



EuXFEL: particle beam is vertically shifted

better spectrum
insensitive to horizontal offset
more freedom for optics

but more laser power needed
heating is non uniform vs. cross-section
needs 3D analysis of parasitic effects



Poisson Solver

the full (non-periodic) problem
(LCLS case)

mesh-lines $N_z \approx \frac{6\sigma_z}{\lambda_{LH}/20} \approx \frac{6 \times 1\,\mathrm{mm}}{760\,\mathrm{nm}/20} \approx 2 \cdot 10^5$ $\quad N_{x,y} \approx \frac{1}{\gamma} \frac{6\sigma_{x,y}}{\lambda_{LH}/20} \approx 70$

particles $N_p \propto \frac{Q_{tot}}{e} \propto 10^9$

is possible; has been done
scans are **time consuming**!

trick 1: reduce bunch length

increasing macro effects
distinguish from micro effects!

trick 2: solve periodic problem

$$N_z \approx 20$$

$$N_p \propto \frac{\hat{I}\lambda_{LH}}{ec} \propto 10^6$$

fast even on single CPU
better resolution possible

Poisson Solver for Periodic Micro Structures

Lorentz transformation

↓

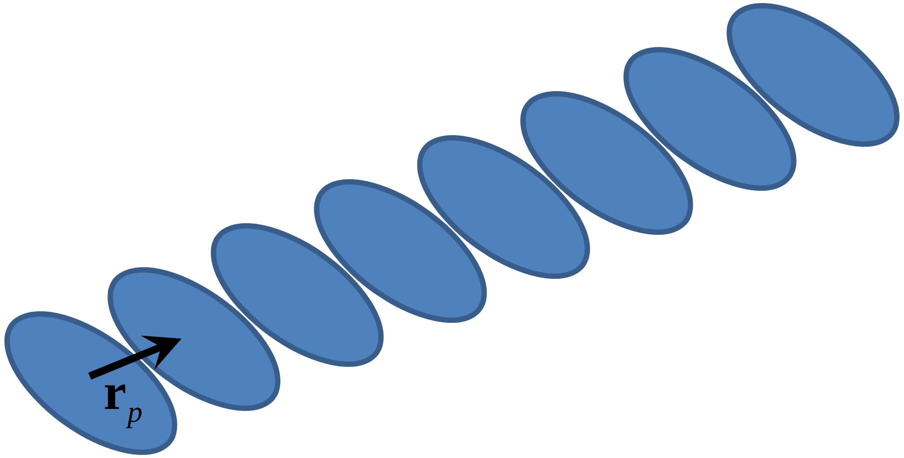
electrostatic problem

Green's function $\mathbf{G}(\mathbf{r}-\mathbf{r}')$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int \rho(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{\left\|\mathbf{r}-\mathbf{r}'\right\|^3}dV'$$

periodic source distribution

$$\rho(\mathbf{r}) = \sum_{n=-\infty}^{\infty}\rho_p(\mathbf{r}-n\mathbf{r}_p)$$



$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int \sum_{n=-\infty}^{\infty}\rho_p(\mathbf{r}'-n\mathbf{r}_p)\frac{\mathbf{r}-\mathbf{r}'}{\left\|\mathbf{r}-\mathbf{r}'\right\|^3}dV' = \frac{1}{4\pi\varepsilon_0}\int \rho_p(\mathbf{r}')\sum_{n=-\infty}^{\infty}\frac{\mathbf{r}-n\mathbf{r}_p-\mathbf{r}'}{\left\|\mathbf{r}-n\mathbf{r}_p-\mathbf{r}'\right\|^3}dV'$$

periodic Green's function $\mathbf{G}_p(\mathbf{r}-\mathbf{r}', \mathbf{r}_p)$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int \rho_p(\mathbf{r}')\mathbf{G}_p(\mathbf{r}-\mathbf{r}',\mathbf{r}_p)dV'$$

implementation: particle-mesh method $\rightarrow \rho_p(\mathbf{r})$

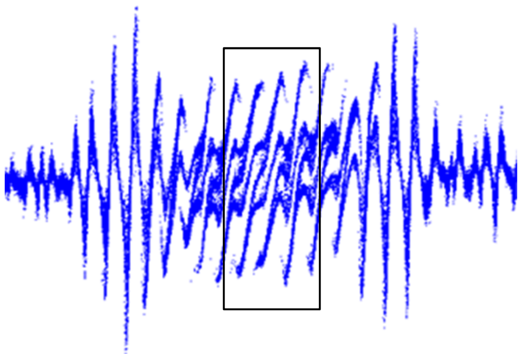
fast convolution (with scalar Green's function) $\rightarrow V(\mathbf{r})$

differentiation (on mesh) $\rightarrow \mathbf{E}(\mathbf{r})$

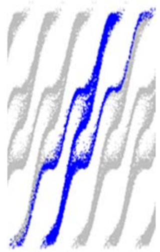
interpolation $\rightarrow \mathbf{E}(\mathbf{r}_\nu)$

example (tracking $\rightarrow$ longitudinal phase space):

full model



periodic model



see session Tu-2: C. Lechner, K. Hacker

LCLS – Trickle Heating, Simulation

beam and setup parameters

from **Suppression of microbunching instability in the linac coherent light source**

Z. Huang,[1,*] M. Borland,[2] P. Emma,[1] J. Wu,[1] C. Limborg,[1] G. Stupakov,[1] and J. Welch[1]

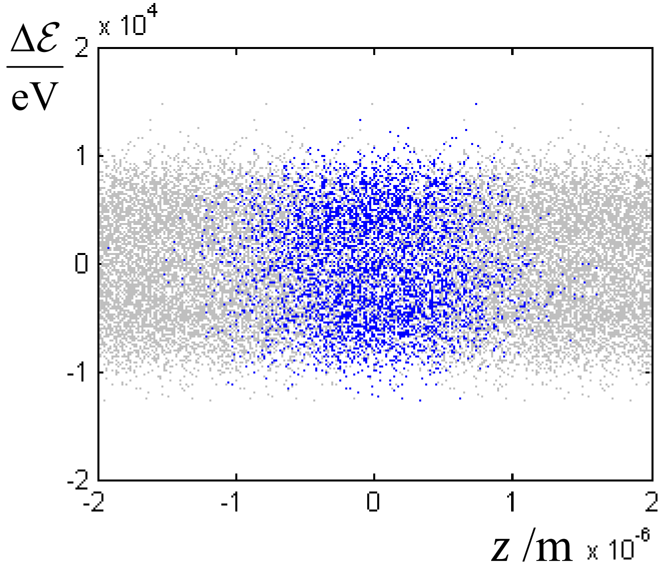
TABLE II. Main parameters for the LCLS laser heater.

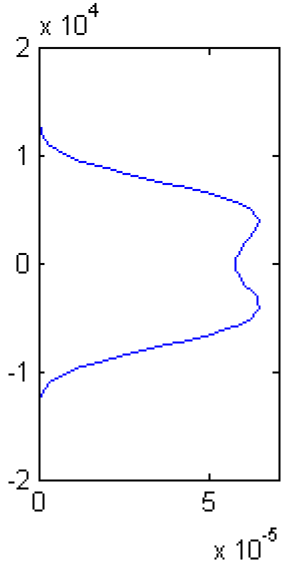
Parameter	Symbol	Value
Electron energy	$\gamma_0 mc^2$	135 MeV
Average beta function	$\beta_{x,y}$	10 m
Transverse rms e-beam size	$\sigma_{x,y}$	190 μm
Undulator period	λ_u	0.05 m
Undulator field	B	0.33 T
Undulator parameter	K	1.56
Undulator length	L_u	0.5 m
Laser wavelength	λ_L	800 nm
Laser rms spot size	σ_r	175 μm (1.5 mm)
Laser peak power	P_L	1.2 MW (37 MW)
Rayleigh range	Z_R	0.5 m (35 m)
Maximum energy modulation	$\Delta\gamma_L(0)mc^2$	80 keV (55 keV)
rms heater-induced local energy spread	$\sigma_{\gamma_L} mc^2$	40 keV

q = 250 pC

numerical parameters

period	800 nm (in z-direction)
particles/period	1E6
longitudinal mesh, dz	800 nm / 50 = 16 nm
transverse mesh	γ dz = 4 µm (about 380 lines)
cpu time	5 min

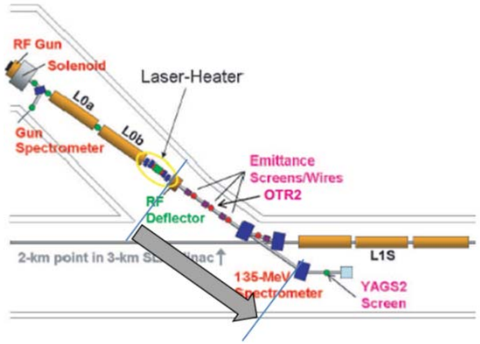


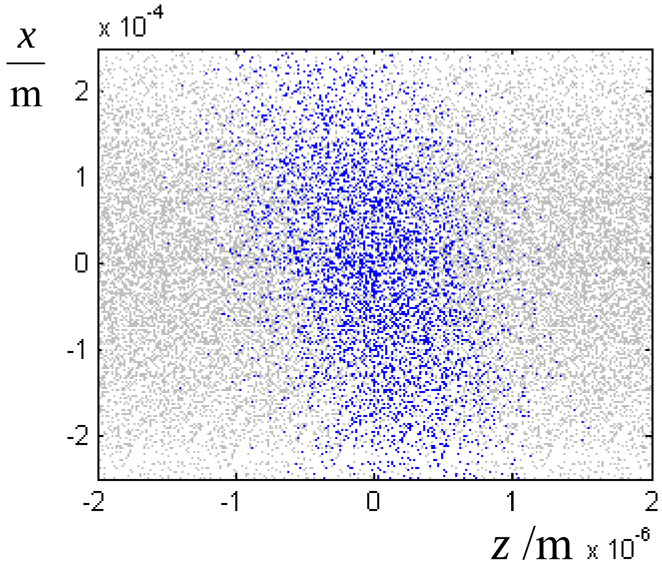


rms energy-spread

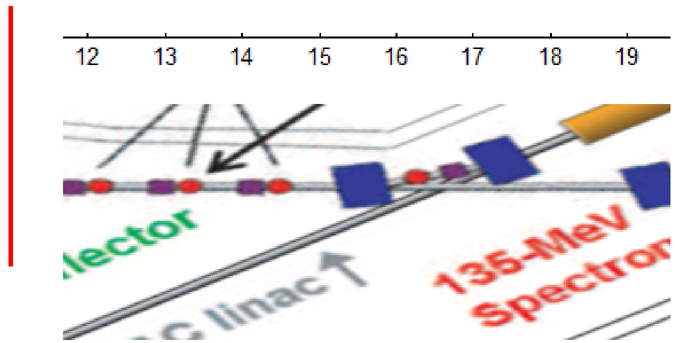
= 2.0 keV before LH

= 5.0 keV after LH undulator

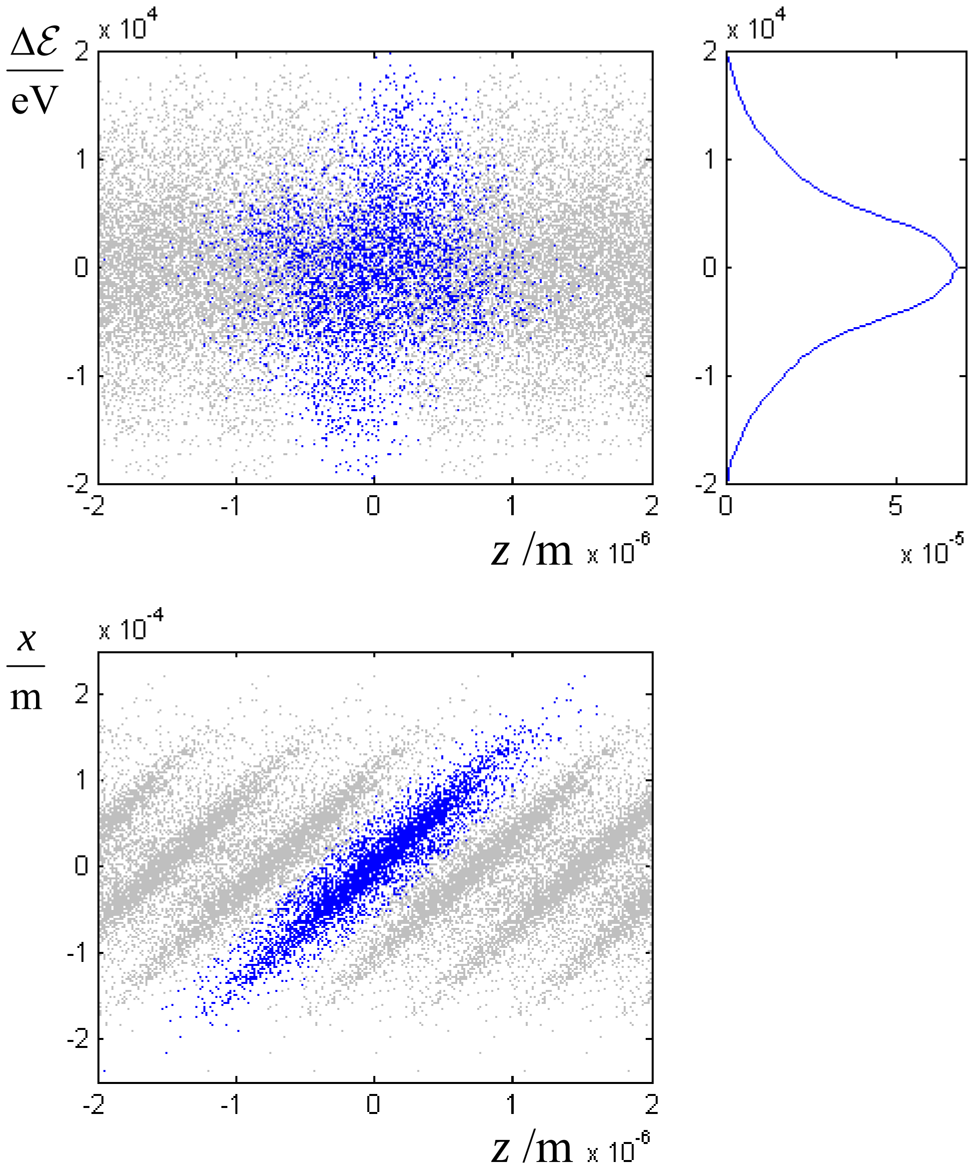




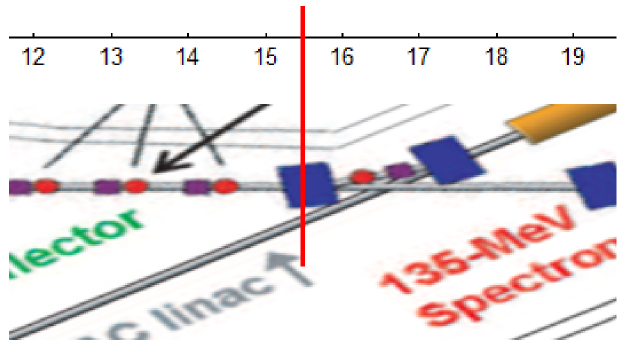
11 m

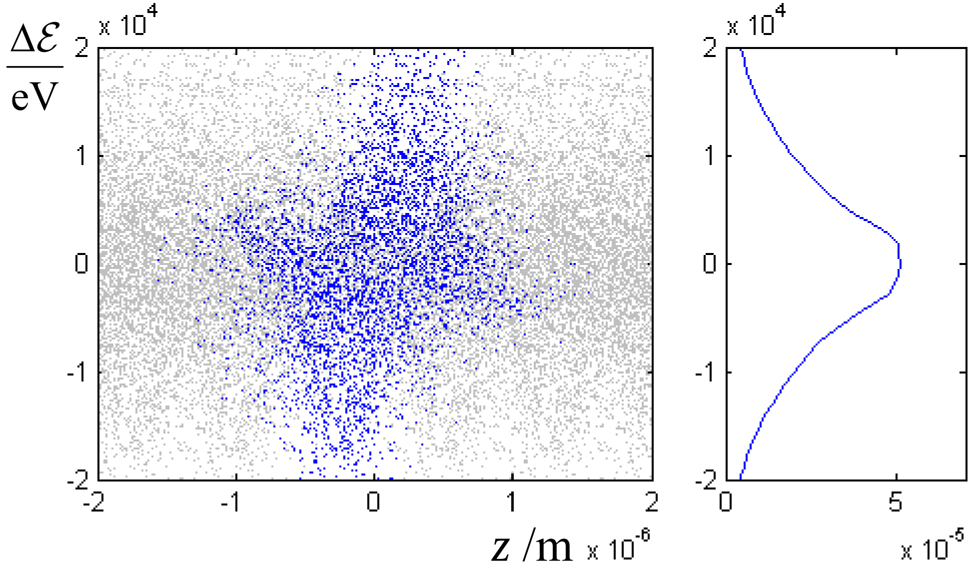


rms energy-spread
= 2.0 keV before LH
= 5.0 keV after LH undulator
≈ 6.5 keV at 15.5m

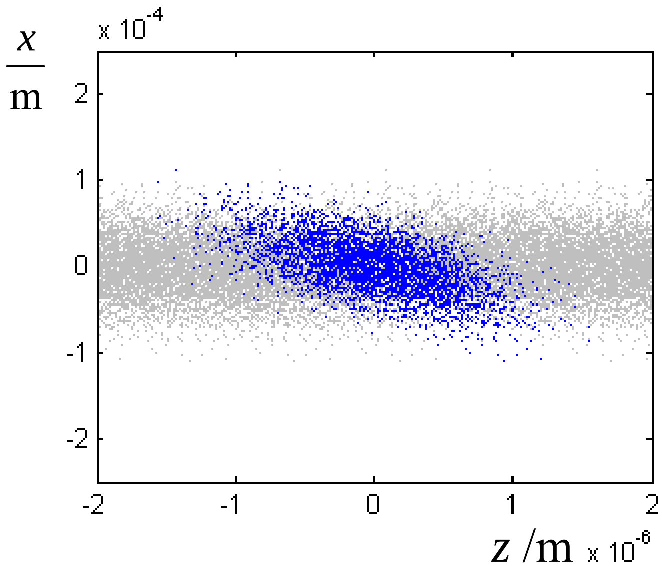


15.5 m

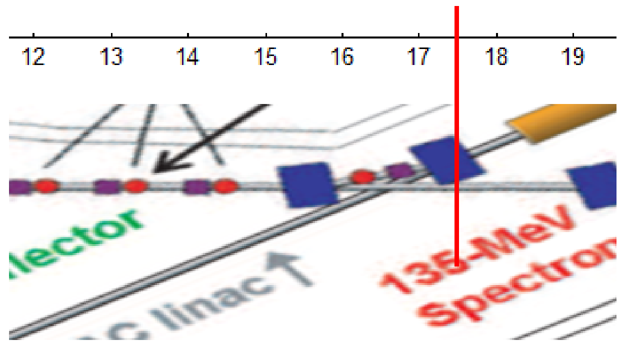




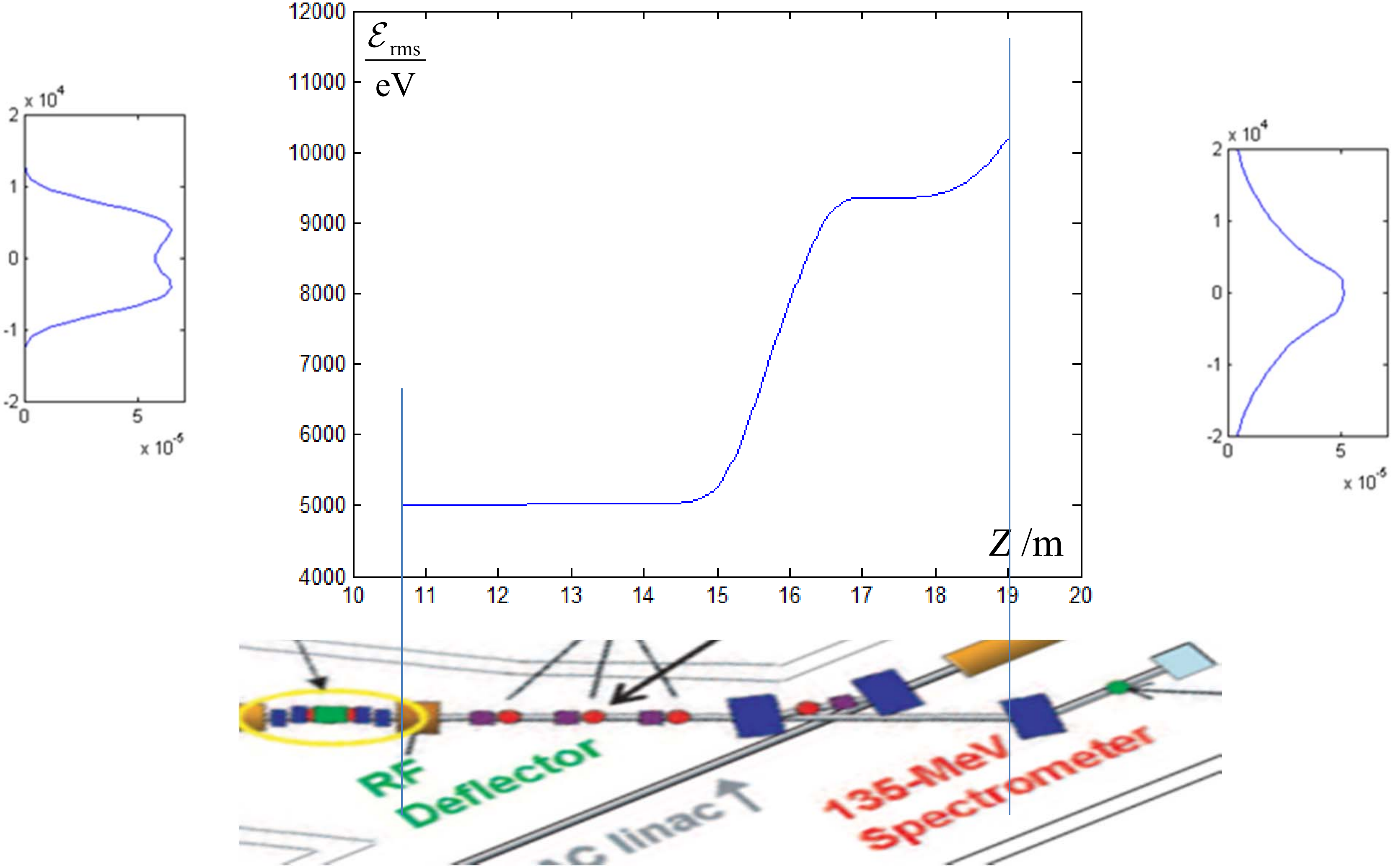
rms energy-spread
= 2.0 keV before LH
= 5.0 keV after LH undulator
≈ 6.5 keV at 15.5 m
≈ 9.5 keV at 17.5 m

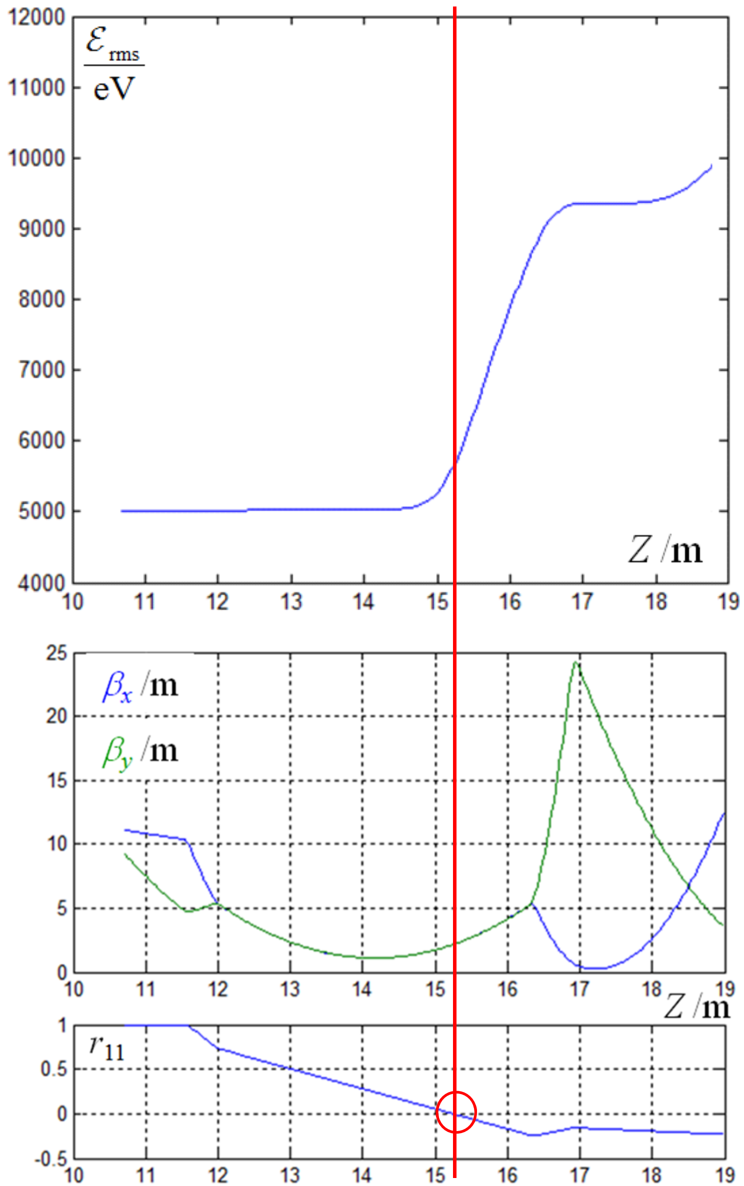


17.5 m

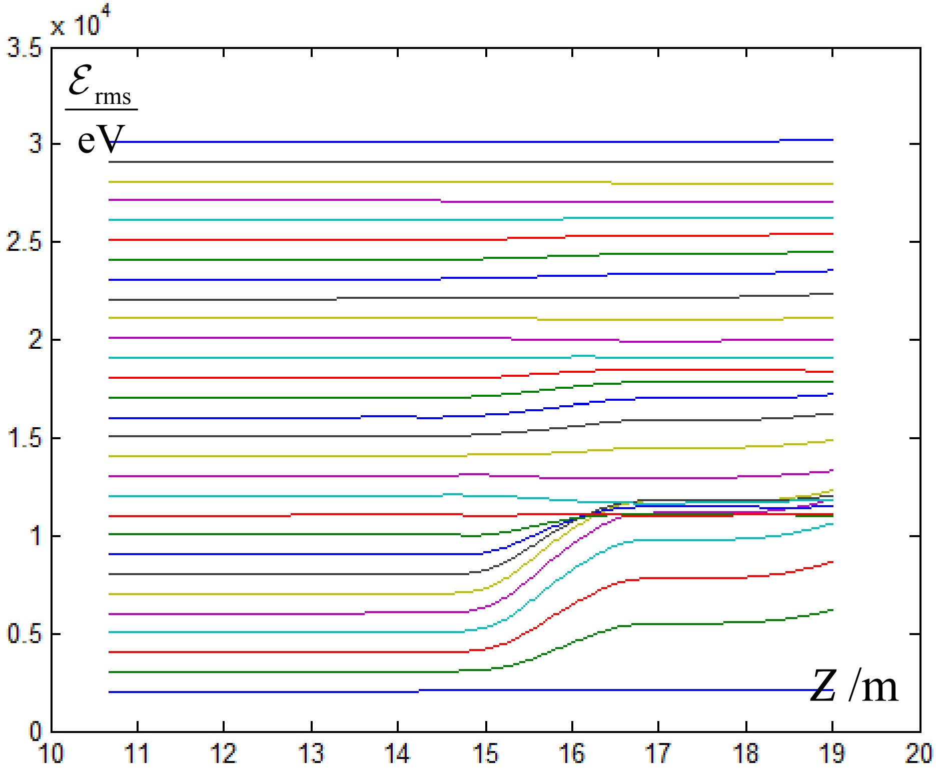


growth of rms energy spread and modification of energy spectrum





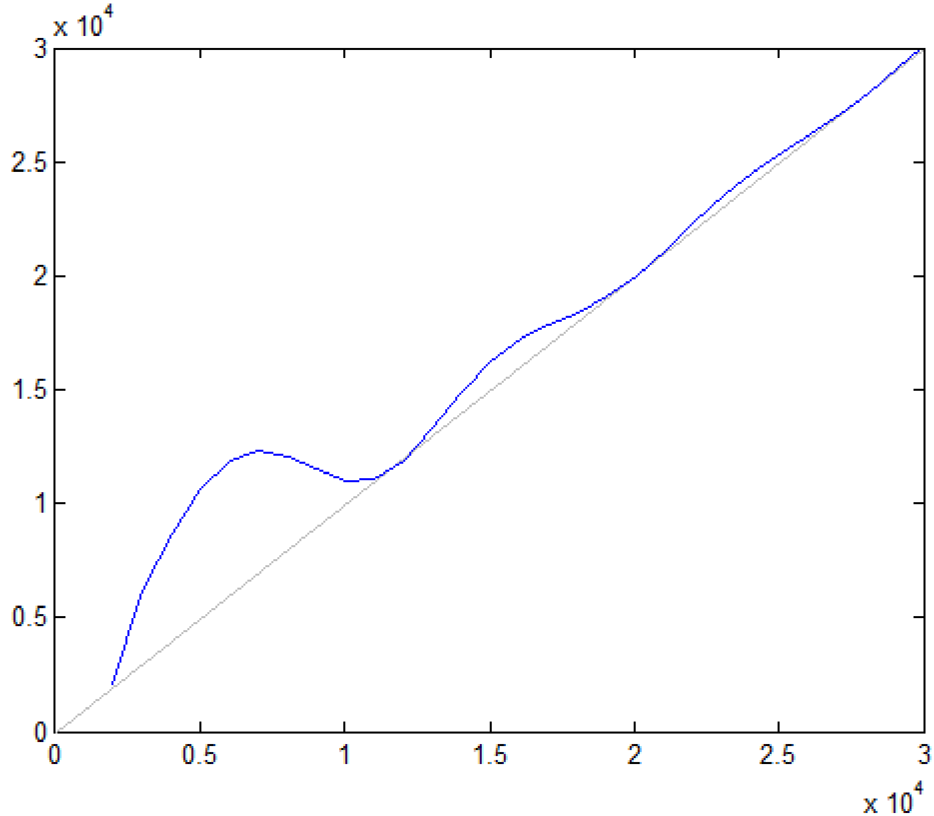
$$\mathcal{E}_{\mathrm{LSC}} = J_1\left(k_0 R_{56}\delta_L\right)\times\frac{2i\mathcal{E}_0}{k_0}\frac{I_0}{I_A}\int dz\times\frac{1}{\sigma_r^2}\exp\left(-\tfrac{\varepsilon}{2\beta}\left(k_0 R_{52} R_{11}\right)^2\right)\frac{1}{1+\gamma^2 R^2}$$



x 10^4
3.5
3
2.5
2
1.5
1
0.5
0
$\mathcal{E}_{rms}$ / eV
10
11
12
13
14
15
16
17
18
19
20
Z /m

scan: rms out versus rms in

rms out / eV



rms in / eV

comparison with measurement

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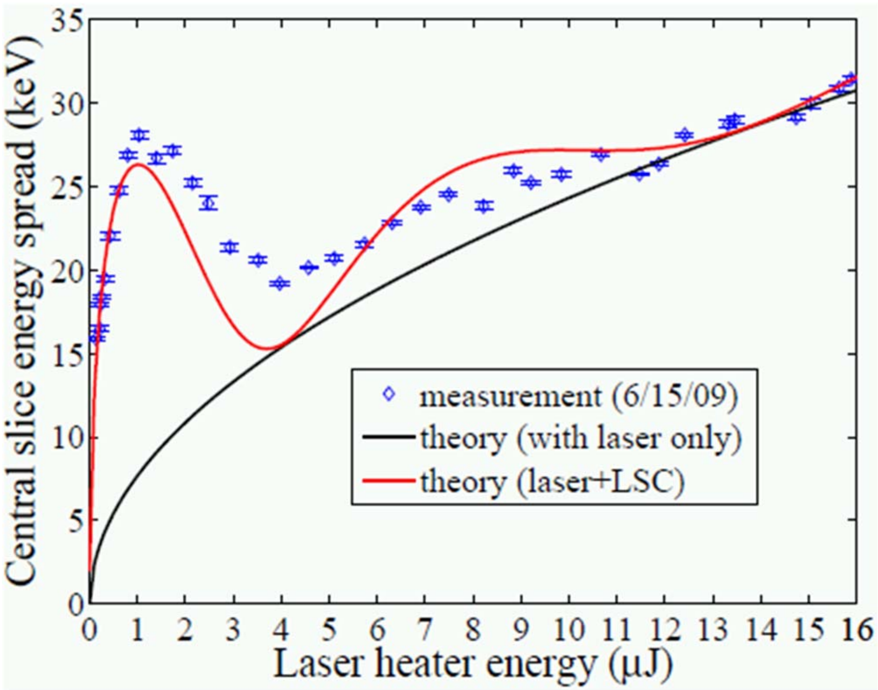
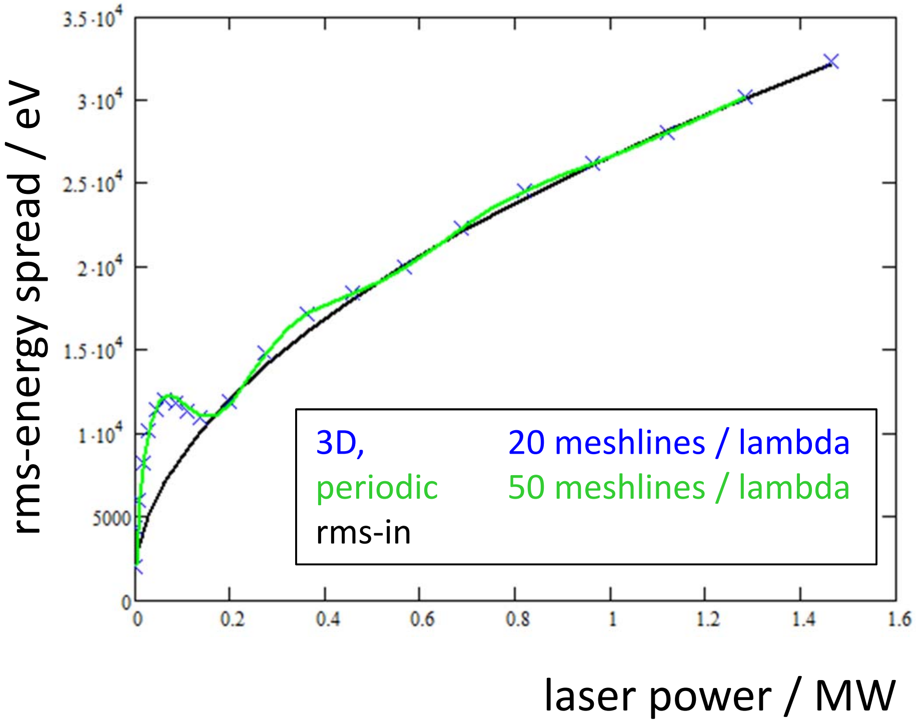
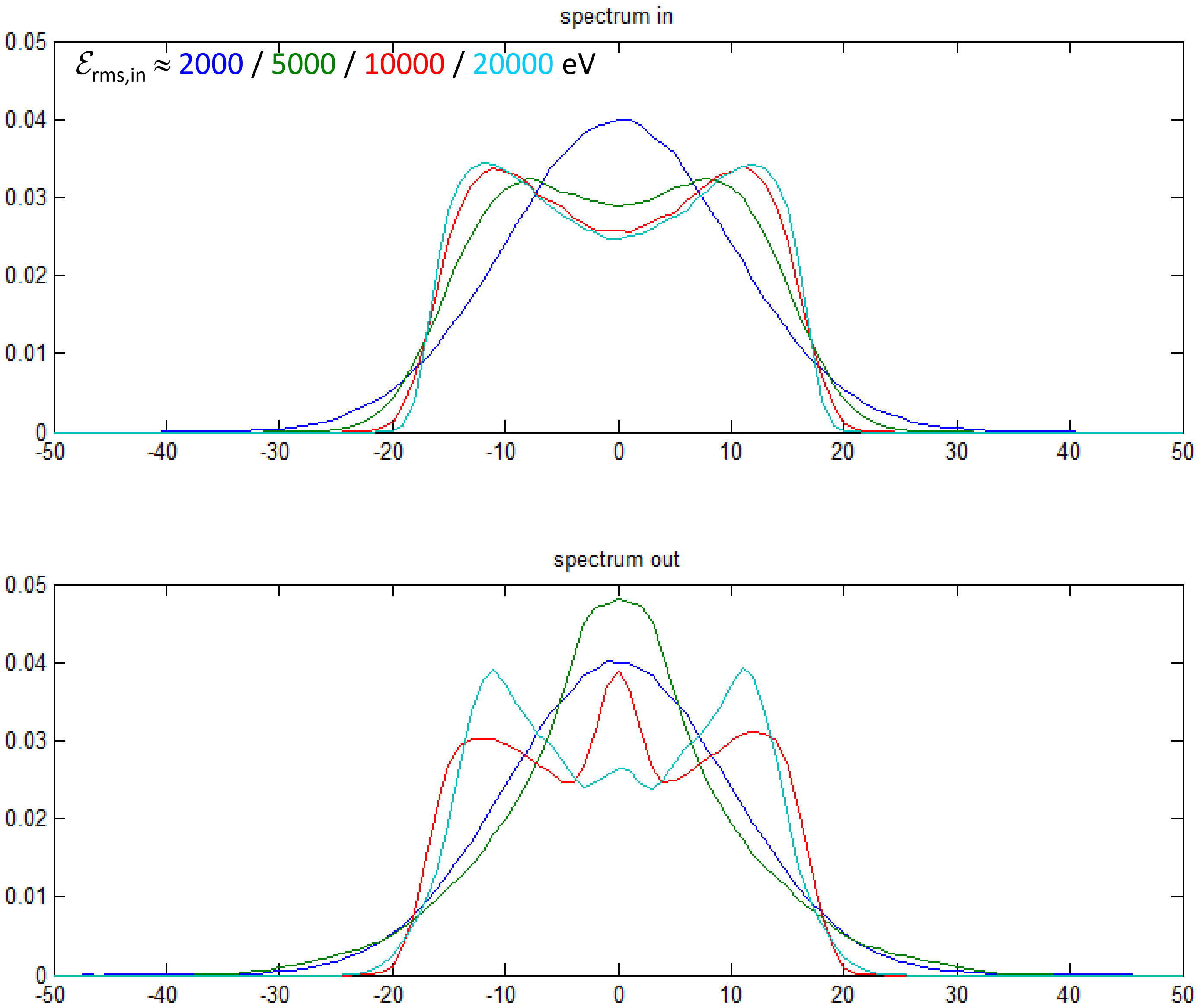


simulation

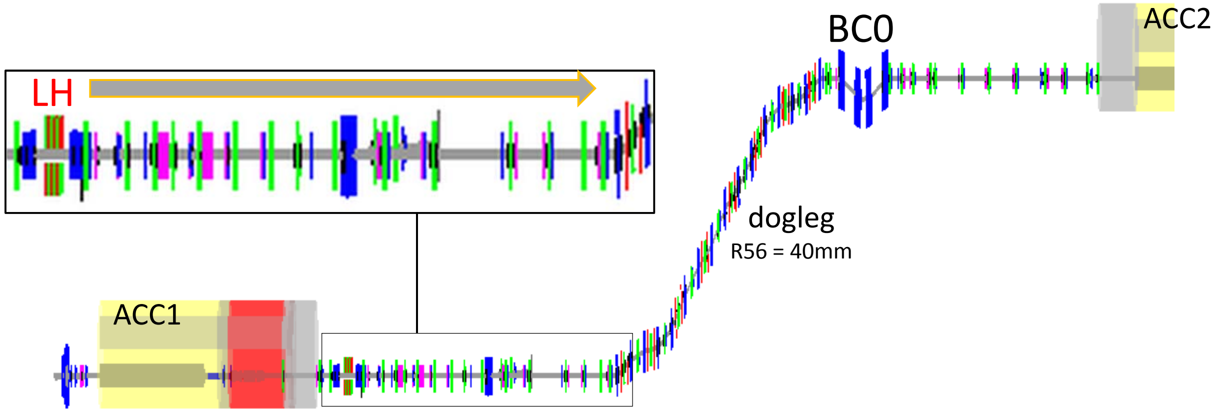


LCLS, 250 pC

normalized spectrum



EuXFEL – Trickle Heating, Simulation

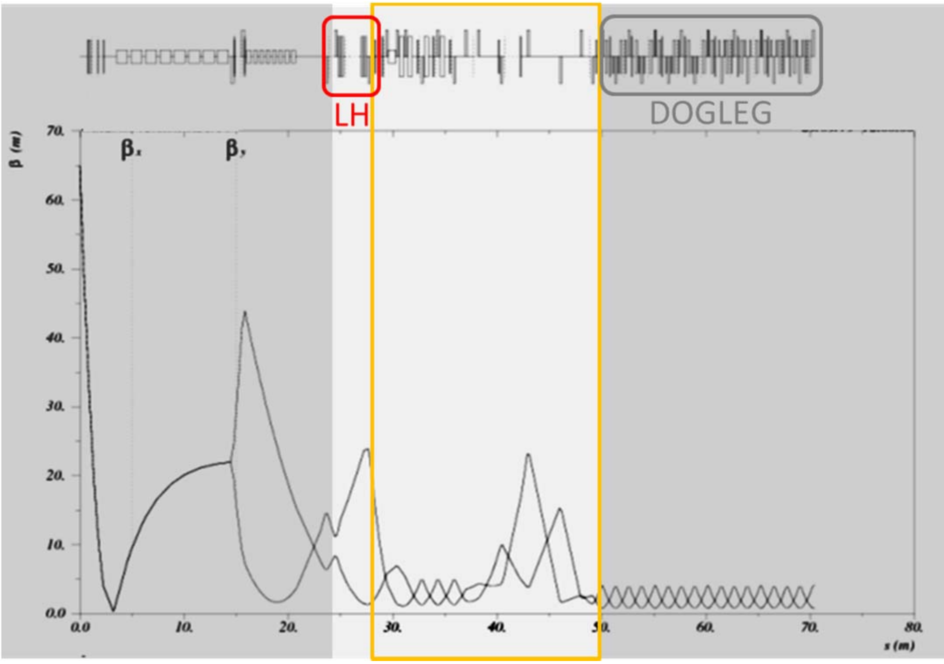


$$\mathcal{E} = 130\ \text{MeV}$$

$$\sigma_\gamma = 2\ \text{keV}\ \frac{q_{\text{bunch}}}{1\ \text{nC}}$$

$$\hat{I} = 50\ \text{A}\ \frac{q_{\text{bunch}}}{1\ \text{nC}} \rightarrow 5\ \text{kA}$$

$$\varepsilon = \frac{1\ \mu\text{m}}{\gamma}\sqrt{\frac{q_{\text{bunch}}}{1\ \text{nC}}}$$



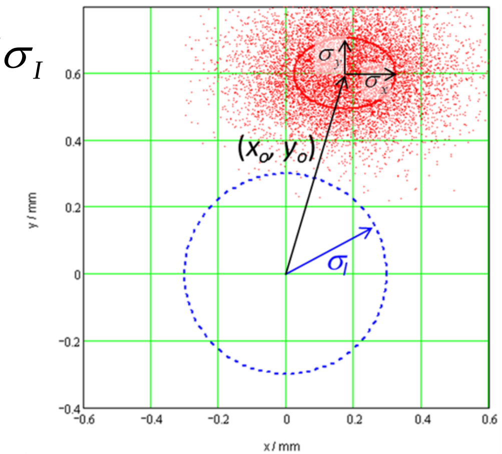
laser: $\sigma_l = 300\ \mu\text{m}$

$\lambda_{LH} = 1064\ \text{nm}$

axial displacement:

$x_o = 0$

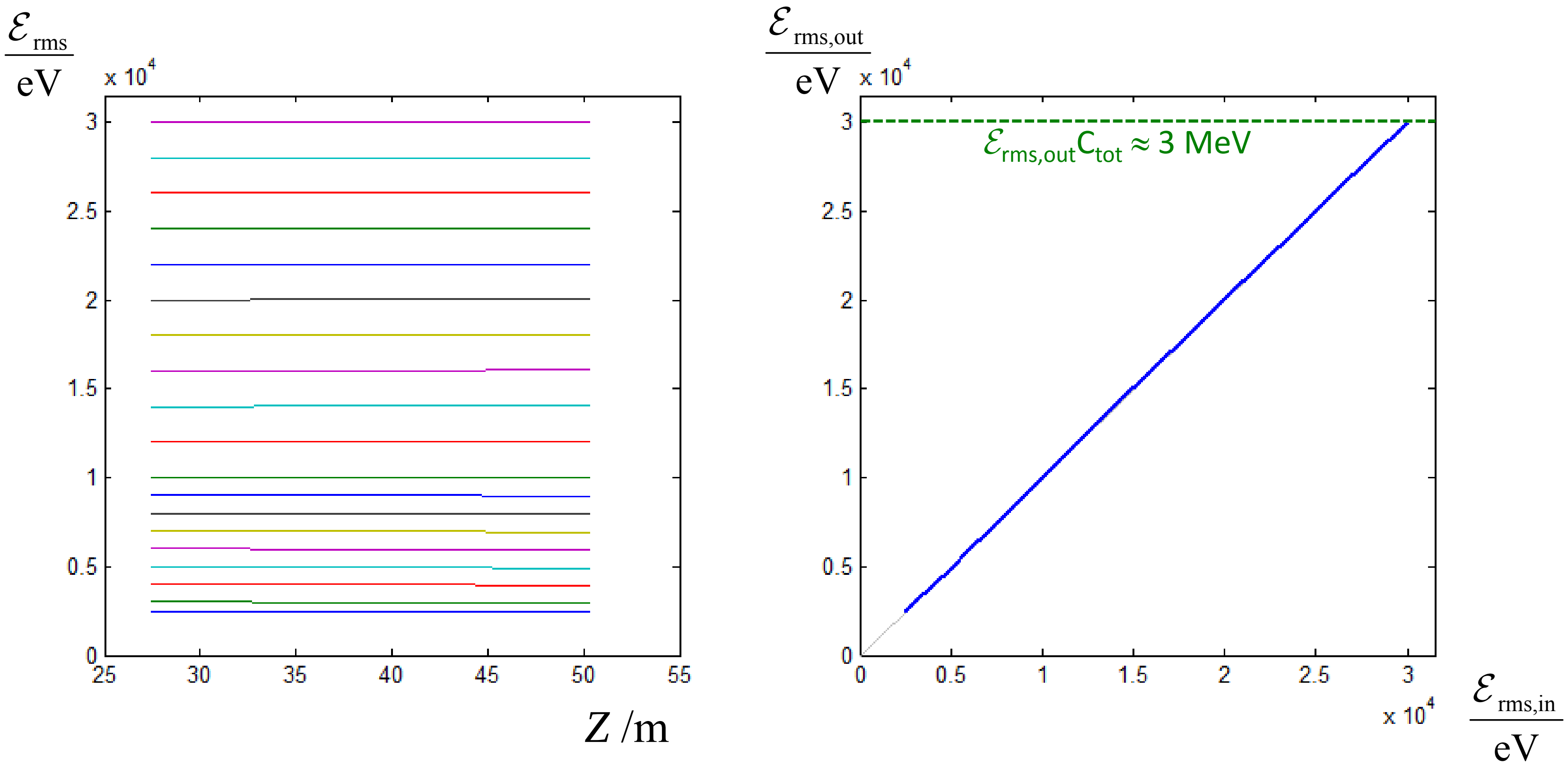
$y_o = 1.5\sigma_l$



XFEL, 1 nC

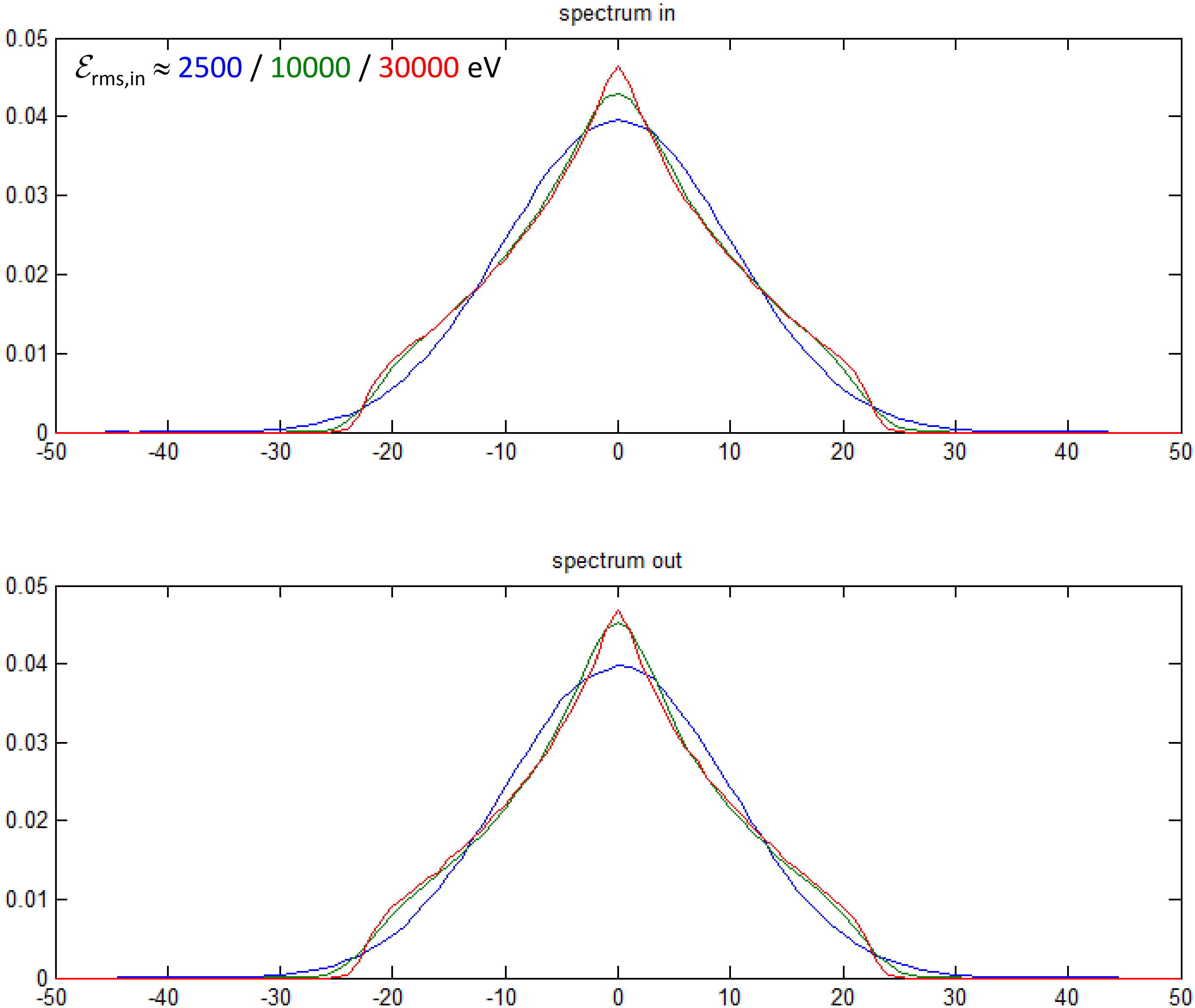
$C_{tot} \approx 100$

rms energy spread



XFEL, 1 nC

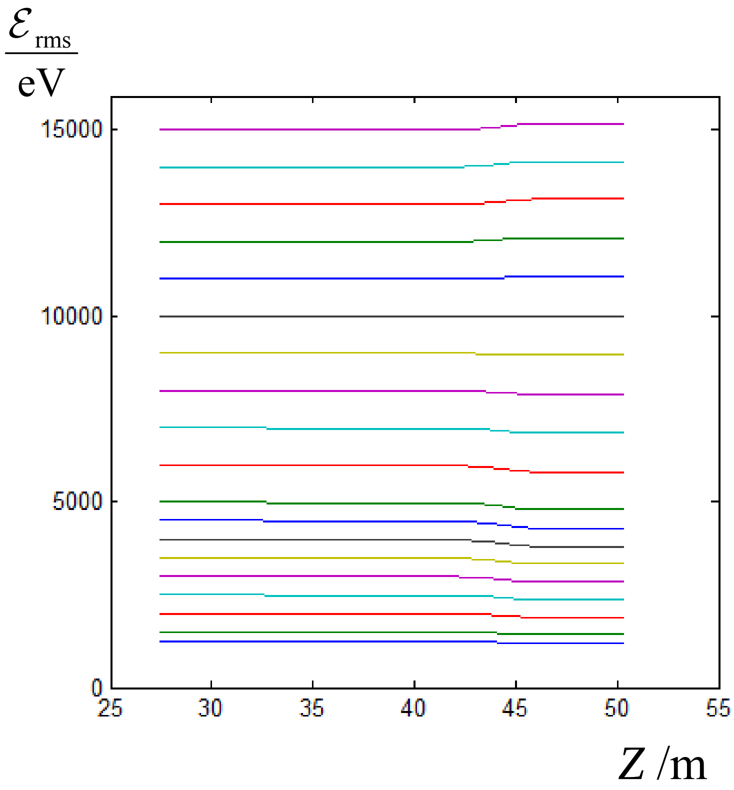
normalized spectrum

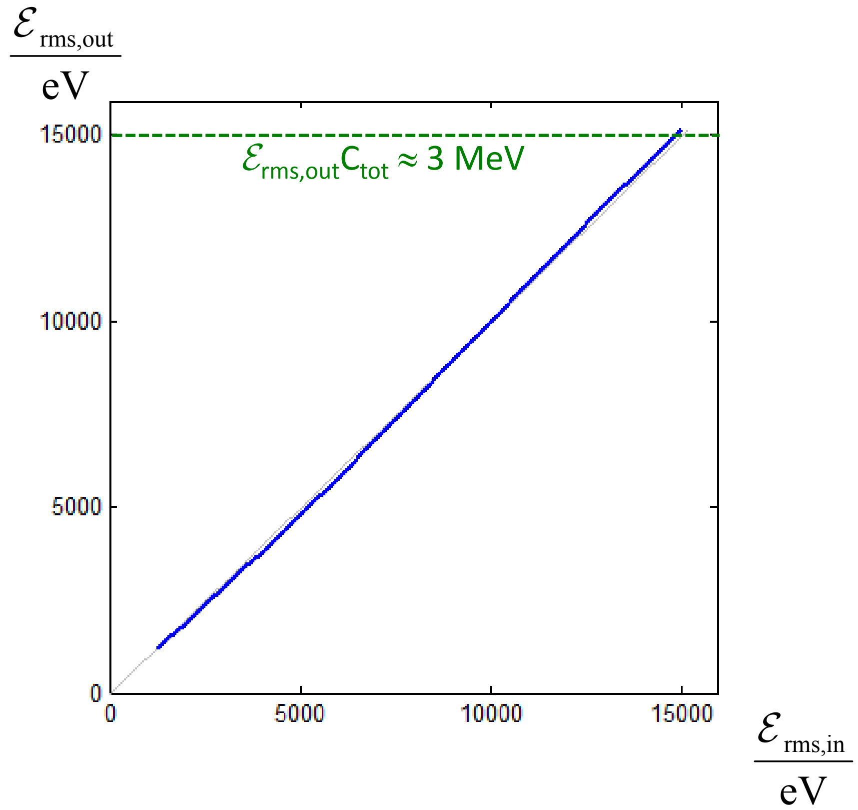


XFEL, 500 pC

$C_{tot} \approx 200$

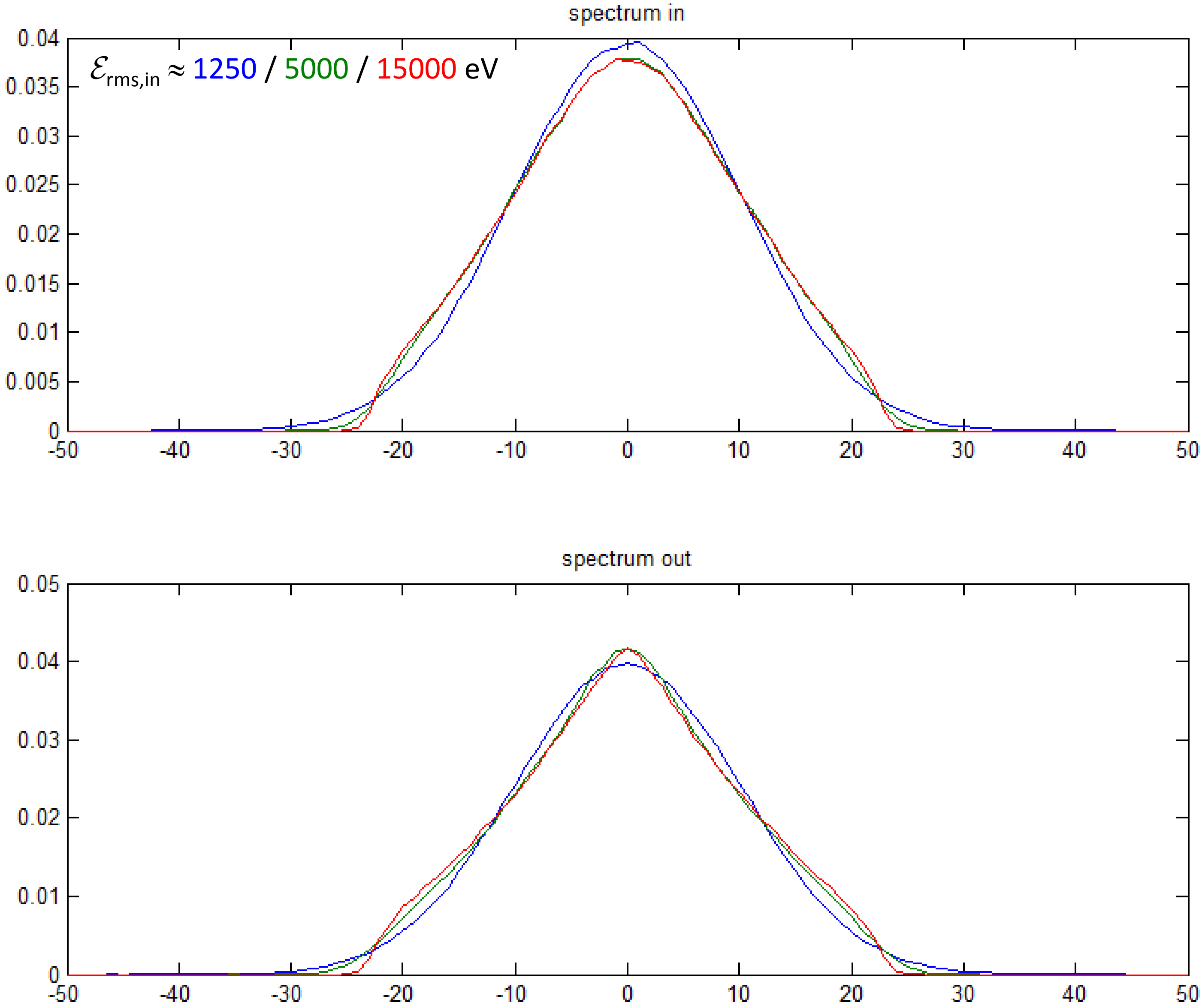
rms energy spread





XFEL, 500 pC

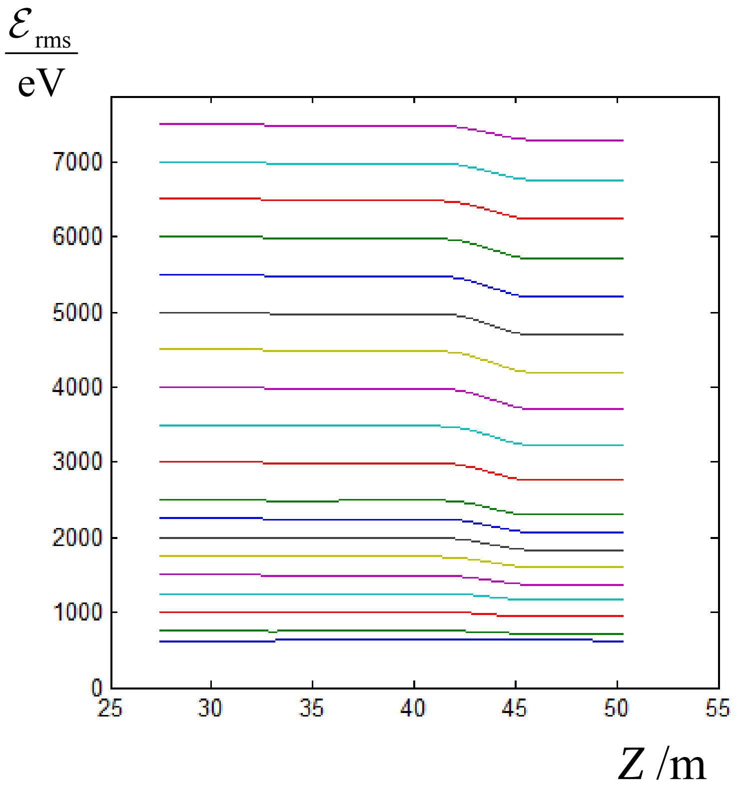
normalized spectrum

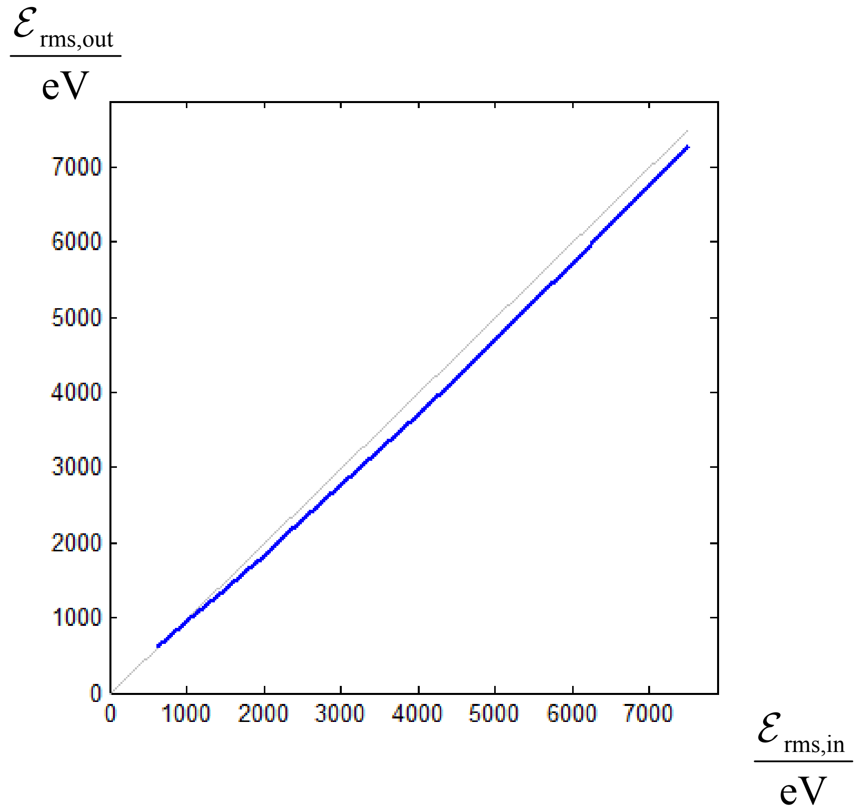


XFEL, 250 pC

$C_{tot} \approx 400$

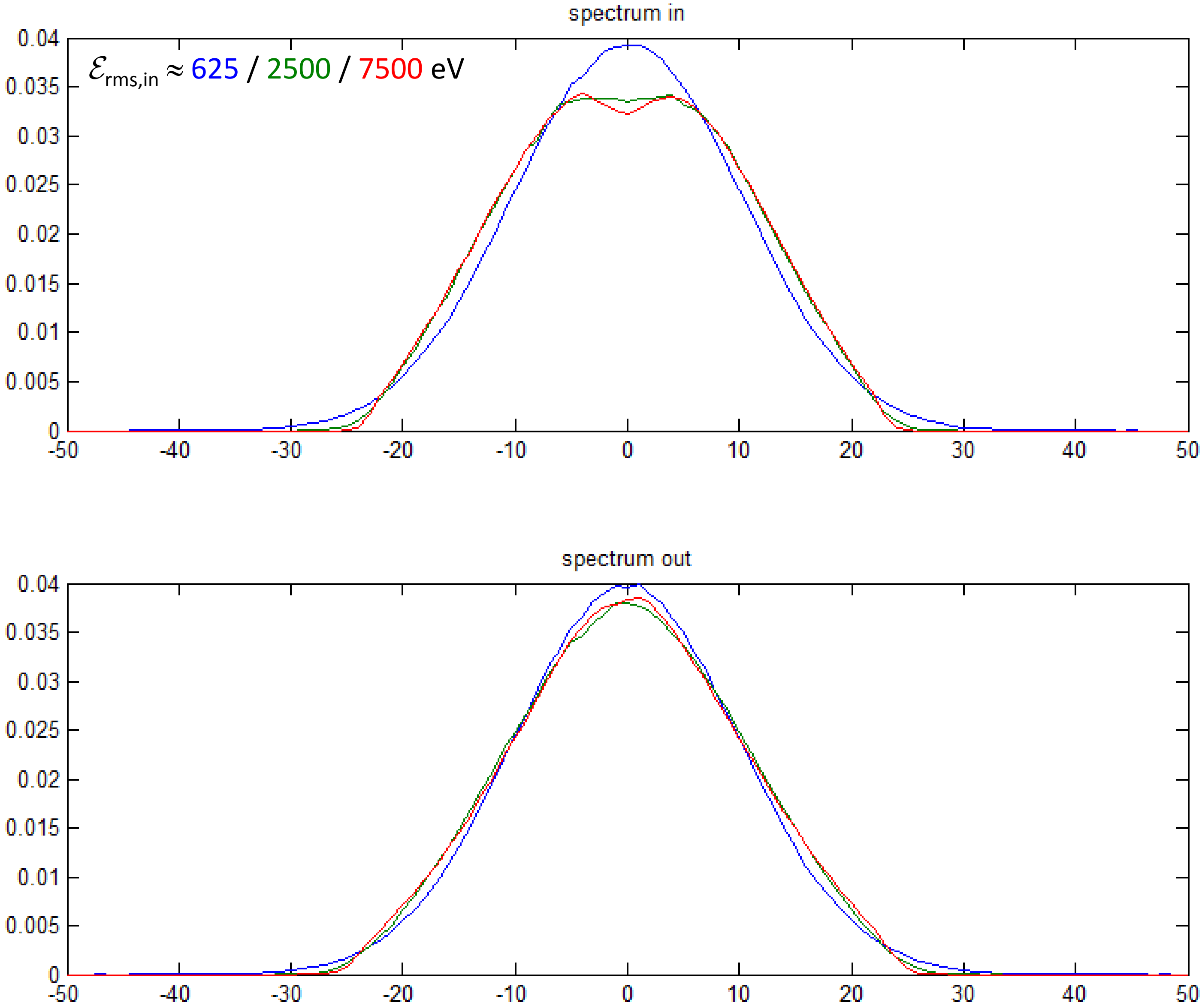
rms energy spread





XFEL, 250 pC

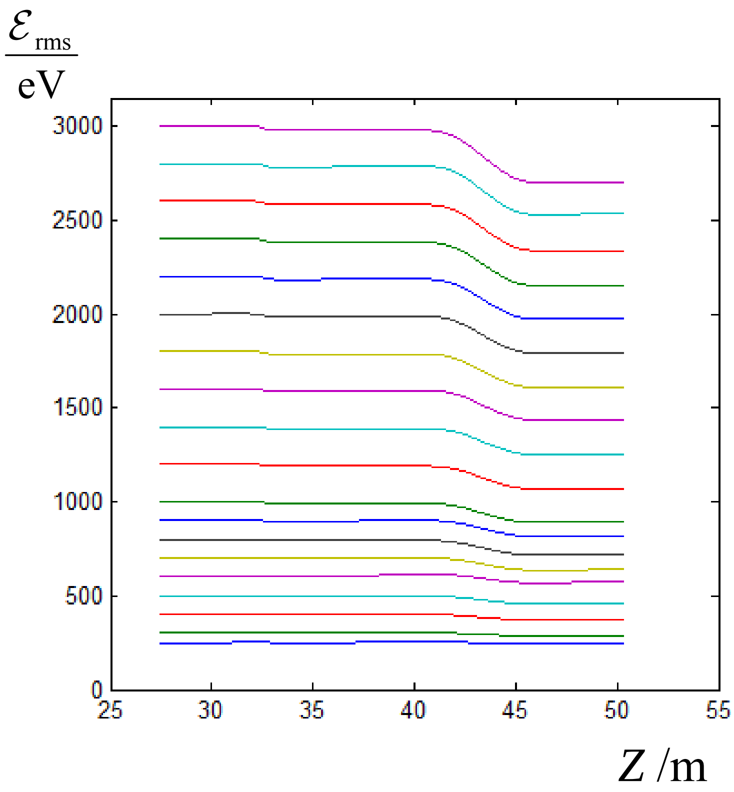
normalized spectrum

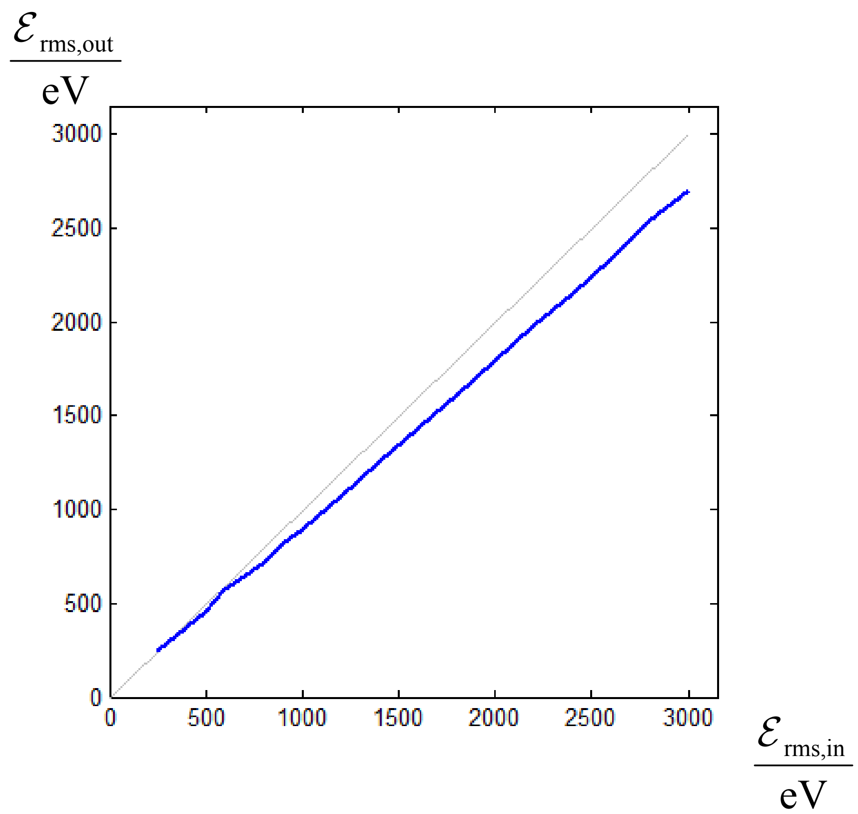


XFEL, 100 pC

$C_{tot} \approx 1000$

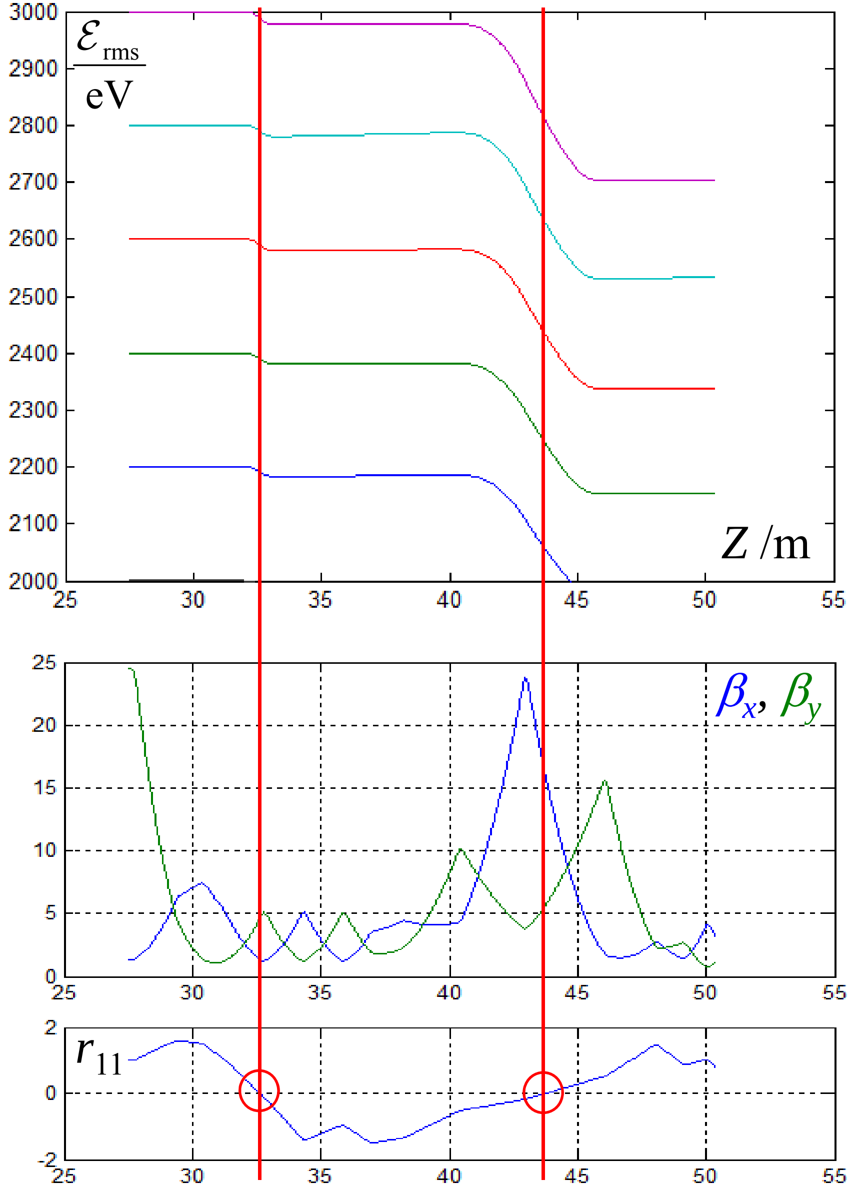
rms energy spread





$$\mathcal{E}_{rms} = \sqrt{\mathcal{E}^2_{rms,before} + \left(\mathcal{E}^2_{rms,L} \ast \mathcal{E}^2_{rms,LSC}\right)}$$

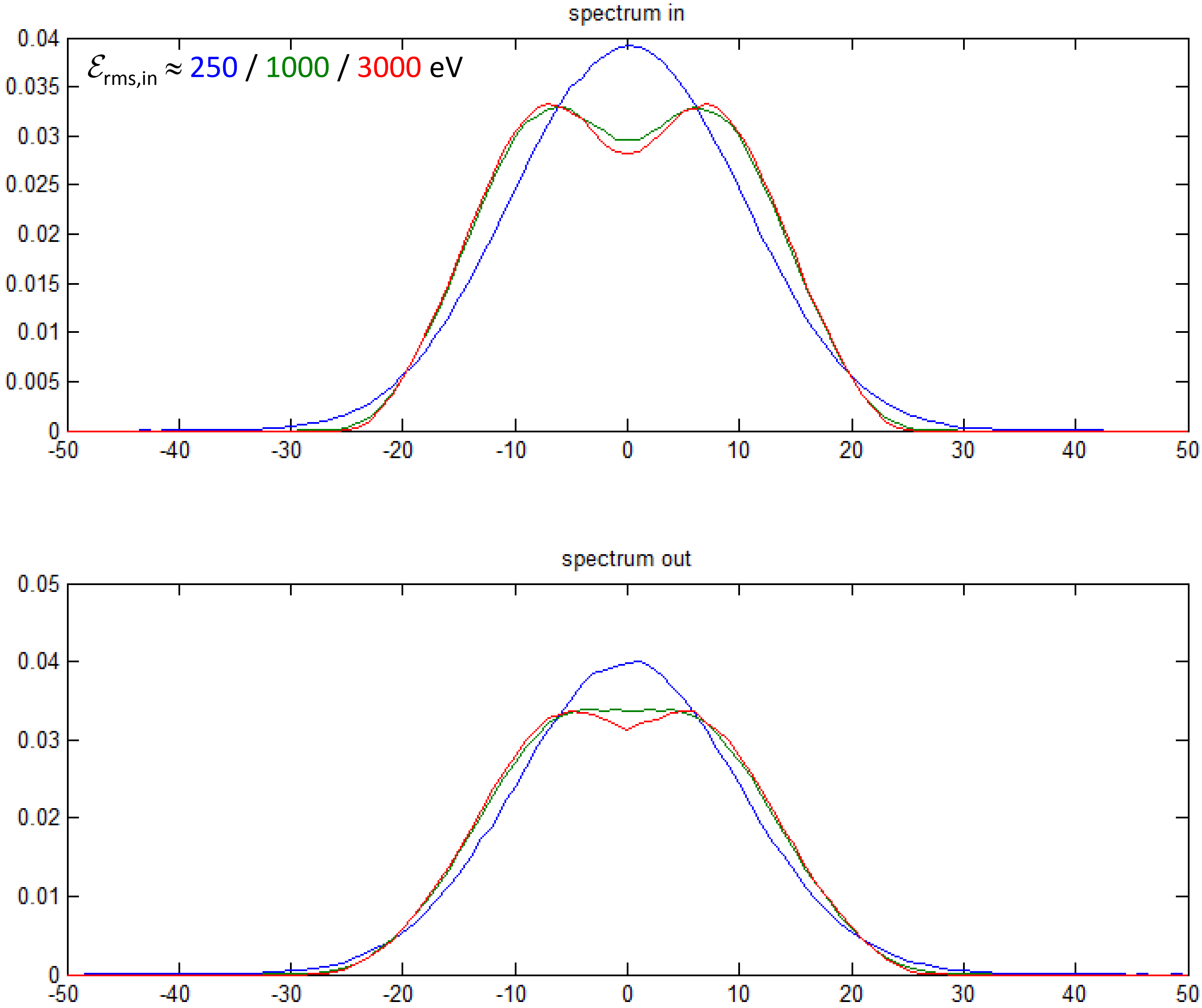
these effects **are** correlated!



$$\mathcal{E}_{\mathrm{LSC}} = J_1\left(k_0 R_{56}\delta_L\right)\times\frac{2i\mathcal{E}_0}{k_0}\frac{I_0}{I_A}\int dz\times\frac{1}{\sigma_r^2}\exp\left(-\tfrac{\varepsilon}{2\beta}\left(k_0 R_{52} R_{11}\right)^2\right)\frac{1}{1+\gamma^2 R^2}$$

XFEL, 100 pC

normalized spectrum



Conclusions, Summary

analytic estimation covers many essential effects

qualitative treatment of shape factors

EuXFEL is really 3D (not rz)

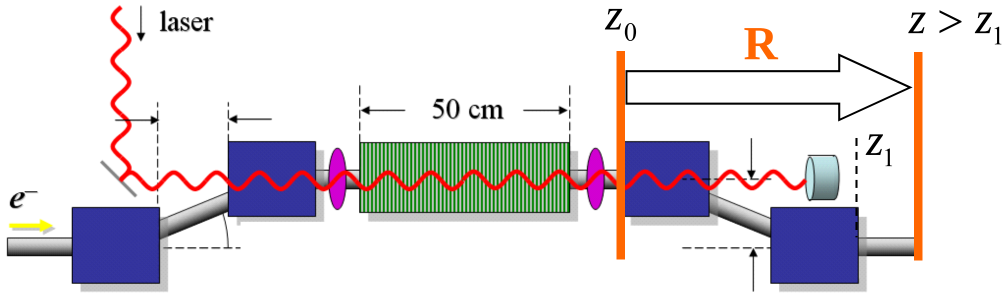
fast and efficient quantitative treatment with periodic SC solver

LCLS case: qualitative agreement (of num. calc.) with measurement

trickle heating changes rms energy spread and energy spectrum
(good {≈ gaussian} spectrum is crucial for effective suppression of μb amplification)

EuXFEL case: weak trickle heating, spectra are not spoiled

Thank You



$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta \end{pmatrix} = \begin{bmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{34} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \\ z_0 \\ \delta_0 \end{pmatrix}$$

$$R_{26} = R_{51} = 0$$

$$R_{16} = R_{52} = const$$

$$x = R_{11} x_0 + \boxed{R_{12}} \, x'_0 + \boxed{R_{16}} \, \delta_0$$

$$z - z_0 = \boxed{R_{16}} \, x'_0 + \boxed{R_{56}} \, \delta_0$$