THE COHERENT SYNCHROTRON RADIATION INFLUENCE ON THE STORAGE RING LONGITUDINAL BEAM DYNAMICS

E.G. Bessonov*, R.M. Feshchenko, Lebedev Physical Institute RAS, Moscow, Russia
V.I. Shvedunov, Moscow state University

Abstract

We investigate influence on the storage ring beam dynamics of the coherent Synchrotron Radiation (SR) self fields produced by an electron bunch. We show that the maximum energy gain in the RF cavity must far exceed the energy loss of electrons due to the coherent SR.

INTRODUCTION

The energy ε of a particle in storage rings oscillates in the vicinity of the equilibrium energyε_s. The difference between equilibrium and nonequilibrium energies is proportional to the derivative of the particle's phase $d\varphi/dt = h(\omega_s - \omega_r)$:

$$\Delta\varepsilon = \varepsilon - \varepsilon_s = \frac{\varepsilon_s}{hK\omega_s}\frac{d\varphi}{dt}, \quad (1)$$

where $K = -\partial \ln \omega_r / \partial \ln \varepsilon = (\alpha\gamma_s^2 - 1)/(\gamma_s^2 - 1)$ is self phasing coefficient; α, the momentum compaction factor; $\varphi = \int \omega_r(t)dt$, the particle's phase; $\gamma = \varepsilon/mc^2$, the relative energy; $\omega_r = 2\pi f$; f, the revolution frequency of a particle in the storage ring. Equilibrium values have lower index s [1-3]. The radio frequency (RF) voltage in the cavity's gap is varying as $V = V_{rf}\cos\omega_{rf}t$, where ω_{rf} is the radio frequency; h, the subharmonic number of radio frequency.

Balance of energy gained by an electron during the period of a single revolution $T = 1/f = C/c = 2\pi R(1+\mu)/c$ in the RF cavity and lost due to synchrotron radiation and Thomson scattering defines an equation for electron phase oscillations in the storage ring:

$$\frac{d\varepsilon}{dt} = \frac{eV_{rf}\cos\varphi}{T} - < P^{rad} >, \quad (2)$$

where $< P^{rad} >= d\varepsilon^{rad}/dt$ is the power of radiation losses averaged over the length of the orbit; C, the length of the orbit; R, the curvature radius of the particle orbit in bending magnets; $\mu = \sum_i l_i/2\pi R$, the ratio of the sum of straight intervals l_i in the storage ring to the path length in the bending magnets. The synchronous phase $\varphi_s$is defined as $d\varepsilon_s/dt = 0$ or $eV_{rf}\cos\varphi_s = \langle P_s^{rad}\rangle T$.

The spontaneous coherent SR doesn't depend on the particle energy but depends on the particle position in the longitudinal direction, the shape of the beam and on the number of particles. For the Gauss longitudinal distribution one can obtain:

$$\langle P_{coh}^{rad}(\varphi)\rangle = -\frac{3^{1/6}\Gamma^2(2/3)Ne^2c}{2^{1/3}\pi R^{2/3}\sigma_s^{4/3}(1+\mu)} *$$
$$\exp\left[-\frac{1}{2}\left(\frac{R(\varphi-\varphi_s)(1+\mu)}{h\sigma_s}\right)^2\right] *$$
$$[1 - \frac{2^{1/6}\sqrt{\pi}}{3\sqrt{3}\Gamma(2/3)}\frac{R(\varphi-\varphi_s)(1+\mu)}{h\sigma_s} -$$
$$\frac{1}{6}\left(\frac{R(\varphi-\varphi_s)(1+\mu)}{h\sigma_s}\right)^2 + \ldots]. \quad (3)$$

It is supposed here that the phase in the center of the bunch is equal to synchronous phase φ_s, σ_s is the bunch mean square length and $\Gamma(2/3) = 1.35$ [4].

If the laser beam is homogeneous and its transversal dimensions far exceed ones of the electron beam, the powers of Thomson scattering radiation and spontaneous incoherent SR obey the simple power dependence as functions of energy$< P_{noncoh}^{rad} >=< P_{s,noncoh}^{rad} > (\varepsilon/\varepsilon_s)^{k_i}$. The difference between radiated power of synchronous and nonsynchronous particles is

$$< P_{noncoh}^{rad} > - < P_{s,noncoh}^{rad} >= \frac{d < P_{s,noncoh}^{rad} >}{d\varepsilon}\Delta\varepsilon =$$
$$k_i < P_{s,noncoh}^{rad} > \frac{\Delta\varepsilon}{\varepsilon_s}, \quad (4)$$

where $k_i = 2$ for the Thompson backscattering, $k_i = 1$ for the Raleigh backscattering by ions and $k_i = 1 \div 1.5$ for the SR.

Subtracting the power balance equation for synchronous particles from the equation for nonsynchronous one (2) and taking into account (1), (4) we obtain equation for phase oscillations in the storage ring:

$$\frac{d^2\varphi}{dt^2} + \frac{k_i < P_{noncoh}^{rad} >}{\varepsilon_s}\frac{d\varphi}{dt} - \frac{he\omega_s^2 K}{2\pi\varepsilon_s}[V(\varphi) - V(\varphi_s)] = 0, \quad (5)$$

where $V(\varphi) = V_{rf}\cos\varphi - 2\pi R(1+\mu)/c\langle P_{coh}^{rad}(\varphi)\rangle$. The synchronous phase is determined by the equation $U(\phi_s) = 0$.

Incoherent synchrotron radiation and Thompson scattering cause slow damping of phase oscillations (the damping time far exceeds the period of oscillations) and can be neglected in the first approximation, so equation (5) can be rewritten as:

* bessonov@x4u.lebedev.ru

$$\frac{1}{2}\frac{d}{dt}(\frac{d\varphi}{dt})^2 - \frac{he\omega_s^2 K}{2\pi\varepsilon_s}[V(\varphi) - V(\varphi_s)]\frac{d\varphi}{dt} = 0. \qquad (6)$$

The first integral, determining particle phase trajectories behavior is

$$\frac{d\varphi}{dt} = \sqrt{\frac{he\omega_s^2 K}{\pi\varepsilon_s}\int[V(\varphi) - V(\varphi_s)]d\varphi}. \qquad (7)$$

The coherent synchrotron radiation force acts similar to the radio frequency accelerating field. The autophasing force of the storage ring is defined by the derivative $dV(\varphi)/d\varphi$. Thus, in accordance with (5), the reaction of the coherent SR makes this force weaker. This weakening reaches maximum when the phase equals $\varphi = \varphi_s + h\sigma_s/R\,(1+\mu)$. Therefore the stability of the electron beam requires that the derivative $dV(\varphi)/d\varphi$ is negative in the interval $|\varphi - \varphi_s| < h\sigma_s/R(1+\mu)$. This phase range corresponds to the stable oscillations of the majority of particles with amplitudes $A \simeq \sigma_s$. Using the formulas for the power of the coherent SR (3) for a beam with Gauss longitudinal distribution of particles one can find:

$$V_{rf} > V_{rf,c} = \frac{2\pi R^2\,(1+\mu)^2\,P_{coh}^{rad}\,(\varphi_s)}{\sqrt{e_n}h\sigma_s}, \qquad (8)$$

where $e_n \approx 2.72$ is the natural logarithm foundation. In reality the coherent SR can be neglected if the value V_{rf} is $2 \div 3$ times higher than $V_{rf,c}$ and $\sin\varphi_s \approx 1$. The maximum energy gains in the RF cavity, according to (8), must far exceed the energy loss of electrons due to the coherent SR.

If the value P_{coh}^{rad} is neglected, the equation (5) is transformed into the equation of small amplitude phase oscillations:

$$\frac{d^2\psi}{dt^2} + \frac{k_i P_{noncoh}^{rad}}{\varepsilon_s}\frac{d\psi}{dt} + \Omega^2\psi = 0, \qquad (9)$$

where $\Psi = \varphi - \varphi_s << 1$ and $\Omega = \omega_s\sqrt{qhKV_{rf}\sin\varphi_s/2\pi\varepsilon_s}$.

The equation (9) has solutions that can be expressed as $\psi = \psi_m(t)\cos\Omega' t$, where $\psi_m = \psi_{m,0}\exp(-t/\tau_{ph})$ is the varying amplitude and

$$\tau_{ph} = \frac{\varepsilon_s}{P_{noncoh}^{rad}}, \qquad (10)$$

the damping time, $\Omega' = \sqrt{\Omega^2 + \tau_s^{-2}}$, the frequency of small particle oscillations.

EXAMPLE

An electron storage ring has the radius R=50 cm, h=10, σ_s= 1 cm, $\mu = 1$, $N = 10^{10}$,$\sin\varphi_s \approx 1$. In this case the losses of a synchronous particle per a revolution is $V_{coh}^{rad}(\varphi_s)$= 9.25 kev, $V_{rf} > 114$ kV. Thus for the stable storage ring operation the RF cavity voltage should be much higher than the coherent radiation losses. The shielding by the vacuum chamber can weaken this requirement [5]. One should also note that the energy losses of a synchronous electron per a revolution are approximately $2^{2/3}$ times greater than average losses of electrons in the beam (see Appendix).

APPENDIX

Suppose that a beam has small angular $\Delta\theta \sim 1/\gamma$ and energy $\Delta\varepsilon/\varepsilon \sim 1/\gamma$ spread (emittance). In such a case electromagnetic fields emitted by different particles are similar to each other but have a temporal shift. The Fourier images of these fields are:$\mathbf{E}_{i,\omega} = \mathbf{E}_{1,\omega}\exp(i\Delta\varphi_i)$ i=1,2,3, ... N, where the phase difference between waves emitted by the first and the i-th particles is $\Delta\varphi = \omega(t_i' - t_1') + \mathbf{k}[\mathbf{r}(t_i') - \mathbf{r}(t_1')]$. The moments of emission t and detection t' are connected as $t = t' - R_0/c - \mathbf{nr}/c$, R_0 is the distance between the points of emission and detection, $\mathbf{k} = \omega\cdot\mathbf{n}/c$, $\mathbf{n}$ is a unit vector pointing in the direction of emission, **r** – the vector lying in the plane perpendicular to the trajectory of a particle. The time difference for ultrarelativistic particles $t_i' - t_1'$ is connected with the space distance by a simple relation $c(t_i' - t_1') = z_i - z_1$. Therefore the Fourier image of the sum of fields of N particles $\mathbf{E}_\omega = \sum_i \mathbf{E}_{i,\omega}$ can be written as (for the electrical field):

$$\mathbf{E}_\omega = N\int_{-\infty}^{\infty}\rho(z,\mathbf{r})\mathbf{E}_{1,\omega}\exp[i\Delta\varphi(z,\mathbf{r})]dzd\mathbf{r}, \qquad (11)$$

where $\rho(z,\mathbf{r})$ – the density distribution of particles normalized to unity.

If the transversal dimensions of the beam are small, the integration in the equation (11) over transversal coordinate **r** can be omitted:

$$\mathbf{E}_\omega = N\cdot\mathbf{E}_{1,\omega}\int_{-\infty}^{\infty}\rho(z)\exp[i\frac{2\pi z}{\lambda}]dz. \qquad (12)$$

In this case the spectra-angular distribution of the emitted energy

$$\frac{\partial^2\varepsilon^{coh}}{\partial\omega\partial o} = cR_0^2|\mathbf{E}_{1,\omega}|^2 = N^2\frac{\partial^2\varepsilon_1}{\partial\omega\partial o}s(\omega), \qquad (13)$$

where ε_1 is the energy of the radiation emitted by a single particle, $s\,(\omega) = \left|\int_{-\infty}^{\infty}\rho\,(z)\exp\left[i2\pi z/\lambda\right]dz\right|^2$, the spectral radiation coherence factor, $\lambda = 2\pi c/\omega$– the wavelength of SR. The spectral energy distribution and the full emitted energy can be found by integration of (13) over angles

$$\frac{\partial\varepsilon^{coh}}{\partial\omega} = N^2\frac{\partial\varepsilon_1}{\partial\omega}s(\omega) \qquad (14)$$

and over frequency

$$\varepsilon^{coh} = N^2 \int_0^\infty \frac{\partial \varepsilon_1(\omega)}{\partial \omega} \cdot s(\omega) \cdot d\omega. \quad (15)$$

From (13) – (15) it follows that for a point-like beam $\rho(z) = \delta(z)$ and therefore $s(\omega) = 1$, $\varepsilon^{coh} = N^2 \int_0^\infty [\partial\varepsilon_1(\omega)/\partial\omega] d\omega = N^2\varepsilon_1(\omega)$, i.e. the energy emitted by the beam is N^2 times larger than the energy emitted by a single particle.

If the beam's motion is periodical one can introduce average radiation power: $P^{coh} = f \cdot \varepsilon^{coh}$, $\partial P^{coh}/\partial\omega = f \cdot \partial\varepsilon^{coh}/\partial\omega$, $f = v/C$ – the revolution frequency, $\nu \approx c$– the particle's velocity and C is the perimeter of the orbit.

The values $\partial\varepsilon_1/\partial\omega$ and $\partial P_1/\partial\omega = f \cdot \partial\varepsilon_1/\partial\omega$ are known. In particular, the spectral power of radiation is

$$\frac{\partial P_1}{\partial \xi} = \frac{3\sqrt{3} e^2 c \gamma^4}{2RC} F(\xi), \quad (16)$$

where $\beta = v/c$ – the relative particle velocity, $\gamma = \varepsilon/mc^2$– the relative energy, $F(\xi) = \xi \int_\xi^\infty K_{5/3}(\xi) d\xi$, $\xi = \omega/\omega_c$, $\omega_c = 3\beta\gamma^3 c/2R$ – the critical radiation frequency, R– the orbit radius in a bending magnet of the storage ring [6,7]. One can also calculate $\int_0^\infty F(\xi)d\xi = 8\pi/9\sqrt{3}$[6]. Thus the full radiation power for one particle can be expressed as:

$$P_1 = \frac{4\pi}{3} \frac{e^2 c \gamma^2}{RC}. \quad (17)$$

In the case under consideration the radiation is coherent if the wavelength is longer than the length of the bunch i.e. $\xi << 1$, $K_{5/3}(\xi) \approx 2^{4/3}\Gamma(2/3)\xi^{-5/3}$,

$$\int_\xi^\infty K_{5/3}(\xi)d\xi = \int_0^\infty K_{5/3}(\xi)d\xi - \int_0^\xi K_{5/3}(\xi)d\xi$$

$$= \pi\sqrt{3} - \int_0^\xi K_{5/3}(\xi)d\xi, F(\xi) = 2^{2/3}\Gamma(2/3)\xi^{1/3}.$$

Now the formula (16) can be written as

$$\frac{\partial P_1}{\partial \xi} = \frac{3\sqrt{3} e^2 c \gamma^4}{2^{4/3}\pi R^2(1+\mu)} \Gamma(\frac{2}{3})\xi^{1/3}. \quad (18)$$

The spectral coherence factor $s(\omega)$ is determined by the particle density distribution law $\rho(z)$ and for the Gaussian distribution

$$\rho(z) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{\frac{-z^2}{2\sigma_x^2}} \quad (19)$$

can be derived from equations (13) and (19) as $s(\omega) = \exp(-4\pi^2\sigma_x^2/\lambda^2)$ [8,9]. The value σ_x is the mean square bunch length.

The full power of the spontaneous coherent SR, the average loss rate for a single particle and the losses over a revolution can be calculated numerically using the formula (15) and the expression $P^{coh} = f \cdot \varepsilon^{coh}$. In the special case when the coherent SR is dominated by the low frequency radiation $\xi << 1$, taking into account (18) and $\int_0^\infty k^{1/3} \exp(-k^2\sigma_x^2)dk = \Gamma(2/3)/2\sigma_x^{4/3}$, one can derive that

$$P^{coh} = c\frac{d\varepsilon^{coh}}{dt} = \frac{3^{1/6}\Gamma^2(2/3) r_e c N^2}{2\pi R^{2/3}\sigma_x^{4/3}(1+\lambda)} mc^2, \quad (20)$$

The energy losses per a revolution are

$$\Delta\varepsilon^{coh} = \frac{d\varepsilon^{coh}}{dt} T = \frac{3^{1/6}\Gamma^2(2/3) r_e R^{1/3} N^2}{\sigma_x^{4/3}} mc^2$$

$$\approx 3.1 \cdot 10^{-7} R^{1/3} N^2/\sigma_x^{4/3} [eV/revolution]. \quad (21)$$

The formula (21) matches with the results of the first work on the coherent SR [10], is $2^{1/6} \approx 1.12$ times lower than one in the reference [4] , $2^{7/3} \approx 5.04$ times lower than the value in the reference [11] and $2^{8/3} \approx 6.35$ times lower than one in the reference [12]. In the last reference the authors used formula from the work of Shiff [10] and erred in converting it to their definition of the value σ_x. They multiplied the Shiff's formula by the $2^{4/3}$ instead of dividing by it. In the remaining references the source of errors is unclear but more probably connected with the same mistake.

The coherence factor is decreasing for the wavelengths $\lambda \geq \lambda_d = 2\pi\sigma_x$ or if $\omega \geq \omega_d = c/\sigma_x$. The expression (21) is justified if the main part of the energy of the coherent SR is emitted in the spectral range $\omega \leq \omega_d \approx \omega_c(\xi << 1)$ i.e. when $\sigma_x > \lambda_c/2\pi$, where $\lambda_c = 2\pi c/\omega_c = 4\pi R/3\gamma^3$. The expression also (21) doesn't take into consideration the shielding of the beam by the vacuum chamber, which leads to the weaker radiation for the wavelengths longer than the vacuum chamber gap.

The vast majority of the energy is emitted in the angular range $\Delta\theta \sim 1/\gamma$ relative to the direction of the particle's motion when $k \cdot r(t') << k \cdot r/\gamma$. So, the condition when one can neglect the transversal beam dimensions is $k_d \cdot r << \gamma$ or $r << (\lambda_d/2\pi)\gamma = \sigma_x\gamma$.

REFERENCES

[1] A.A. Kolomensky and A.N. Lebedev, Theory of Cyclic Accelerators. North Holland Publ., , 1966.

[2] H. Bruk, Accelerateurs Circulaires de Particules (Press Universitaires de France, 1966).

[3] C H. Wiedemann, Particle Accelerator Physics I \& II (Springer-Verlag, New York, 1993).

[4] L.V.Ioganson, M.S.Rabinovich, JETP, v.37, 118, 1959 (in Russian).

[5] J.B.Murphy, S.Krinsky, R.L.Gluckstern, Particle Accelerators, 1997, v.57, pp.9-64.

[6] D.Ivanenko, A.Sokolov, Classical Theory of field, GITTL, 1951 (in Russian).

[7] L.D.Landau, E.M.Lifshits, The classical Theory of Fields, Pergamon Press, Oxford, UK, 1975.

[8] H.Wiedemann, Particle Accelerator Physics, v.I, Springer – Verlag, NY, 1993, p.319.

[9] A.Andersson, M.Johnson, B.Nelander, Coherent synchrotron radiation in the far infrared from 1 mm electron bunch, Opt. Eng. V.39, p. 3099-3115, 2000.

[10] L.Schiff, Rev. Sci. Instr., v.17, 7, 1946.

[11] E.L.Saldin, E.A.Schneidmiller, M.V.Yurkov, TESLA FEL Report 96-14, November 1996 (corrected in Nucl. Instr. Meth. v.A398 (1997), p.373).

[12] J.S.Nodvic, D.S.Saxon, Phys. Rev., v.96, No 1, p. 180-184, 1954.