Updates of 2D CSR Effects

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Outline

- Introduction
- Early studies/predictions of 2D effects
- Canonical formulation
- Behavior of residual 2D effects
- Implication for simulation and experiments
- Summary

Thanks to J. Bisognano, Ya. S. Derbenev, B. Carlsten and G. Geloni for discussions on this topic in past years.

Introduction

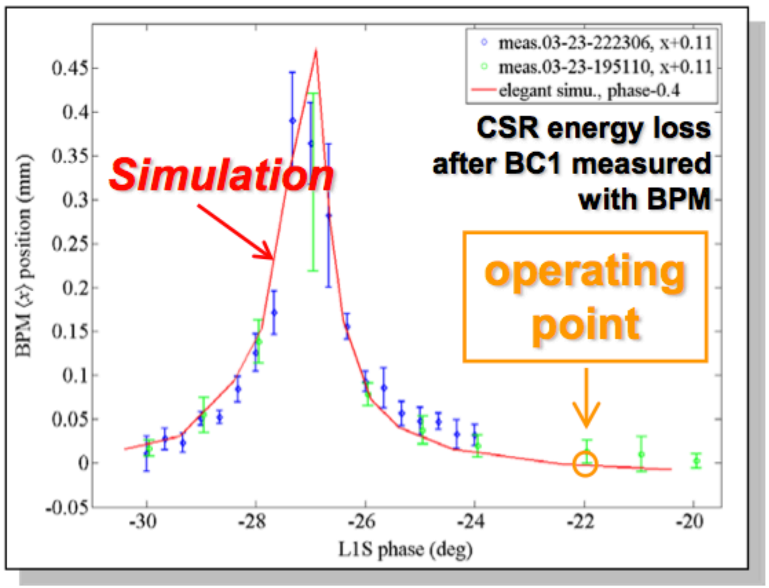
CSR plays important role in perturbing the dynamics of high brightness electron beam when transported through magnetic dipoles. It can cause

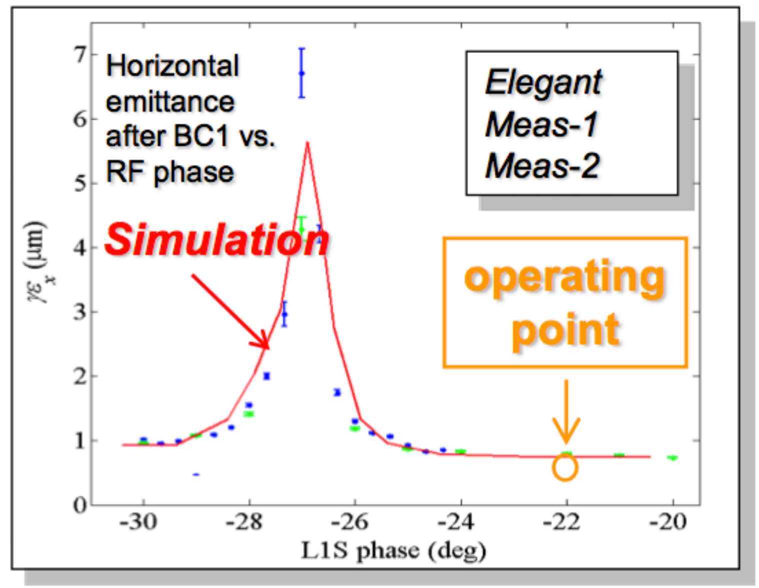
- Energy loss, emittance growth
- Filamentation of longitudinal phase space
- Microbunching instability

There have been extensive studies of the CSR effects. Most studies are based on 1D CSR interaction model

- Theory: CSR force generated by 1D rigid-line bunch (steady state and transient)
- Simulation: ELEGANT (based on 1D CSR model)
- Experiment: good agreement with ELEGANT results on beam phase space degradation
- Microbunching Instability: linearized Vlasov analysis

Examples of measured CSR effects on LCLS BC1 and their with Elegant simulation





K. Bane, et al. PRSTAB 12 (2009) 030704

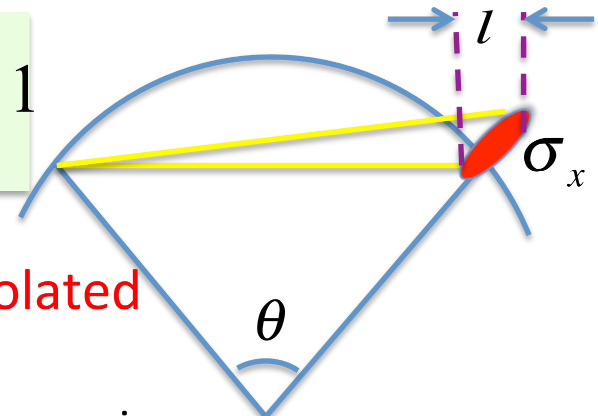
Questions: Are there any 2D Effects in these experiments?
If not, why? When 2D effects are supposed to show up?

Derbenev's Criterion for 1D CSR Effect

Derbenev et al, TESLA-FEL-Report 1995-05 (1995)

- For a bunch with rms length σ_z, the overtaking fields $(\lambda : \sigma_z)$ will have transverse coherence if across σ_x the phase difference is much less than 2π

$$l \approx \sigma_x \theta = \sigma_z \quad , \quad \text{or} \quad \alpha = \frac{\sigma_x}{\sigma_z^{2/3} R^{1/3}} = 1$$



- Situations when Derbenev criterion may be violated

(1) During roll-over compression

Large σ_x due to dispersion and small σ_z due to compression

(2) Microbunching when modulation wavelength is very small

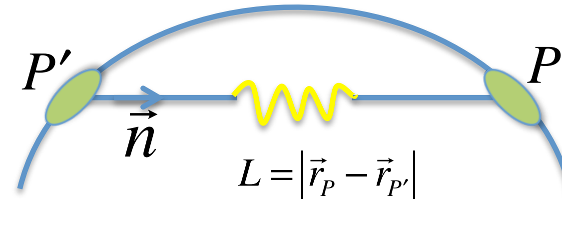
For COTR observed at LCLS BC1 Loos et al, SLAC-PUB-13395 (2008)

$$\sigma_x = 80\mu\text{m}, R = 2.1\text{m}, \lambda \approx 500\text{nm} \longrightarrow \lambda^{2/3} R^{1/3} : \sigma_x$$

(3) Magnetized electron beam in Circulator Cooler Ring of MEIC has large size in x-y plar itts microbunching needs 3D description.

Question: Are there any other 2D effects in addition to the transverse decoherence?

Basic Equations



- Lienard Wiechert Fields

$$\vec{E}^{col}(\vec{x},t) = \sum_{i=1}^{N} \vec{E}_0(\vec{x},t;i); \qquad \vec{E}_0(\vec{x},t;i) = \underbrace{e\left[\frac{\vec{n}-\vec{\beta}}{\gamma^2\left(1-\vec{\beta}\cdot\vec{n}\right)^3 L^2}\right]_{ret}}_{\text{Coulomb part}} + \underbrace{\frac{e}{c}\left[\frac{\vec{n}\times\{(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}\}}{\left(1-\vec{\beta}\cdot\vec{n}\right)^3 L}\right]_{ret}}_{\text{Radiative part}}$$

$$\vec{B}^{col}(\vec{x},t) = \sum_{i=1}^{N} \vec{B}_0(\vec{x},t;i); \qquad \vec{B}_0(\vec{x},t;i) = \left(\vec{n}\times\vec{E}_0\right)_{ret}$$

- Lorentz Force

$$\frac{d\vec{p}}{dt} = \vec{F}^{ext} + \vec{F}^{col} = e\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right) \qquad \text{with } \vec{E} = \vec{E}^{ext} + \vec{E}^{col}, \quad \vec{B} = \vec{B}^{ext} + \vec{B}^{col}$$

$$\vec{F}^{col} = F_r^{col}\,\vec{e}_r + F_s^{col}\,\vec{e}_s$$

Basic Equations (con'd)

- Longitudinal particle dynamics in dipoles

$$\Delta\mathcal{E}(t) = \mathcal{E}(t) - \mathcal{E}_0 = \Delta\mathcal{E}(t=0) + \int_0^t dt \left(E_s^{col} v_s + E_r^{col} v_r \right)$$

- Transverse particle dynamics in dipoles

$$\frac{d^2 x}{c^2 dt^2} + \frac{x}{R^2} = \frac{\Delta\mathcal{E}(t)}{R\mathcal{E}_0} + \frac{F_r^{col}(t)}{\mathcal{E}_0}$$

Nominal optics: $F_r^{\mathrm{col}} = F_s^{\mathrm{col}} = 0$, $\Delta\mathcal{E}$ gives the dispersion correction to the betatron oscilations.

1D CSR model: $F_r^{\mathrm{col}} = 0$, F_s^{col} for a rigid-line bunch

Questions: Why does the 1D approximation work so well?
When will the transverse CSR force be needed? How does it behave?

Early Studies/Predictions of 2D Effects

Transverse CSR Force

(Talman, PRL 56, 1429 (1986))

- For a bunch with line charge density λ on circular orbit of radius R, the radial component of space charge does not have relativistic cancellation between E and B

$$F_r^{col} = \frac{\lambda}{4\pi\varepsilon_0 R}\left[\left(1-\frac{1}{2\gamma^2}\right)\ln\frac{\tan(\theta_{max}/4)}{\tan(\theta_{min}/4)}-1\right]$$

Centrifugal space charge force (CSCF) due to radiative part of LW

This is the first time the behavior and effect of transverse CSR force was pointed out

- Behavior of F_r^{col}
  - Logarithmic divergence at $\theta = 0$ entails a rapid spatial variation over transverse and longitudinal beam distribution
  - Independent of energy

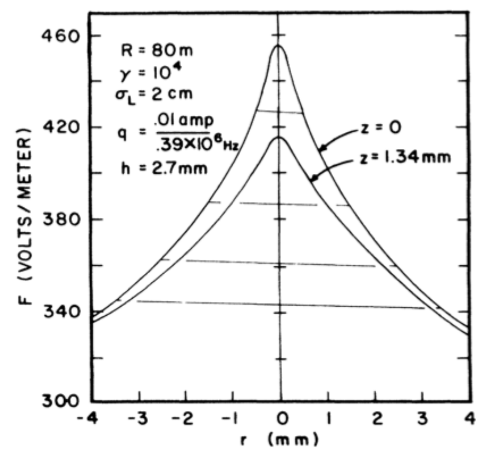


FIG. 1. Dependence of the transverse force F on radial position r and vertical position z.

- Possible effects of F_r^{col} on beam dynamics:

$$\frac{d^2r}{dt^2}+2\alpha\frac{dr}{dt}+Q_r^2\omega_0^2 r = F_r^{col}$$

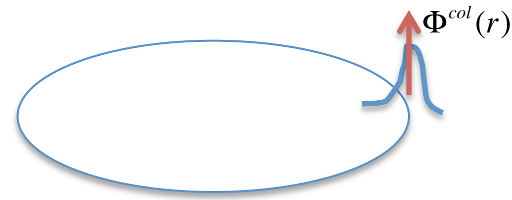
Equivalent to temporary shift up of momentum---chromatic effect

A potent driver of complicated nonlinear resonances, such as $Q_r = Q_z + Q_s$

Cancellation of the CSCF

(Lee, Particle Accelerators 25, 241 (1990))

A particle undergoing betatron oscillations has simultaneously oscillations of its kinetic energy due to its motion through the beam's electric potential.



$$\frac{d\mathcal{E}}{dt} = q\vec{E}\cdot\vec{v} \quad \text{or} \quad \delta(\gamma mc^2) = -eE_r^{col}(r)\delta r$$

In curved geometry, the kinetic-energy oscillation results in a first-order dynamical term in the horizontal equation of motion that cancels the effect of CSCF.

$$\frac{d(\gamma m\vec{v})}{dt} = \vec{F} \quad \text{or} \quad \frac{d(\gamma m\dot{r})}{dt} - \gamma m r\dot{\theta}^2 = F_r^{ext} + F_r^{col} \qquad (\text{for } F_r^{col} = -e(E_r + B_z)^{col})$$

At equilibrium: $\dot{r} = 0$

$$\boxed{\gamma(R)mc^2 = R\left[eB^{ext} - F_r^{col}(R)\right]}$$

1st order dynamics:

$$\boxed{\frac{d^2\delta r}{c^2dt^2} + \frac{\delta r}{R^2} = \frac{\delta\gamma}{\gamma R} + \frac{1}{\gamma mc^2}\left(\left.\frac{\partial F_r^{col}}{\partial r}\right|_R \delta r\right)}$$

or

$$\frac{d^2\delta r}{c^2dt^2} + \omega_r^2\delta r = 0 \quad \text{for} \quad \omega_r^2 = \frac{c^2}{R^2} + \frac{1}{\gamma m}\left[\frac{eE_r^{col}(R)}{r} - \frac{\partial F_r^{col}}{\partial r}\right]$$

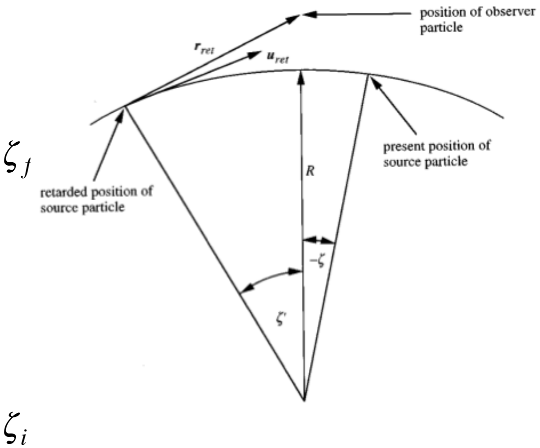
Cancellation of effect of potential energy depression with CSCF:

$$\frac{eE_r^{col}(R)}{r} - \frac{\partial F_r^{col}}{\partial r} : O\left(\frac{e\lambda}{R_0^2}\right) : (10^{-3} \text{ to } 10^{-5})\frac{\partial F_r^{col}}{\partial r}$$

Noninertial Space Charge Force (B. Carlsten, PRE 54, 838 (1996)

The longitudinal LW electric field on an off-axis particle from the interaction of a line bunch on design orbit:

$$E_\theta = \frac{\lambda}{4\pi\varepsilon_0}\left\{\frac{1}{\vec{r}_{ret} - \vec{r}_{ret}\cdot\vec{u}/c}\left[\overbrace{\frac{1}{\gamma^2}}^{\text{Coulomb}} \overbrace{-\beta^2\frac{x}{R} + \beta^2(1-\cos\zeta')}^{\text{radiative LW part}}\right]\right\}\Bigg|_{\zeta_i}^{\zeta_f}$$



Usual Coulomb force:

- $\propto \gamma^{-2}$, no net energy loss
- Logarithmic divergence at $\zeta = 0$

Usual CSR force:

- net energy loss
- Independent of energy
- no logarithmic divergence

Noninertial space charge force (NSCF):

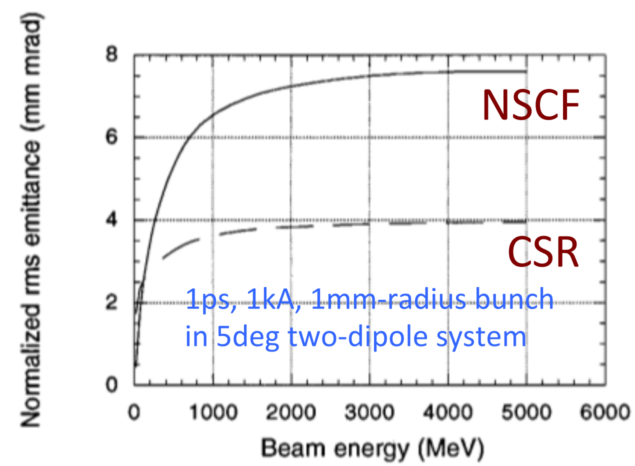
Similar to space charge

- Logarithmic divergence at $\zeta = 0$
- No net energy loss

Similar to CSR

- Independent of energy

Effect of NSCF on Emittance Growth



Longitudinal space charge effect in slowly converging relativistic beam

K. Bane and A. Chao, PRSTAB 5, 104401 (2002)

In a 4-dipole chicane, the relativistic bunch converges drastically during the drift before the last dipole.

$$\mathcal{E}_f - \mathcal{E}_i = \int \vec{E}\cdot\vec{v}\,dt = -q(\varphi_f - \varphi_i) + (\text{term } \propto \gamma^{-2})$$

Independent of γ, mainly caused by LSC force

$$\langle \Delta\phi^2 \rangle = \int \ldots \int \mathbf{dx}_0 dx_{d0} dy_{d0} dz_{d0} \psi_0(\mathbf{x}_0)[\tilde{\phi}_G(x, y, z, x_d, y_d, z_d) - \tilde{\phi}_G(x_0, y_0, z_0, x_{d0}, y_{d0}, z_{d0})]^2,$$

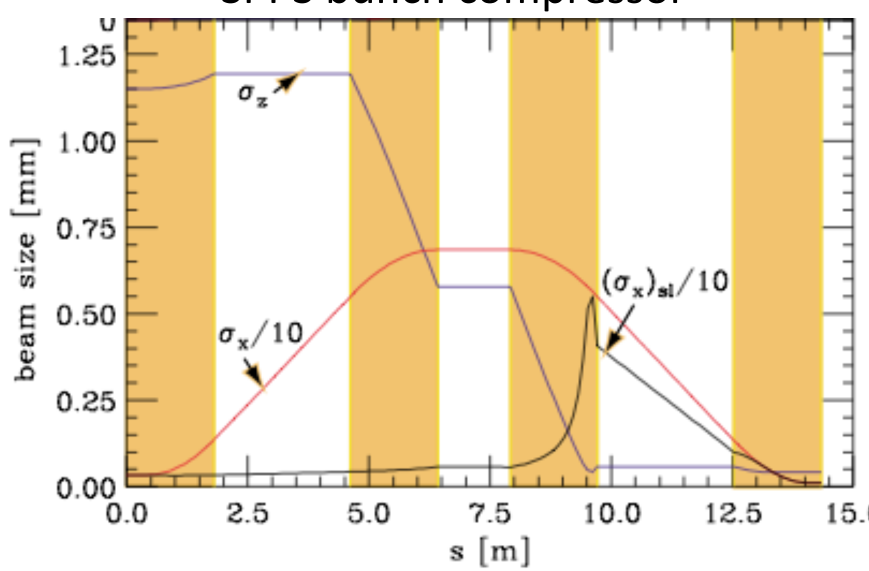
φ_G : Green's function of potential for a rod current in a circular beam pipe

For SPPS bunch, $\mathcal{E} = 9\,GeV, N_e = 2.1\times 10^{10}$

$\sigma_{z0} = 1.15$ mm, $\sigma_{zf} = 41\ \mu$m

LSC Effects	CSR Effects
$\langle\delta\rangle = -27\times 10^{-5}$	$\langle\delta\rangle = -37\times 10^{-5}$
$\delta_{rms} = 11.5\times 10^{-5}$	$\delta_{rms} = 16\times 10^{-5}$
$\Delta\varepsilon_x/\varepsilon_x$: 13%	

SPPS bunch compressor



Transverse self-field in an arc of a circle

G. Geloni *et al.*, DESY 03-44 (2002)

- Analysis of Coulomb and radiative contributions to F_r^{col} using Lienard-Wiechert fields
- A parallel approach of the analysis for F_s^{col} by Saldin *et al.* in NIM A 398, 373 (1997)
- Careful studies of F_r^{col} in various parameter regime, including entrance and exit behavior

Major findings:

Unlike the usual F_s^{col}, the transverse force F_r^{col} has a head-tail contribution due to radiative parts of the LW fields:

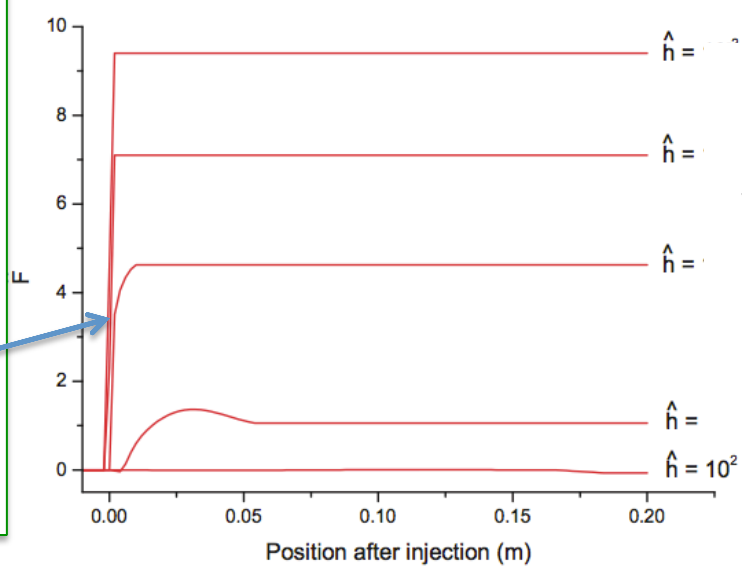
$$F_r^{col}\Big|_{\text{head-tail}} \simeq \frac{e^2}{4\pi\varepsilon_0}\frac{1}{R\Delta s}$$

Features:

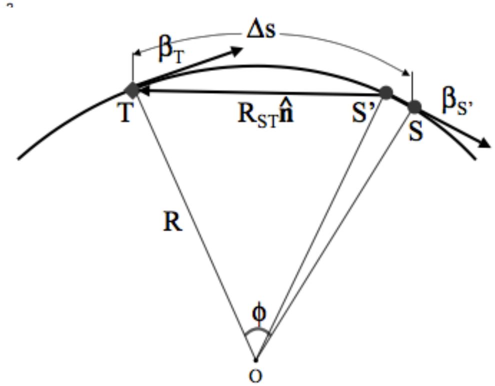
(1) $\left\|F_r^{col}\right\|_{\text{head-tail}}$ is 1 : 2 times of $\left\|F_r^{col}\right\|_{\text{tail-head}}$

(2) Sudden turn-on of $\left|F_r^{col}\right|_{\text{head-tail}}$ at arc entrance

Normalized radial force vs. s



Head-tail interaction



Kinetic energy change $\Delta\mathcal{E}(t)=\mathcal{E}(t)-\mathcal{E}_0=\Delta\mathcal{E}(t=0)+\int_0^t dt\ \left(E_s^{col}v_s+E_r^{col}v_r\right)$

Transverse dynamics $\frac{d^2x}{c^2dt^2}+\frac{x}{R^2}=\frac{\Delta\mathcal{E}(t)}{R\mathcal{E}_0}+\frac{F_r^{col}(t)}{\mathcal{E}_0}$

The history of studies of transverse effects from F_r^{col}, F_s^{col} and $\Delta\mathcal{E}$ (1985-now) reflects the step-by-step endeavors toward a complete understanding of the EM interaction on a curved orbit.

Talman (1985)	F_r^{col} from LW fields, (on an arc)	Strong dependence on x due to local logarithmic divergence
Lee (1990)	$\Delta\mathcal{E}_k=e\Delta\varphi^{col}=-e\int E_r^{col}(r)dr$ (coasting beam on circular orbit),	Cancels F_r^{col} effects on dynamics
Carlsten (1995):	E_s^{col} from LW fields (on an arc for off-axis particles)	Non-inertial term sensitive to local interaction
Bane and Chao (2002):	$\Delta\mathcal{E}_k=e\Delta\varphi^{col}=-e\int E_s^{col}(r)ds$ (on straight section),	LSC for converging beam, No γ^{-2} dependence
Geloni *et al.* (2002):	F_r^{col} from LW fields (straight + arc)	Head-tail contribution, Sudden turn-on at entrance

Questions

- We see disparate terms, on straight or on an arc, due to Coulomb fields or radiative fields, all with strong dependence on transverse particle coordinates. How do they relate to each other?

- Is the cancellation pointed by E. Lee for a coasting beam a general property of EM interaction on a curved orbit?

Cancellation and the Canonical Formulation

- Radial force in terms of potentials   Derbenev and Shiltsev, SLAC-PUB-7181 (1996)

Instead of expressing F_r^{col} in terms of Coulomb and Radiative parts, here the Cylindrical component of fields are expressed in terms of potentials $\left(t' = t - |\vec{r} - \vec{r}'|/c\right)$

$$\begin{cases} \varphi = \int \frac{\rho(\vec{r}',t')}{|\vec{r}-\vec{r}'|} d\vec{r}' \\ \vec{A} = \int \frac{J(\vec{r}',t')}{|\vec{r}-\vec{r}'|} d\vec{r}' \end{cases} \Rightarrow \begin{cases} \vec{E} = -\nabla\varphi - \frac{\partial \vec{A}}{c\partial t}, \\ \vec{B} = \vec{\nabla}\times\vec{A} \end{cases} \Rightarrow \begin{cases} E_x = -\frac{\partial\varphi}{\partial x} - \frac{\partial A_x}{c\partial t} \\ B_y = \frac{1}{1+\kappa x}\frac{\partial A_x}{\partial y} - \frac{\partial A_s}{\partial x} - \frac{A_s}{r} \end{cases}$$

No relativistic Cancellation Between E and B

effective radial force

$$F_r^{col} = \boxed{-\frac{\partial V_0}{\partial x} - e\frac{dA_x}{cdt}} + e\frac{A_s}{r}$$

with

$$V_0 = e(\varphi - \beta A_s) = e^2 \int d\vec{r}' \frac{\gamma^{-2} + \theta^2/2}{|\vec{r}-\vec{r}'|}$$

γ^{-2} dependence

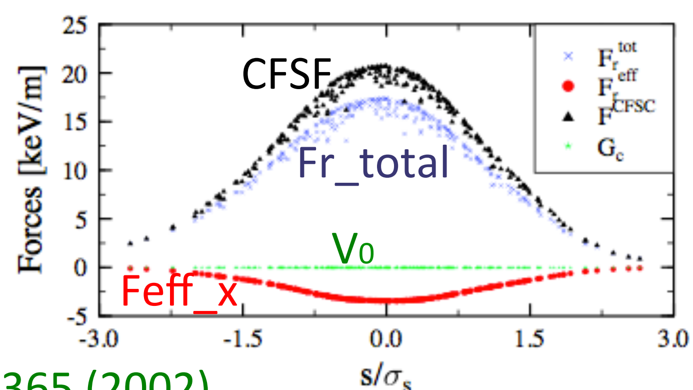
No local singularity

$$\frac{d\mathcal{E}}{cdt} = \boxed{\frac{\partial V_0}{c\partial t}} - e\frac{d\varphi}{cdt}$$

Centrifugal Space charge force

Effective longitudinal force

From effect on the SC invariants, Derbenev and Shiltsev stated that the cancellation shown by E. Lee for a coasting beam is valid for any relativistic bunch.



Li, EPAC 2002, 1365 (2002)

Cancellation of the Potent Driving Terms in the Transverse Dynamics on a Circular Orbit

Li, Proc. of HBHB workshop, 369 (1999)

- Equation of Motion

$$\frac{d^2x}{c^2dt^2}+\frac{x}{R^2}=\frac{\Delta\mathcal{E}(t)}{R\mathcal{E}_0}+\frac{F_r^{col}(t)}{\mathcal{E}_0}$$

$$\boxed{\frac{\mathcal{E}(t)-\mathcal{E}_0}{R\mathcal{E}_0}=\frac{1}{R\mathcal{E}_0}\left((\mathcal{E}(0)-\mathcal{E}_0)-e(\cancel{\varphi(t)}-\varphi(0)+\int\frac{\partial V_0}{\partial t}dt\right)}$$

$$\boxed{\frac{F_r^{col}(t)}{\mathcal{E}_0}=\frac{1}{\mathcal{E}_0}\left(-\frac{\partial V_0}{\partial x}-e\frac{dA_x}{cdt}+\cancel{e\frac{A_s(t)}{r}}\right)}$$

Potent terms Cancelled !

- Reorganize Eq. of Motion

$$\frac{d^2x}{c^2dt^2}+\frac{x}{R^2}=\frac{\delta_{tot}}{R}+\frac{1}{\mathcal{E}_0}\left(\underbrace{\frac{1}{R}\int\frac{\partial V_0}{\partial t}dt}_{\text{dominant}}-\frac{\partial V_0}{\partial x}\underbrace{-e\frac{dA_x}{cdt}+\frac{V_0}{R}}_{\text{small}}\right)$$

$$\delta_{tot}=\frac{[\mathcal{E}(0)+e\varphi(0)]-\mathcal{E}_0}{\mathcal{E}_0}$$

Act as initial energy spread

$$V_0=e(\varphi-\beta A_s)=e^2\int d\vec{r}'\frac{\gamma^{-2}+\theta^2/2}{|\vec{r}-\vec{r}'|}$$

Kinetic energy change

$$\Delta E_0(t) = E(t) - E_0 = \Delta E_0(t=0) + \int_0^t dt\,\left(E_s^{col} v_s + E_r^{col} v_r\right)$$

Transverse dynamics

$$\frac{d^2 x}{c^2 dt^2} + \frac{x}{R^2} = \frac{\Delta\mathcal{E}(t)}{R\mathcal{E}_0} + \frac{F_r^{col}(t)}{\mathcal{E}_0}$$

We now look at the previous studies from the new perspective:

Talman (1985)

$$F^{CSCF} = e\frac{A_s}{r} = \frac{\lambda}{4\pi\varepsilon_0 R}\ln\frac{\tan(\theta_{max}/4)}{\tan(\theta_{min}/4)}$$

cancelled by $-e\frac{\varphi - \varphi_0}{R}$ in $\frac{\Delta\mathcal{E}}{R}$

Lee (1990)

$$\gamma(R)mc^2 + e\varphi(R) = eRB^{ext}$$

(Equilibrium orbit)

$$\omega_r^2 \approx \frac{c^2}{R^2} + \frac{e}{\gamma m}\left(\frac{1}{R}\frac{\partial\varphi}{\partial r} - \frac{1}{R}\frac{\partial A_s}{\partial r}\right)$$

Carlsten (1995):

$$F^{NSCF} = -e\frac{d\varphi}{cdt}, \qquad \frac{1}{R}\int_0^t F^{NSCF}(t')v_s dt' = -e\frac{\varphi(t) - \varphi(0)}{R}$$

cancelled by $e\frac{A_s}{R}$ in F_r

Bane and Chao (2002):

$$\frac{\Delta\mathcal{E}_k}{R} = \frac{e(\varphi - \varphi_0)}{R} = -\frac{e}{R}\int E_s^{col}(r)ds$$

cancelled by $e\frac{A_s}{R}$ in F_r

(In the 4th dipole)

Geloni *et al.* (2002):

$$F_r^{col}\Big|_{\text{head-tail}} \simeq \frac{e^2}{4\pi\varepsilon_0}\frac{1}{R\Delta s} = \frac{eA_s}{R}$$

cancelled by $-e\frac{\varphi - \varphi_0}{R}$ in $\frac{\Delta\mathcal{E}}{R}$

That explains the good agreement of Elegant simulation with experiment.

Li, Proc. of HBHB workshop, 369 (1999)   Li, PAC 2003, 208 (2003)   Li, PAC 2005, 1631 (2005)

Canonical Formulation of EM Dynamics on a Curved Trajectory

Li, EPAC 2002, 1365 (2002)

Li and Derbenev, JLAB-TN-02-054 (2002)

- Least action principle

$$S = -\int_{\tau_1}^{\tau_2} P^\mu dx_\mu = \int_{\tau_1}^{\tau_2} \left[\underbrace{-mc\sqrt{U^\mu U_\mu}}_{L_0} \underbrace{-\frac{e}{c} A^\mu U_\mu}_{L_{\text{int}}} \right] d\tau$$

Canonical momentum: $$P^\mu = p^\mu + eA^\mu / c = \left(\frac{\mathcal{E} + e\varphi}{c}, \vec{p} + e\vec{A} / c \right)$$

- Euler-Lagrangian Equation

$$\frac{dP^\mu}{d\tau} = -\frac{\partial L_{\text{int}}}{\partial x_\mu} \xrightarrow{\text{Conventional approach}} \frac{dp^\mu}{dt} = \frac{e}{c} F^{\mu\nu} U_\nu$$

$$\begin{cases} \dfrac{d(\vec{p} + e\vec{A}/c)}{dt} = \nabla L_{\text{int}} = -e\left(\nabla\varphi - \beta_i \nabla A_i\right) \\[2ex] \dfrac{d(\mathcal{E} + e\varphi)}{dt} = -\dfrac{\partial L_{\text{int}}}{\partial t} = -e\left(\dfrac{\partial \varphi}{\partial t} - \vec{\beta} \cdot \dfrac{\partial \vec{A}}{\partial t}\right) \end{cases}$$

(Cartesian frame)

$$\begin{cases} \dfrac{d\vec{p}}{dt} = \vec{F} = e\left(\vec{E} + \dfrac{\vec{v}}{c} \times \vec{B}\right) \\[2ex] \dfrac{d\mathcal{E}}{dt} = \vec{F} \cdot \vec{v} \end{cases}$$

$$\vec{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \nabla \times \vec{A}$$

Canonical Formulation on Straight Path

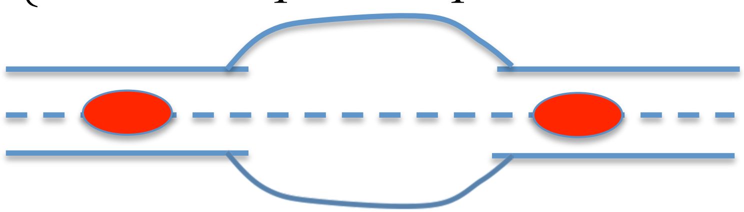
Some not-so-easy to prove theorem for dynamics in EM field can be straightforwardly shown in terms of canonical formulation

- Wake function on straight path (for E, B being RF fields or wake field)

$$\begin{cases} w_{\parallel}(\vec{\rho},s) = -\dfrac{c}{q}\Delta p_{\parallel} = -\dfrac{c}{q}\displaystyle\int dt\ \ E_z\Big|_{z=ct-s}, \\ w_{\perp}(\vec{\rho},s) = \dfrac{c}{q}\Delta p_{\perp} = \dfrac{c}{q}\displaystyle\int dt\ \left(\vec{E}_{\perp} + \vec{z}\times\vec{B}\right)_{z=ct-s} \end{cases}$$

$$\frac{\partial w_{\perp}}{\partial s} = \nabla_{\rho} w_{\parallel}$$

Panofsky-Wenzel Theorem



For the canonical momentum $P = p + eA/c$

$$\frac{d\vec{P}}{dt} = -e\nabla(\varphi - \vec{\beta}\cdot\vec{A})$$

$$\begin{cases} \Delta p_{\perp} + \cancel{e\Delta A_{\perp}/c} = -e\displaystyle\int \nabla_{\perp}(\varphi - \vec{\beta}\cdot\vec{A})\ dt \\ \Delta p_{\parallel} + \cancel{e\Delta A_{\parallel}/c} = -e\displaystyle\int \nabla_{\parallel}(\varphi - \vec{\beta}\cdot\vec{A})\ dt \end{cases}$$

$$\frac{\partial \Delta p_{\perp}}{\partial s} = \frac{\partial \Delta p_{\parallel}}{\partial r_{\perp}}$$

This can hold only when $F_{\perp}$ and $F_{\parallel}$ are calculated accurately

Stupakov, SLAC-PUB-8683 (2002)

Canonical Formulation on Curved Orbit

Li, EPAC 2002, 1365 (2002)   Li and Derbenev, JLAB-TN-02-054 (2002)

Projecting on Eular-Lagrangian equation on the radial basis:

$$\frac{d\left(p_x + eA_x/c\right)}{dt} = \frac{d(\vec{P}\cdot\vec{e}_x)}{dt} = \frac{d\vec{P}}{dt}\cdot\vec{e}_x + \vec{P}\cdot\frac{d\vec{e}_x}{dt}$$

$$\frac{d\left(p_x + eA_x/c\right)}{dt} = -e\frac{\partial(\varphi - \vec{\beta}\cdot\vec{A})}{\partial x} + v_s\frac{p_s + eA_s/c}{r}$$

Usual centrifugal force

Centrifugal Space charge force

Generalized centrifugal force:

$$F^{GCF} = v_s\frac{p_s + eA_s/c}{r} \approx \frac{\mathcal{E} + e\varphi}{r} + e\frac{A_s - \varphi}{r}$$

$$\frac{d(\mathcal{E} + e\varphi)}{dt} = e\left(\frac{\partial\varphi}{\partial t} - \vec{\beta}\cdot\frac{\partial\vec{A}}{\partial t}\right), \quad (\mathcal{E} + e\varphi)_t = (\mathcal{E} + e\varphi)_0 + \int dt\ e\left(\frac{\partial\varphi}{\partial t} - \vec{\beta}\cdot\frac{\partial\vec{A}}{\partial t}\right)$$

Transverse equation of motion:

$$\frac{d(p_x + eA_x/c)}{dt} = F_x^{ext} + \frac{\mathcal{E}_{tot}(t=0)}{r} - \frac{1}{r}\int_{t_0}^{t}\frac{\partial L_{\text{int}}}{\partial t'}dt' + \frac{\partial L_{\text{int}}}{\partial x}$$

No local singularity

This can only hold when Fr and Fs are calculated accurately, and dynamics are advanced self-consistent

Discussions

- The cancellation is explicit in terms of the geometric effect of canonical momentum
- The usual approach using Lorentz force and Lienard-Wiechert is equivalent to the canonical one: the cancellation can be taken care of implicitly if
  - BOTH longitudinal and transverse CSR/LSC forces are included
  - dynamics advanced self-consistently.
- This may explain why 1D CSR model works well, since in 1D rigid-line bunch model,
  - both $e\varphi(t)/r$ and eA_s/r are not included
  - Often 1D CSR force is approximately the effective longitudinal CSR force
  - The transverse effective CSR force has much smaller effects
  - The initial total energy offset (including potential energy) does not cause emittance growth for achromatic bending systems
- The canonical momentum $P^\mu = p^\mu + eA^\mu / c$ works as a dynamical entity is natural since EM field is fundamentally quantum gauge field.

Minimal coupling in QM: $p^\mu \rightarrow p^\mu + eA^\mu / c \quad \text{or} \quad ih\partial_\mu \rightarrow ihD_\mu = ih\partial_\mu + eA_\mu / c$

- Busch's theorem: conservation of canonical angular momentum $\vec{L} = \vec{r} \times \vec{P}$

Behavior of Residual Force Terms

- Complete Transverse dynamical equation

dominant

$$\frac{d^2x}{c^2dt^2}+\frac{x}{R^2}=\frac{\Delta\delta_{tot}(t=0)}{R}+\frac{1}{R\mathcal{E}_0}\int_0^t F_v^{eff}(t')c\,dt'+\frac{F_r^{eff}(t)}{\mathcal{E}_0}$$

Correlated perturbations causing $(\Delta x_c,\Delta x_c',\Delta z_c,\Delta\delta_c)$

Driving Factors:

$$F_v^{eff}=e\frac{\partial(\varphi-\beta\cdot A)}{c\partial t},$$

Effective longitudinal CSR force

$$F_r^{eff}=-e\frac{\partial(\varphi-\beta\cdot A)}{\partial x}-e\frac{dA_x}{cdt}+\frac{e(A_s-\varphi)}{r}$$

Effective transverse CSR force

$$\delta_{tot}(t=0)=\frac{\mathcal{E}(0)+e\varphi(0)-\mathcal{E}_0}{\mathcal{E}_0}=\delta_{k0}+\delta_{\varphi0}$$

Canonical relative energy offset

$$(\text{for } \delta_{\varphi0}=e\varphi(0)/\mathcal{E}_0)$$

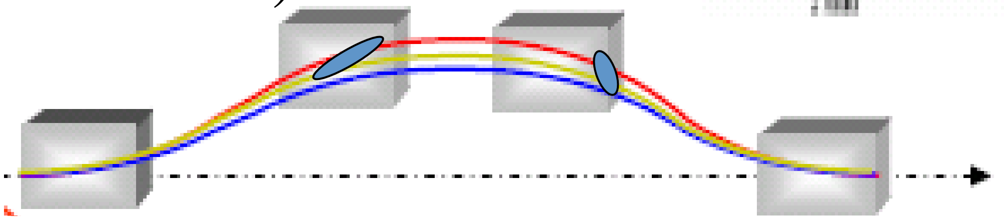
Perturbation to linear optics:

$$\begin{cases} x=R_{11}x_0+R_{12}x_0'+R_{16}(\delta_{k0}+\delta_{\varphi0})+\Delta x_c \\ x'=R_{21}x_0+R_{22}x_0'+R_{26}(\delta_{k0}+\delta_{\varphi0})+\Delta x_c' \\ z=z_0+R_{51}x_0+R_{52}x_0'+R_{56}(\delta_{k0}+\delta_{\varphi0})+\Delta z_c \\ (\delta_k+\delta_\varphi)=(\delta_{k0}+\delta_{\varphi0})+\Delta\delta_{kc} \end{cases}$$

$\delta_{\varphi0}$ acts as initial energy spread and does not cause emittance growth in achromatic system

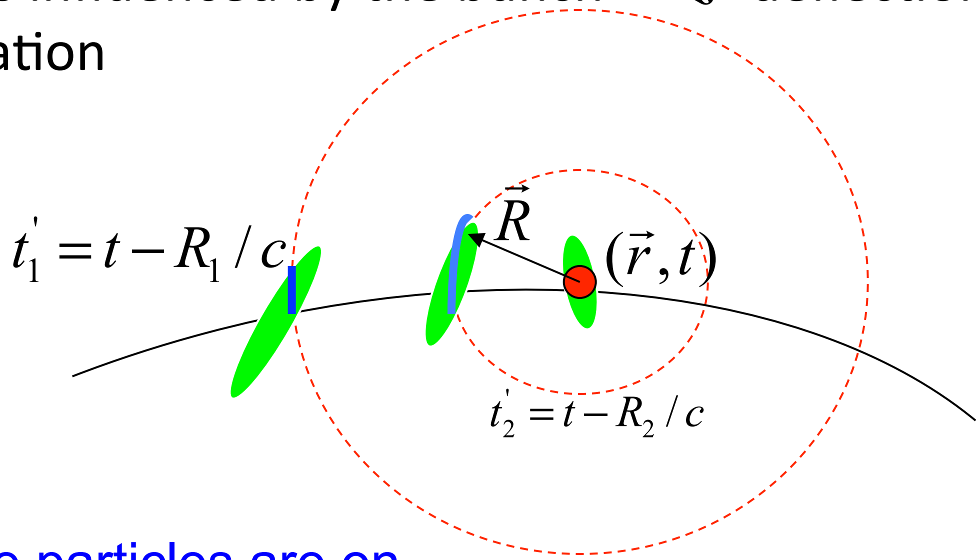
Longitudinal Effective Force F_v^{eff}

- x-z correlation for an energy-chirped (δ-z correlated) bunch in dispersive region (x- δ correlated)



(Drawing from P. Krejcik, SLAC)

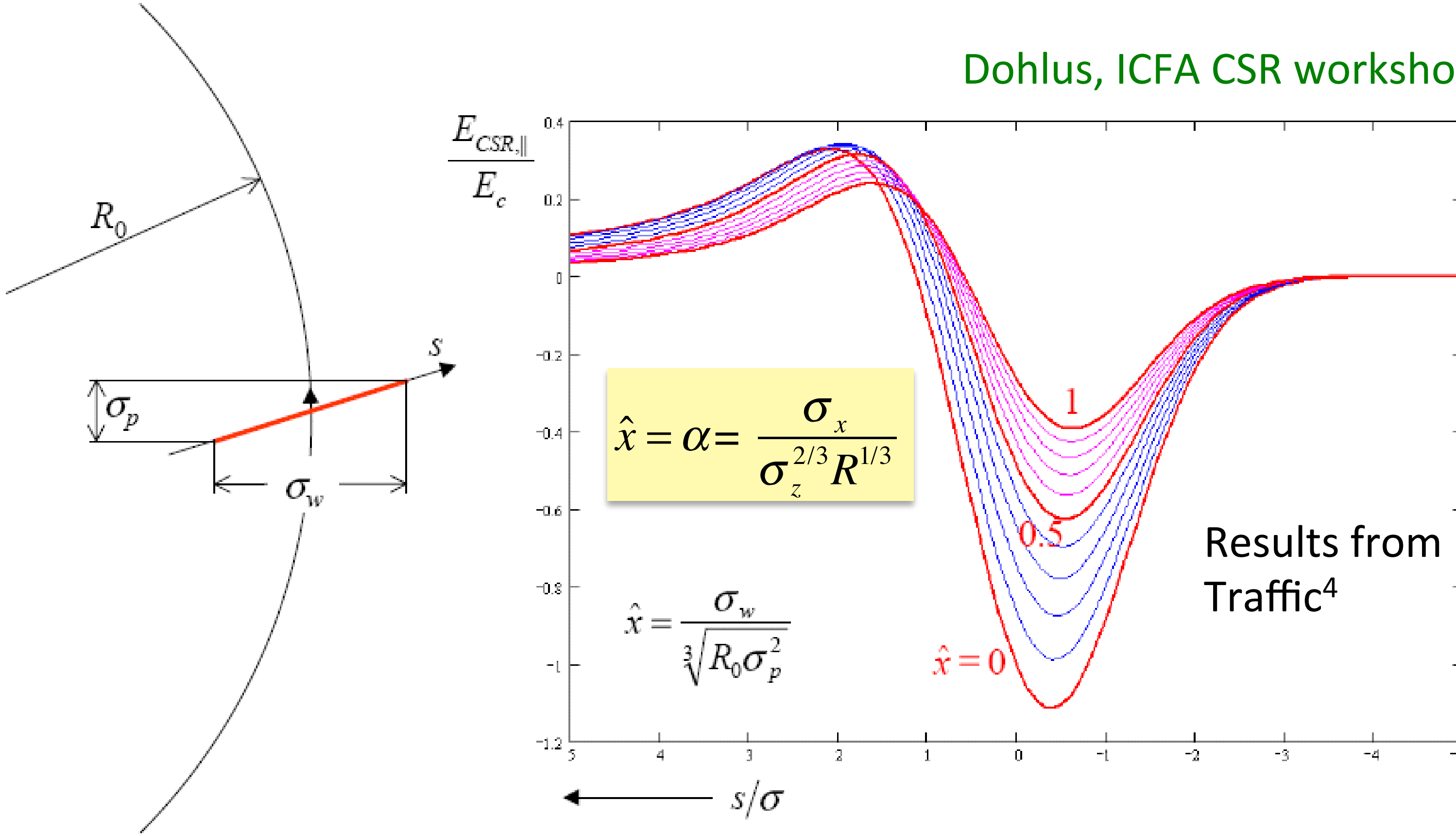
- The CSR force is influenced by the bunch x-z deflection mainly through retardation



For each t', the source particles are on a cross-section of light cone with the x-s correlated bunch

CSR Field of a Tilted Thin Beam

Dohlus, ICFA CSR workshop (2002)



e.g. $R_0 = 10\,\mathrm{m}, \sigma_p = 100\,\mu\mathrm{m}, \sigma_w = 2\,\mathrm{mm} \rightarrow \hat{x} = 0.43$

Martin Dohlus Deutsches Elektronen Synchrotron ICFA Workshop Jan. 2002

As the deflection gets bigger, the CSR force is smaller in amplitude as compared to the Rigid-bunch result (with the same projected bunch length)

Analytical Result of Effective Longitudinal CSR Force (for a thin Gaussian bunch)

$$\tilde{F}_H(\tilde{z}_0,\alpha) \simeq \frac{2Nr_e}{\gamma_0 3^{1/3}\sqrt{2\pi}[\sigma_z(s)]^{4/3}|R_0|^{2/3}} I(\tilde{z}_0,\alpha),$$

$$I(\tilde{z}_0,\alpha) = \frac{3^{1/3}}{4}\int_0^\infty d\Delta\tilde{s}\frac{\Delta\tilde{s}}{\Lambda_1(\Delta\tilde{s},\alpha)}(\tilde{z}_0-\Delta\tilde{z}_0)\exp\left[-\frac{(\tilde{z}_0-\Delta\tilde{z}_0)^2}{2}\right]$$

$$\Lambda_1(\Delta\tilde{s},\alpha) = \left(1+\frac{\alpha\Delta\tilde{s}}{2}\right)\sqrt{\left(1+\frac{\alpha\Delta\tilde{s}}{2}\right)^2+\frac{(\alpha\Delta\tilde{s})^2}{12}}.$$

$$\alpha = \frac{\sigma_x}{(\sigma_z^2 R)^{1/3}}$$

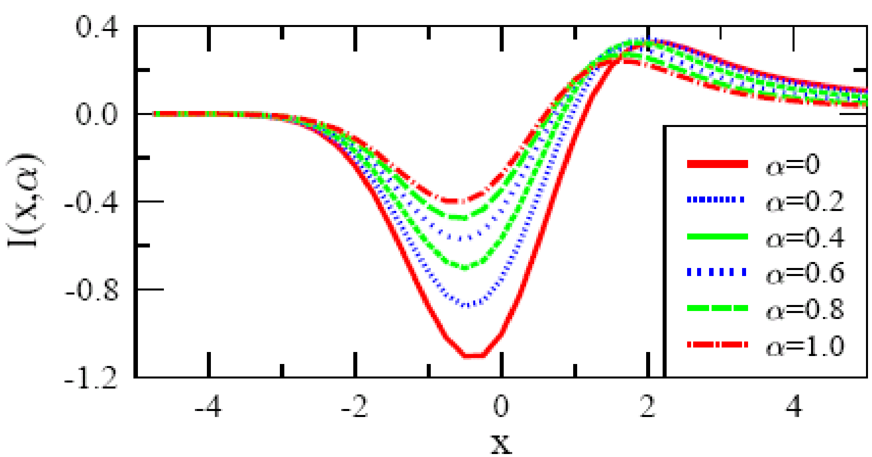


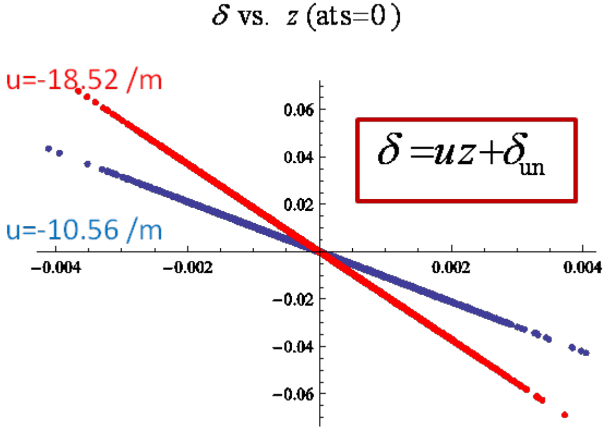
FIG. 2: $I(x,\alpha)$ vs. x for various α given by Eq. (122).

agree with Dohlus' Trafic4 result

Li, PRST-AB 11, 024401 (2008)

•Roll-over (parasitic) compression around end of middle dipole

Initial energy chirp of the bunch at entrance of the chicane



The Model Chicane :
total path length=2m

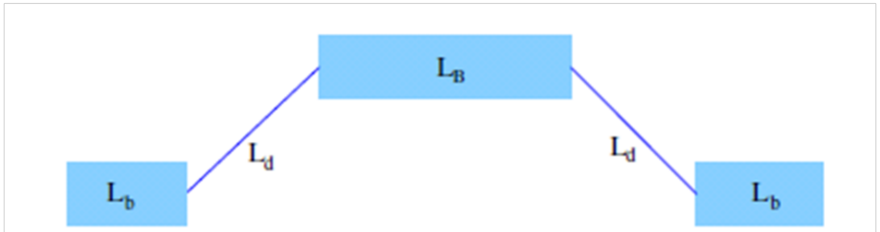
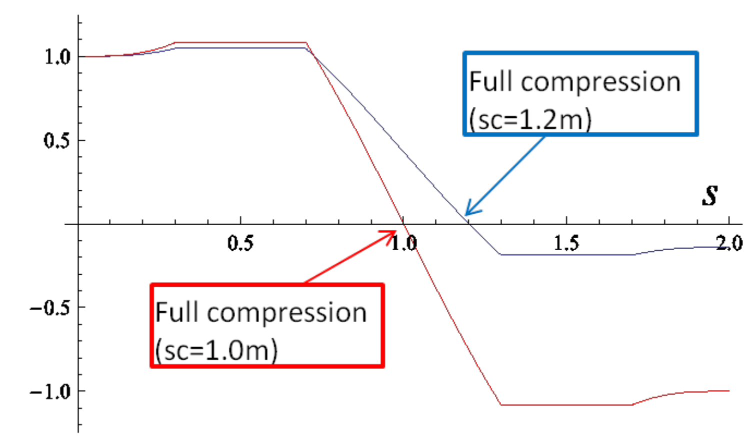
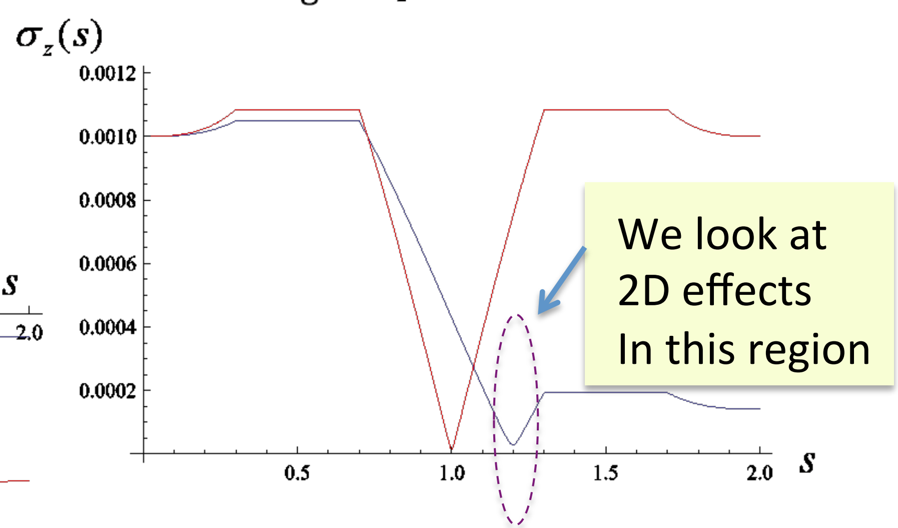


FIG. 3. (Color) Model chicane: $R = 1$ m, $L_b = 0.3$ m, $L_B = 0.6$ m, and $L_d = 0.4$ m.

Bunch compression factor vs s



Bunch length $\sigma_z(s)$ vs s



Effective longitudinal CSR force over bunch distribution during roll-over compression

$E = 70\ \mathrm{MeV}$

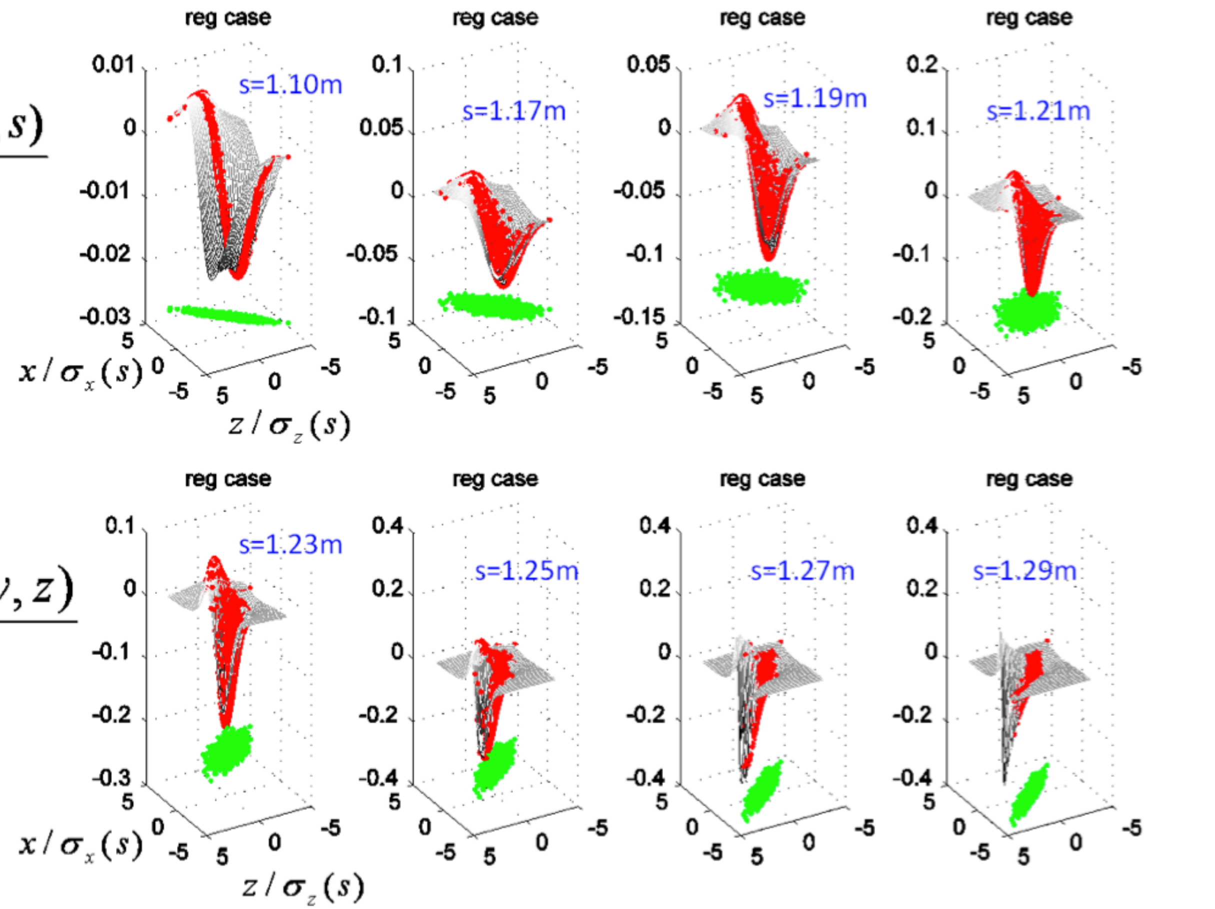
$\varepsilon_x = 1.0\mu\mathrm{m}$

$\delta_{un} = 10^{-4}$

Li, PRST-AB 11, 024401 (2008)

$$\frac{F_z^{CSR}(x,z,s)}{F_0}$$

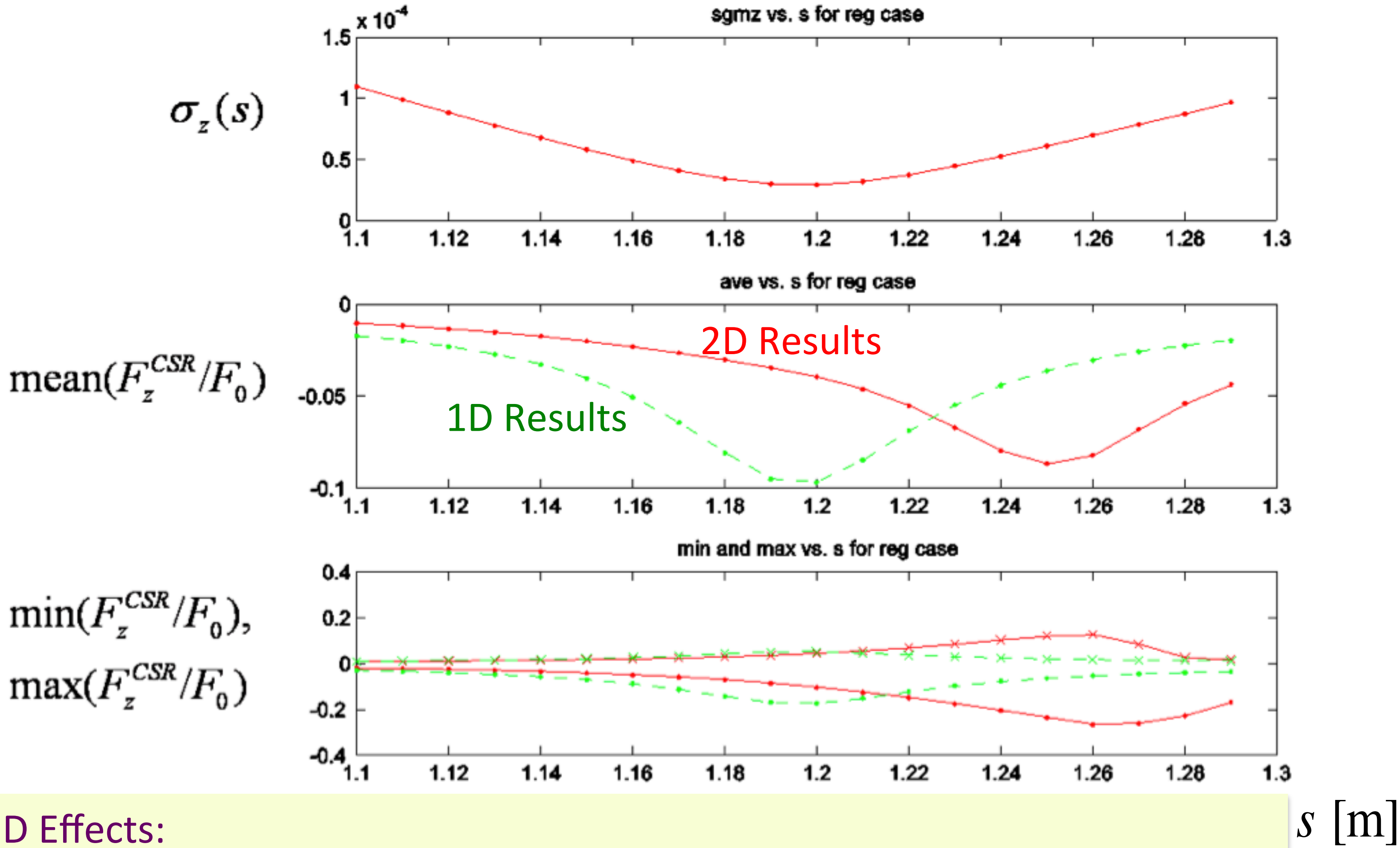
$$\frac{F_z^{CSR}(x,y,z)}{F_0}$$



Features of 2D Effects:

- Effective longitudinal CSR force could have strong dependence on x-coordinates during roll-over compression
- This may act as an impulse and cause slice emittance growth during roll-over for an over-compressed bunch

Bunch length and total energy loss of the bunch during roll-over compression



Features of 2D Effects:

- Delayed response of both CSR force and energy loss to the variation of bunch length due to retardation, this explains Dohlus' observation.
- Integrated total energy loss through roll-over compression for 2D CSR is similar to the 1D case

Effective Transverse Force F_r^{eff}

- Compare the driving terms in the transverse dynamical equation,

$$\frac{d^2x}{c^2dt^2}+\frac{x}{R^2}=\frac{\delta_{tot}(t=0)}{R}+\frac{1}{\mathcal{E}_0}\left(\frac{1}{R}\int_0^t F_v^{eff}(t')c\,dt'+\boxed{F_r^{eff}(t)}\right)$$

For a Gaussian bunch with angular distribution $\lambda_\phi=\frac{1}{\sqrt{2\pi}}e^{-\phi^2/2}$

$$F_r^{eff}\simeq-\frac{2Ne^2}{R\sigma_s}\lambda_\phi(\phi),\qquad F_s^{eff}\simeq\frac{2Ne^2}{\left(3R^2\sigma_s^4\right)^{1/3}}\frac{d}{d\phi}\int_0^\infty\frac{d\phi_1}{\phi_1^{1/3}}\lambda_\phi(\phi-\phi_1)$$

$$\left\|\frac{F_r^{eff}}{F_v^{eff}}\right\|:\left(\frac{\sigma_s}{R}\right)^{1/3}\ :\ \text{overtaking angle}$$

Derbenev et al, TESLA-FEL-Report 1995-05 (1995)

Li, Proc. of HBHB workshop, 369 (1999)

F_r^{eff} is not negligible compared to $\frac{1}{R}\int_0^t F_v^{eff}(t')c\,dt'$ during formation length,

but the overalll driving forces are dominated by F_v^{eff}

Relative Potential Energy $\delta_{\varphi 0}$

$$\frac{d^2x}{c^2dt^2}+\frac{x}{R^2}=\frac{\delta_{k0}+\delta_{\varphi 0}}{R}+\frac{1}{\mathcal{E}_0}\left(\frac{1}{R}\int_0^t F_v^{eff}(t')cdt'+F_r^{eff}(t)\right)$$

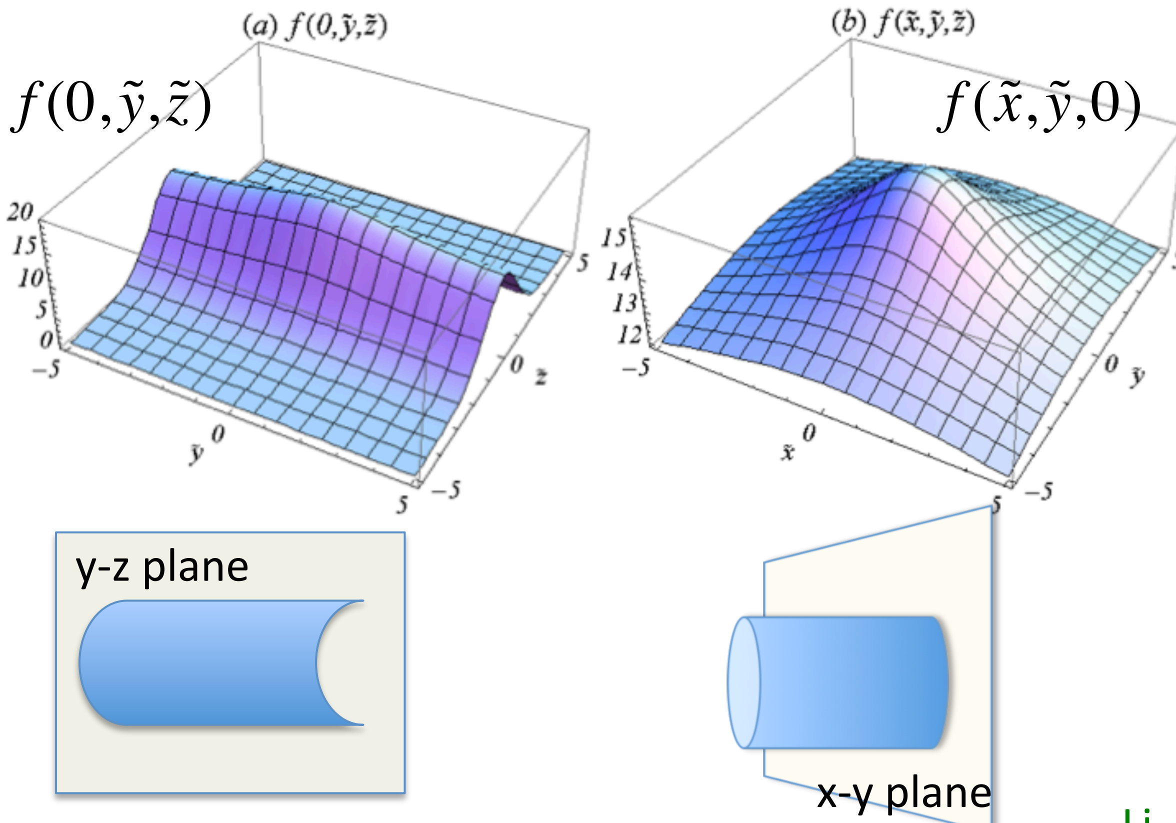
with $\delta_{\varphi 0}=\mathcal{E}_{\varphi 0}/\mathcal{E}_0$ for $\mathcal{E}_{\varphi 0}=e\varphi(0)$

- For a bunch with low peak current, the effect of $\delta_{\varphi 0}$ is negligible compared to δ_{k0}
- It is called pseudo energy spread because it appears in all transverse and longitudinal measurement as effects of kinetic energy spread
- It should also play a role of Landau damping for microbunching instability
- Unlike δ_{k0} , $\delta_{\varphi 0}$ has strong correlation with transverse and longitudinal density distribution

Potential Energy for a Gaussian Bunch

For a cylindrically symmetric 3D Gaussian bunch with rms size (σ_r, σ_z), the potential is

$$\mathcal{E}_\varphi = e\varphi(x,y,z) = \left(mc^2 \frac{I_p}{I_A} \right) \cdot f\left(\frac{x}{\sigma_r}, \frac{y}{\sigma_r}, \frac{z}{\sigma_z} \right)$$

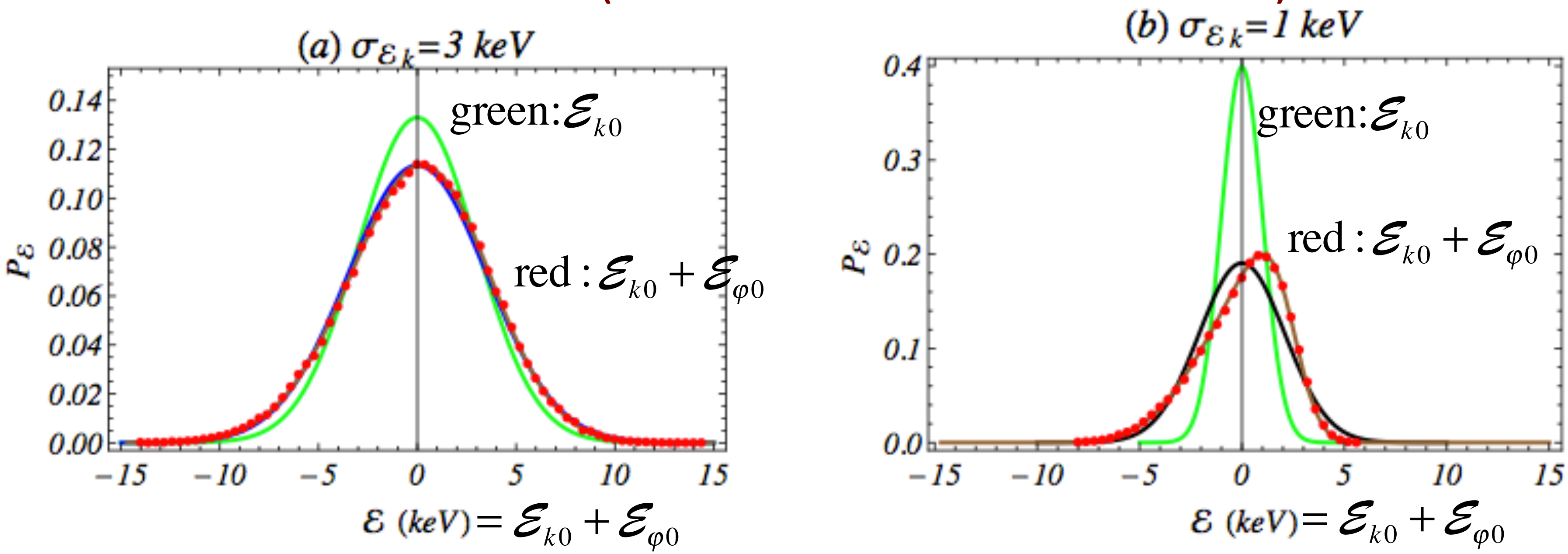


Li, arXiv: 1401.2868 (2014)

Pseudo Slice Energy Spread

Example: $\mathcal{E}_0 = 135\ \mathrm{MeV},\quad \sigma_z = 750\ \mu\mathrm{m},\ \gamma_0\varepsilon_x = \gamma_0\varepsilon_y = 1\ \mu\mathrm{m},\ I_p = 120A$

Contribution of potential energy to the total energy distribution for z=0 slice ($\mathcal{E}_{k0}$ has Gaussian distribution)



When I_p is high and $\sigma_{\mathcal{E}_k}$ is low, σ_φ could be appreciable and Its effects could be measurable.

Implication to Simulation and Experiments

Experiments:

F_v^{eff}

- Has strong x-dependence during roll-over compression, can cause increase of slice emittance

$\delta_{\varphi 0}$

- Can cause observable effects only at high peak current
- Enters into bunch energy spread observed in dispersive region
  - Increase of observed slice energy spread (important for HGHG, EEHG and HHG)
  - Modifies longitudinal phase space curvature
  - May affect the minimum bunch length $R_{56}\sigma_{\varepsilon_{tot}}$ at full compression

Presently machines are designed and operated at bunch charge of hundred pC

Detecting these phenomena require bunches with high charge and small transverse emittance and slice kinetic energy spread.

Simulations

The canonical formulation is helpful for theoretical understanding of the interplay of F_r^{col} and F_s^{col} , but numerical modeling still need to use Lorentz force in terms Of E and B fields.

1D CSR Model

- Take care cancellation in CSR by not including F_r^{col} and the x-dependence of F_s^{col}
- Keeping only the dominant $F_s^{col}(z)$ term

2D/3D CSR Model

- 2D model takes care effect of large σ_x in dispersion region on the retardation by correctly identify the source particles
- All terms related to cancellation, including residual δ_φ , depends on 3D bunch distribution, are dominated by local interaction contributions.
- To include all effects and to let the cancellation play out, through correct treatment of EM fields and dynamical advances, a fully self-consistent 3D CSR and space charge code is needed.

Summary

- The Lorentz force derived from E and B fields indeed contains potent terms that has strong dependence on the particles' transverse position. But its effect on transverse dynamics cancels with that from potential energy effect.

- To correctly model the cancellation in the simulation, the F_r^{col} and $\Delta\varphi^{col}$ needs to be calculated with the same accuracy

- The cancellation is a manifestation that $\vec{p} + e\vec{A}/c$ experience the centrifugal geometric effect together as a dynamical entity

- The residual driving terms influence the dynamics when the bunch peak current is high and slice emittance and energy spread is small