

## MEASUREMENT AND CALCULATION OF THE 'ELECTRON EFFICIENCY' ON THE 'CLIO' FREE-ELECTRON LASER

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### Abstract

This paper describes the recent measurements of 'Electron efficiency' which has been done with the 'CLIO' Free-electron laser (FEL). The 'Electron efficiency' is the relative energy loss of the electron beam in the undulator during the FEL interaction. It gives an absolute measurement of the optical power produced by the FEL, which is then compared to the direct measurement using a power meter at the exit of the FEL. An analytical expression of the 'Electron efficiency' is given here, and it is compared to the measurements.

### THE CLIO FREE ELECTRON LASER

The 'CLIO' Free Electron Laser (FEL) is an infrared tuneable laser source [1]. It uses a linear accelerator, which produces an electron beam of adjustable energy from 13MeV to 50MeV. The laser spectral range is tuneable from 5 $\mu$ m to about 120 $\mu$ m. Such a wide broadband is achieved by using, for  $\lambda > 100\mu$ m, a special set of toroidal mirrors. Also, in order to reduce the cavity losses at large wavelength, a waveguide is installed in place of the vacuum chamber in the undulator section. The laser extraction of the cavity is achieved by hole coupling in the front mirror.

The best configuration of radius of curvature for the cavity mirrors has been obtained with a numerical code [2] [3]. This code calculates the propagation of the laser wave front  $A(x,y)$  in the optical cavity. It uses an iterative process of wave propagation in the cavity, which converges to a steady state laser mode  $A_s(x,y)$  corresponding to the FEL saturation regime. It takes into account the design of the cavity (mirrors, vacuum chamber,...) including the waveguide effect in the undulator section, and the hole coupling in the front mirror. This code gives the amplitude distribution  $A_s(x,y)$  of the laser mode at saturation, in any point of the cavity, and it gives all related parameters : the cavity losses  $L$ , the 'extraction rate'  $T_x$  of hole coupling, the optical mode cross-section  $\Sigma_o, \dots$

### 'ELECTRON EFFICIENCY', AND FEL POWER

In order to measure the energy distribution of the electrons after FEL interaction, an electron spectrometer [1] is installed at the exit of the undulator. It gives an experimental value of the 'Electron efficiency'  $\eta$ , which corresponds to the percentage of energy of the electron beam which is transferred to the laser optical mode, during the FEL interaction :

$$\eta = \Delta We / We \quad (1)$$

where  $We = Q \cdot (\gamma mc^2 / e)$  is the electron bunch energy, with  $Q$  the charge of each electron bunch, and  $\Delta We$  is the

amount of energy produced in FEL interaction by each electron micro-bunch. The average extracted power (energy per second) of the FEL can be deduced from  $\eta$  by

$$\langle P_x \rangle \cong W_e \cdot \eta \cdot \Delta T_{sat} \cdot \frac{T_x}{L} \cdot f_\mu \cdot f_M \quad (2)$$

where  $f_M$  and  $f_\mu$  are respectively the repetition rates of macro-pulses and micro-pulses, and  $\Delta T_{sat}$  is the time duration of the FEL saturation in the macro-pulse. The ratio  $T_x/L$  is the 'extraction ratio' between the losses by hole coupling  $T_x$  and the total losses  $L$  of the laser cavity. It depends on the intracavity laser mode, and it is obtained here by numerical simulation. The power  $\langle P_x \rangle$ , deduced from expression (2), is compared here to the direct measurement using a detector at the exit of the FEL.

In order to fit the measurements of 'Electron efficiency' with the theory, we have written an analytical expression [4] for  $\eta$ , which is based on the analytical expression [5] [6] of the FEL intensity at saturation:

$$\eta \approx \frac{1}{4N_u} \cdot (\alpha + 0.22 \cdot g_o \alpha^2 + 4.85 \cdot 10^{-3} \cdot g_o^2 \alpha^3) (1 - e^{-\eta}) F_f \left( \frac{\Sigma_o \cdot \Delta t_o}{\Sigma_e \cdot \Delta t_e} \right) \quad (3)$$

where  $g_o$  is the gain coefficient for linearly polarized undulators :

$$g_o = \frac{16\pi}{\gamma} \lambda_R L_u \frac{J_e N_u^2}{1.7 \cdot 10^4} \xi [J_0(\xi) - J_1(\xi)]^2 \quad (4)$$

where  $\xi = \frac{1}{4} \frac{K^2}{1 + K^2/2}$  and  $K$  is the undulator parameter,

$N_u$  and  $L_u$  are respectively the number of periods and length of the undulator,  $\lambda_R$  is the resonance wavelength and  $J_e$  is the density current of electrons. The coefficient  $\alpha = F_{inh} \cdot F_S$  in expression (3) is an attenuation factor for the gain coefficient  $g_o$ . It depends on the inhomogeneous broadening factor  $F_{inh}$  [5] :

$$F_{inh} = \frac{1}{1 + 1.7 \cdot \mu_e^2} \cdot \frac{1}{1 + \mu_y^2} \quad (5)$$

with  $\mu_e = 4N_u (\sigma_y / \gamma)$  depending on the relative RMS energy spread ( $\sigma_y / \gamma$ ) of the electron beam, and

$$\mu_y = \frac{N_u \sqrt{2}}{\lambda_u} \frac{K}{1 + K^2/2} \cdot \mathcal{E}_{nor} \quad \text{corresponding [7] to the}$$

normalized emittance  $\mathcal{E}_{nor} = 4\pi\gamma\sigma\sigma'$  along vertical axis ( $y$ ). The longitudinal mode coupling factor  $F_S$  is [6] :

$$F_S = \frac{1}{1 + \mu_c/3} \quad (6)$$

where  $\mu_c = N_u \lambda_R / c\sigma_e$  is the ratio between slippage length  $N_u \lambda_R$  and electron bunch length  $c\sigma_e$ , with  $\sigma_e$  the RMS time duration of electron bunch. The last part of expression (3) involves the size of the laser pulse :  $\Sigma_o = \pi\sigma_{ox}\sigma_{oy}$  and  $\Delta t_o = \sqrt{\pi}\sigma_o$ , which are respectively the cross-section and time duration of intensity, with  $\sigma_o$  the

RMS time duration of the laser mode amplitude. The size of the electron bunch is :  $\Sigma_e = 2\pi\sigma_{ex}\sigma_{ey}$  for the cross-section and  $\Delta t_e = \sqrt{2\pi}\sigma_e$  for the time duration with  $\sigma_e$  the RMS time duration. The ‘filling factor’, used in expression (3), is depending on the transverse cross sections:  $F_f \cong 1/(1+\Sigma_o/\Sigma_e)$ . The coefficient  $h = [1.8/(1+G_p^*)] \cdot [(1-L)G_p^* - L]/L$  in expression (3) depends on the ‘Small Signal Gain’ of power  $G_p^*$ , and on the optical losses  $L$ . The ‘Small Signal Gain’ of power is :

$$G_p^* = F_f \cdot \left[ 0.85 \cdot (g_o \alpha) + 0.19 \cdot (g_o \alpha)^2 + 4.12 \cdot 10^{-3} \cdot (g_o \alpha)^3 \right] \quad (7)$$

where  $\alpha = F_{inh} \cdot F_s$ . Note that this expression includes the non linear components which are present for large values of the gain [5]. The expression (3) of ‘Electron efficiency’ also takes into account the non linear behaviour.

In first approximation, for small values of gain coefficient  $g_o$ , the expression (3) of  $\eta$  is close to  $1/4N_u$  multiplied by the attenuation factor  $\alpha$  which lies generally in the range 0.5 to 1. The factor  $[1-\exp(-h)]$  becomes important when the ‘small signal gain’  $G_p^*$  is close to the losses  $L$ . The last two terms of (3) are dependant on the volume of optical and electron bunches. Now, in high gain regime, i.e. for large values of gain coefficient  $g_o$ , the non linear behaviour of the gain  $G_p^*$  may increase strongly the ‘Electron efficiency’. Note that, the third order correction in (7) works [5] in a large range of  $g_o$  values ( $g_o \leq 20$ ) ; and this always keeps the ‘Electron efficiency’  $\eta < 1$ .

## MEASUREMENTS

An example of electron spectra is shown in figure 1, with  $\gamma mc^2 = 15.2 \text{ MeV}$  and laser wavelength  $\lambda = 72 \mu\text{m}$ . The horizontal axis of electron spectra is the time scale, and the vertical axis is the variation, in %, of the electron energy  $\gamma mc^2$  during the macro-pulse. Two spectra are displayed here: for ‘laser OFF’ and for laser ‘ON’. The time evolution of electron energy centroid  $\langle \gamma mc^2 \rangle(t)$  is represented by a curve on each spectrum. The two curves on bottom of figure 1 represent a Y-cut of the energy distribution. The dashed vertical lines represent the centroid of electron energy  $\langle \gamma mc^2 \rangle$ . The difference ( $\langle \gamma mc^2 \rangle_{\text{OFF}} - \langle \gamma mc^2 \rangle_{\text{ON}}$ ), between the centroids, corresponds to the ‘Electron efficiency’  $\eta(t) = \Delta W_e / W_e$ . At low electron beam energy (about 15 MeV), the whole series of measured spectra show, as observed in figure 1, an important slope of electron energy versus time at macro-pulse time scale : about 1.5% of energy variation in 10  $\mu\text{s}$ . Indeed, the electron beam at low energy is much more sensitive to all perturbations than at high energy ( $> 40 \text{ MeV}$ ). The slope of electron energy reduces the power at laser saturation, because the wavelength centroid is continuously shifted along the macro-pulse, by about  $\Delta\lambda/\lambda = 3\%$  (twice of electron energy shift), whereas the FEL gain line width is only about  $\delta\lambda/\lambda = 1/2N_u = 1.3\%$ . Therefore, the laser saturation is not fully obtained during the macro-pulse.

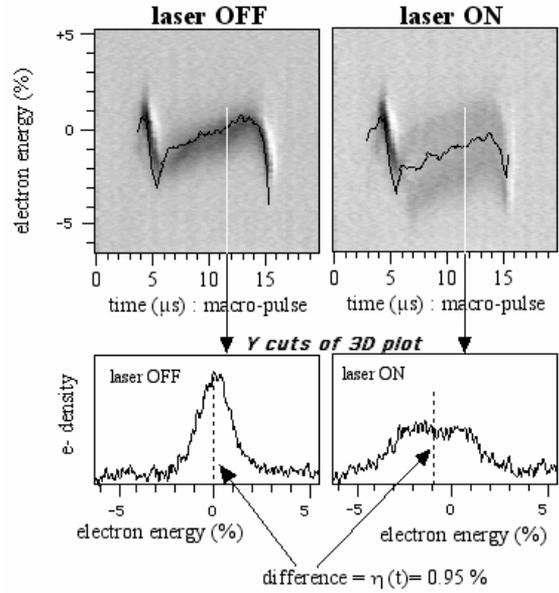


Figure 1: Electron energy spectrum at the undulator exit, with laser OFF and laser ON, at laser wavelength  $\lambda = 72 \mu\text{m}$ .

The loss of energy of the electrons during the FEL interaction is well represented by a subtraction between density spectra of figure 1 : ‘laser ON’-‘laser OFF’. This subtraction is represented in figure 2 by a series of Y-cut, performed in the middle of the electron macro-pulse. Each curve displays the subtraction of electron density (laser ON-laser OFF) as a function of the relative electron energy (in %). Each curve corresponds to a different undulator gap, from 18mm to 27mm, corresponding to laser wavelengths from  $\lambda = 49 \mu\text{m}$  to  $92 \mu\text{m}$ . The hollow which is observed here corresponds to a lack of electrons of nominal energy  $\gamma_o mc^2$  (energy at undulator entrance). The bump corresponds to an accumulation of electrons which have lost energy during the FEL interaction. The whole set of curves in figure 2 exhibit a variation of amplitude, but they are keeping the same shape and same position of the peaks. This last feature means that the energy loss  $\Delta\gamma mc^2 / \gamma_o mc^2$  of the electrons, during the FEL interaction, is independent on wavelength: it is close to 2% in any case. Note that this parameter  $\Delta\gamma mc^2 / \gamma_o mc^2$  is not equivalent to the ‘Electron efficiency’  $\eta$ , which values are always less than 2% (see figure 3). Indeed,  $\eta$  represents an average over the whole number of electrons, and it corresponds to the difference of centroid of the energy distributions.

At first sight, the similitude of the curves in figure 2 would lead to the conclusion that the ‘Electron efficiency’  $\eta$  is also independent on wavelength. However, as it will be shown below (in figure 3), it is not the case. Indeed, the variations of  $\eta$ , according to wavelength, are well represented in figure 2 by a variation of amplitude of the curves of electron density. These amplitude variations correspond to a difference in the number of electrons

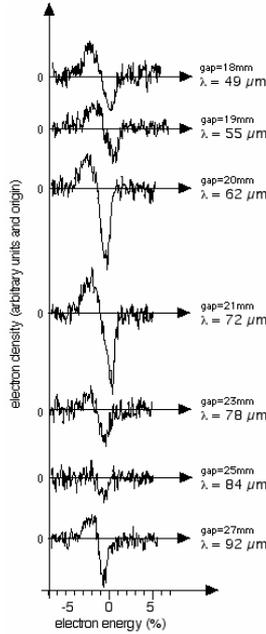


Figure 2 : Subtraction of electron density (laser ON-laser OFF) versus electron energy (in %). Each curve corresponds to a different undulator gap, and a different laser wavelength.

which are involved in the FEL interaction and which are losing energy. As a conclusion, a variation of  $\eta$  corresponds to a variation in the number of electrons involved in the FEL interaction, and not to a variation in the energy loss of the electrons. This conclusion is in good agreement with the fundamental theory [5] of the electron trapping in FEL interaction, which shows that the energy loss of the electrons is governed by the line width of the gain distribution, which only depends on  $1/N_u$  and not on the laser wavelength.

The figure 3 represents the ‘Electron efficiency’  $\eta$  as a function of the laser wavelength. The black dots represent an experimental estimation of the ‘Electron efficiency’  $\eta$  which is deduced from the electron energy spectra (such as of figure 1). The estimation of the electron energy centroid  $\langle \gamma mc^2 \rangle$ , from the electron spectra, is rather sensitive to the noise background of these spectra. This represents the main source of error in the estimation of the ‘Electron efficiency’  $\eta$ , because this parameter is obtained by a small difference between large and imprecise quantities  $\langle \gamma mc^2 \rangle_{OFF} - \langle \gamma mc^2 \rangle_{ON}$ . The experimental data in figure 3 are compared to a theoretical model, using the analytical expression (3). This expression is dependant on the electron beam cross-section  $\Sigma_e$ , which is not known with a good precision. Therefore, two theoretical curves are represented in figure 3. The line curve on the top of the grey area has been calculated with  $\sigma_{ex}=1.5\text{mm}$  and  $\sigma_{ey}=1\text{mm}$ ; and the line curve on the bottom of area has been calculated with  $\sigma_{ex}=2\text{mm}$  and  $\sigma_{ey}=1.5\text{mm}$ . The optical losses  $L$ , the

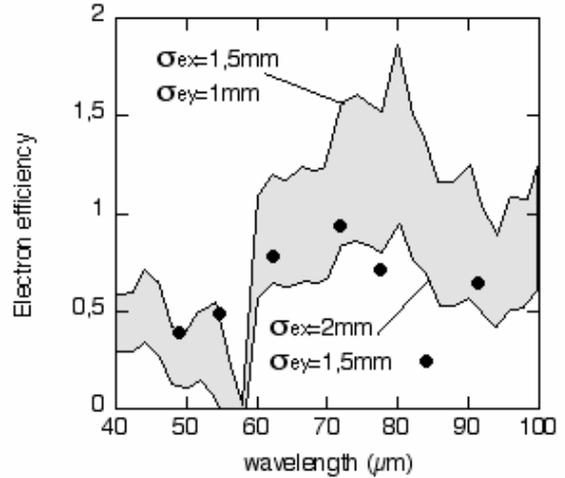


Figure 3: ‘Electron efficiency’  $\eta$  versus laser wavelength. The dots are experimental data, and the gray area is theoretical from the analytical expression (3).

‘Filling factor’  $F_f$  and the cross-section  $\Sigma_o$  of the optical mode are calculated by the numerical code [2] [3]. The optical pulse length  $\Delta t_o$  of the FEL is deduced from the spectrum line width  $\Delta\lambda/\lambda$  of the laser. Taking into account the various sources of error for both theory and experiment, the figure 3 shows that the measurement of the ‘Electron efficiency’ is in rather good agreement with the analytical expression (3). In addition, this means that the numerical simulation, which gives  $L$ ,  $F_f$  and  $\Sigma_o$ , is rather reliable.

The average extracted power  $\langle P_x \rangle$  of the FEL can be calculated from the value of ‘Electron efficiency’  $\eta$ , using the expression (2). Three curves  $\langle P_x \rangle = f(\lambda)$  are displayed in figure 4 : (A) a purely experimental measurement of  $\langle P_x \rangle = f(\lambda)$  using a power meter at the exit of the FEL ; (B) a semi-experimental estimation of  $\langle P_x \rangle$  using the experimental values of  $\eta$  displayed in figure 3 ; (C) a theoretical estimation of  $\langle P_x \rangle$  using the theoretical values of  $\eta$  displayed in figure 3. The behaviour is similar for the three plots. The theoretical curve C fits rather well the curve B. Both of these curves are using the expression (2) for  $\langle P_x \rangle$ . The fit between these two curves B and C, in log scale here, corresponds to the fit of  $\eta$  (theoretical and experimental) in figure 3. The ‘extraction ratio’  $T_x/L$  has no influence in the comparison between B and C because it is used in both curves. Now, a comparison between B or C, and A, shows a larger discrepancy: this means that the ‘extraction ratio’  $T_x/L$ , obtained by numerical simulation, does not correspond to the experiment and is responsible of the rather bad fit of curve A. The figure 5 corresponds to the numerical simulation, and it shows  $L$  and  $T_x$  as a function of wavelength. Taking into account the above conclusion about the reliability of the numerical simulation for  $L$ , we can suppose that  $T_x$  is the parameter which gives the most important error. This conclusion is easy to understand, because  $T_x$  must be very sensitive to the alignment of the cavity mirrors and to the transverse

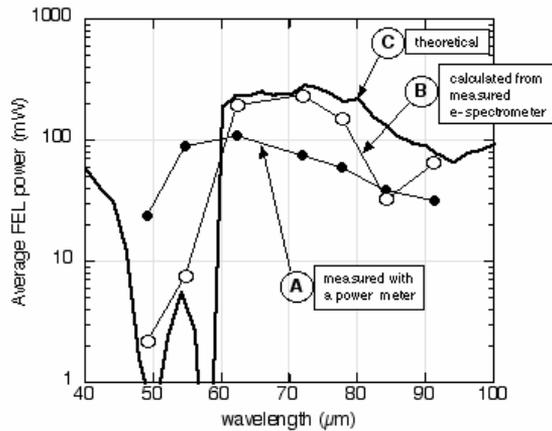


Figure 4: Average extracted laser power  $\langle P_x \rangle$  versus wavelength. The curve A is purely experimental: obtained with a power meter at exit of the FEL. The curve B is semi-experimental, using the measurement for  $\eta$  and the numerical simulation for  $T_x/L$ . The curve C is purely theoretical:  $\eta$  is obtained by analytical expression (3), and  $T_x/L$  is obtained by numerical simulation

profile of the laser mode. A small error of these parameters may change strongly the coefficient  $T_x$ . Note that the figure 5 exhibits, at  $\lambda=50\mu\text{m}$ , a strong decreasing of the laser extraction  $T_x$  and an increasing of the cavity losses  $L$ . This has already been commented in the past [8], and it is due to a special transverse distribution of the laser mode in the cavity, which has zero intensity on longitudinal axis. This effect can be observed in the experimental curve (A) in figure 4, which shows a decreasing of the power  $\langle P_x \rangle$  by a factor 5 between  $\lambda=54\mu\text{m}$  and  $50\mu\text{m}$ .

As a summary, the estimation of  $\langle P_x \rangle$  is less accurate than  $\eta$  because of the uncertainty on  $T_x$ . Nevertheless, the three curves of  $\langle P_x \rangle$  in figure 4 exhibit the same behaviour and they fit the same order of magnitude. Taking into account the various sources of error, both in measurements and simulation, we can consider that the estimation of laser power from the direct power measurement, from the analysis of electron beam spectra, and from the analytical expression, are in rather good agreement.

## CONCLUSION

The electron energy spectra show the scattering due to the FEL interaction between the electron bunch and the laser pulse. This gives a measurement of the 'Electron efficiency'  $\eta$  of the FEL, corresponding to the ratio between the energy produced by the FEL and the input energy of the electron beam. The measurements of  $\eta$  are in rather good agreement with the analytical expression of 'Electron efficiency' which is written here. This expression involves the electron beam parameters, which some of them are only imprecisely estimated, and the optical cavity parameters, which are obtained by numerical simulation.

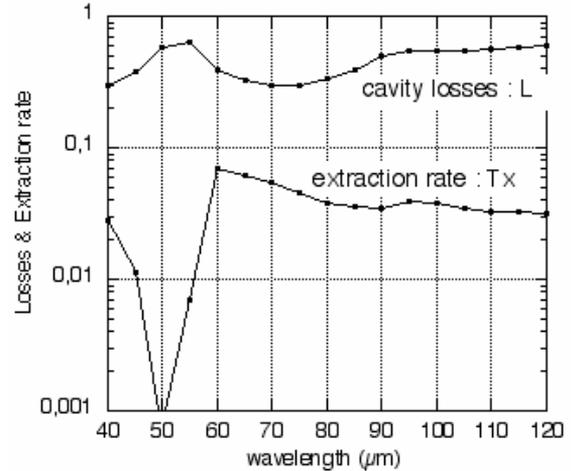


Figure 5: Numerical simulation : cavity losses  $L$  and 'extraction rate'  $T_x$  versus wavelength

The extracted laser power  $\langle P_x \rangle$  is deduced from the values obtained for  $\eta$ . We have made a comparison between the power  $\langle P_x \rangle$  and the direct measurement using a power meter at the exit of the FEL. These data are in rather good agreement, and the remaining error is mainly due to the error in the calculation of the 'extraction rate'  $T_x$  of hole coupling in the optical cavity. The calculation of the FEL power is less reliable than the calculation of the 'Electron efficiency', but both of them are still sufficiently reliable to give the right order of magnitude. The perfect knowledge of electron beam characteristics would give a much better result.

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