ISOCHRONOUS BEND FOR HIGH GAIN RING FEL

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Abstract

The recently proposed ring free electron laser (FEL) consists of several undulators with isochronous bends between them. Isochronous bends are necessary to preserve the beam bunching between undulators. Such FEL configuration may be used as an independent soft X-ray source or as a master oscillator for an X-ray FEL (high gain harmonic generator or other type). The lattice of the compact 500-MeV 60-degree bend for a soft X-ray (50 nm) FEL is proposed. Fundamental restrictions due to quantum fluctuations of synchrotron radiation and technically achievable fields to construct isochronous bends of a shorter wavelength ring FEL are discussed.

INTRODUCTION

A mirror-free ring Free Electron Laser (FEL) configuration consisting of several undulator sections separated by isochronous bends is considered to be a possible solution for an X-ray FEL [1, 2]. In this configuration the beam bunching grows from undulator to undulator and after a 360-degree turn radiation from the last undulator illuminates the beam in the first undulator and induces bunching in the incoming fresh beam. Such a configuration, driven by an energy recovery linac, is shown in Fig. 1.

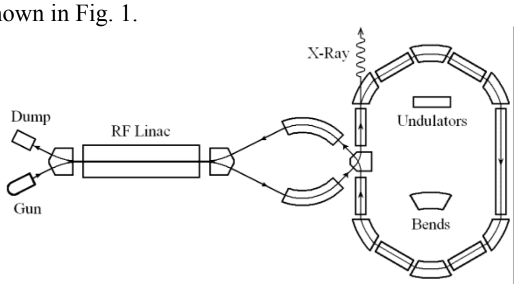

Figure 1. Possible layout of ring FEL driven by an energy recovery LINAC.

One of the critical parts of the scheme is the isochronous bend section between undulators. Their aim is to preserve the beam bunching along the ring. If we express the time delay of an electron from a reference particle moving along the design orbit in terms of the first order transport matrix *Rij* we get (planar bend is assumed)

$$c\Delta t = R_{51}x + R_{52}x' + R_{56}\frac{\Delta E}{E} \qquad (1)$$

In an isochronous bend we must make all three matrix elements in Eq. (1) equal to zero. Let $d(s)$ is the (horizontal) dispersion function with zero initial conditions at the bend entrance s = 0, s is the coordinate along the reference trajectory Then R_{52} and R_{51} are proportional to dispersion $d(L) = R_{16}$ and it's derivative $d'(L) = R_{26}$ at the bend exit $s = L$, so the bend must be achromatic in the first order. R_{56} can be expressed in terms of dispersion

$$R_{56}(0,L) \approx \int_0^L h(s)d(s)ds \qquad (2)$$

Here $h(s)$ is the orbit curvature. This matrix element can be easily made zero in a symmetric three-bend achromat.

It is worth noting, that bunching conservation in the achromatic bend for the visible wavelength range was demonstrated in [3].

In this paper we discuss two causes for bunching degradation and propose a compact 500-MeV 60-degree bend for a soft X-ray (50 nm) ring FEL. Quantum fluctuations of synchrotron radiation and second order aberrations in the lattice are taken into account. Limiting factors to construct an X-ray ring FEL are discussed further.

EFFECT OF QUANTUM FLUCTUATIONS OF SYNCHROTRON RADIATION

An electron, moving in the bending magnet, emits synchrotron radiation. In classical electrodynamics it leads to the slow change of the beam energy. But really the quantum fluctuations of radiation increase the beam energy spread. Due to the energy loss particles arrive the bend exit slightly later. If quantum nature of radiation is taken into account, the time delay spread appears*.

The delay caused by the emission of photon with energy $\varepsilon = \hbar\omega$ at the point s of the trajectory and measured at the point L (exit of achromat) is

$$c\Delta t = -R_{56}(s,L)\frac{\varepsilon}{E} \qquad (3)$$

The probability of this photon emission may be expressed through the spectral power density of synchrotron radiation $dI/d\omega$

$$dp = \frac{dI}{d\omega}d\omega\frac{ds}{\varepsilon c} \qquad (4)$$

Then the square of the delay dispersion $\sigma_{c\Delta t}^2 = \left\langle (c\Delta t)^2 \right\rangle - \left\langle c\Delta t \right\rangle^2$ is the sum of contributions from different parts of the trajectory ds and frequency intervals $d\omega$

* The debunching due to quantum fluctuations was pointed out by V. N. Litvinenko.

$$\sigma_{c\Delta t}^2 = \int_0^\infty\int_0^L \left(R_{56}(s,L)\frac{\varepsilon}{E}\right)^2 \frac{dI}{d\omega}\frac{ds}{\varepsilon c}d\omega \qquad (5)$$

After the integration over frequencies it gives

$$\sigma_{c\Delta t}^2 = \frac{55}{24\sqrt{3}}\frac{r_0^2}{\alpha}\gamma^5 \int_0^L R_{56}^2(s,L)\cdot h^3(s)\cdot ds \qquad (6)$$

where r_0 is the classical electron radius, α = 1/137 is the fine-structure constant, γ is the relativistic factor. For the mirror symmetric isochronous systems $R_{56}(s,L)$=-$R_{56}(0,s)$. Then

$$\sigma_{c\Delta t}^2 = \frac{55}{24\sqrt{3}}\frac{r_0^2}{\alpha}\gamma^5 \int_0^L R_{56}^2(0,s)\cdot h^3(s)\cdot ds \qquad (7)$$

The reduction of the current Fourier harmonics at frequency ω = ck is given by the factor $\left\langle e^{i\omega\Delta t}\right\rangle$. If $k\sigma_{c\Delta t} << 1$, the current spectral density reduction after the pass through the bend may be estimated as

$$\left|\left\langle e^{ikc\Delta t}\right\rangle\right|^2 \approx \left|1 + ik\langle c\Delta t\rangle - \frac{k^2}{2}\left\langle (c\Delta t)^2\right\rangle\right|^2 \approx$$
$$1 - k^2\left(\left\langle (c\Delta t)^2\right\rangle - \langle c\Delta t\rangle^2\right) = 1 - k^2\sigma_{c\Delta t}^2 \qquad (8)$$

As it is seen from Eq. (8), $k\sigma_{c\Delta t} < 1$ is required to keep significant bunching after the bend.

Consider, for simplicity, a three-bend isochronous achromat. It may be shown, that the main (and uneliminable) contribution to the delay dispersion Eq. (7) is given by two side magnets. Let α_0 is the bend angle for each of these magnets, and α_0<<1. At the first magnet we have

$$R_{56}(0,s) = \frac{h^2 s^3}{6} \qquad (9)$$

and then, neglecting the contribution of the central magnet,

$$\sigma_{c\Delta t}^2 = \frac{55}{24\sqrt{3}}\frac{r_0^2}{\alpha}\gamma^5\frac{\alpha_0^7}{126}.$$

Comparing it to $(\lambda/2\pi)^2$, we get an estimation of the maximum permissible bend angle in the system

$$\alpha_0 \sim \left(\frac{\lambda^2}{4\pi^2 r_0^2\gamma^5}\right)^{1/7} \qquad (10)$$

It is worth noting, that this limitation does not depend on the orbit curvature h, therefore high-field superconducting magnets may be used.

If we assume the beam emittance to be a limiting factor for a short-wavelength FEL, the beam energy should be large enough, $\gamma > k\varepsilon_n$, where ε_n is the normalized beam emittance. Therefore,

$$\alpha_0 < \frac{\lambda}{2\pi}\varepsilon_n^{-5/7} r_0^{-2/7} \approx \frac{\lambda}{200\,\mathring{A}} \qquad (11)$$

(ε_n=10^{-6} m·rad is taken for the estimation).

Thus, the debunching, caused by quantum fluctuations of synchrotron radiation, can be neglected for the wavelengths, larger then 200 Å. For shorter wavelengths maximal bend angle scales as λ and going deep below 100 Å can be a problem.

SHORT WAVELENGTH RING FEL ISOCHRONOUS BEND

For wavelength shorter then 100 Å the bend section can be made up of a number of isochronous bends on a smaller angle. Such a system can be thought of as a bended undulator. However, the focusing of an undulator is too weak to make its period isochronous and additional quadrupole focusing is necessary.

Total debunching in the section, consisting of N isochronous bends, is

$$(\sigma_{c\Delta t}^2)_N \sim \sigma_{c\Delta t}^2 N \qquad (12)$$

Taking N=1/α_0 for the bend section of 1 radian and comparing it to $(\lambda/2\pi)^2$ we get an estimation of the maximum permissible bend angle in the system for this case

$$\alpha_0 \sim \left(\frac{\lambda^2}{4\pi^2 r_0^2\gamma^5}\right)^{1/6} \text{ or}$$

$$\alpha_0 < k^{-7/6}\varepsilon_n^{-5/6} r_0^{-1/3} \approx \left(\frac{\lambda}{200\,\mathring{A}}\right)^{7/6} \qquad (13)$$

Making the bend angle small we can decrease the synchrotron radiation effects to a desirable level. However, the length of the bend section can become too long. The bend section total length can be estimated in the following manner. Let us denote the maximal achievable gradient of the focusing lenses G. In order to make the dispersion zero in each bend its length must be

$$l_{min} \sim 2\pi\sqrt{\frac{pc}{eG}} \qquad (14)$$

The total one-radian section length is given then by

$$L \sim l_{\min}N \sim 2\pi\sqrt{\frac{pc}{eG}}\left(\frac{4\pi^2 r_0^2\gamma^5}{\lambda^2}\right)^{1/6}, \qquad (15)$$
$$\approx \left(\frac{140\,\mathring{A}}{\lambda}\right)^{5/3} m$$

G = 10 kGs/cm was assumed. It is worth noting that for short wavelengths this length is much longer then the orbit curvature radius in a bending magnet, i.e. the bend section length of an X-ray ring FEL is limited by the maximal achievable field gradient.

SECOND ORDER ABERRATIONS

The finite energy spread and transverse emittances will also cause debunching. The delay longitudinal dependence for a particle is given by

$$\frac{d}{ds}c\Delta t=\frac{1}{\beta_0}\left(hx-\frac{\delta}{\gamma_0^2}-\frac{hx\delta}{\gamma_0^2}+\frac{\delta^2}{\gamma_0^2}\left(1+\frac{\beta_0^2}{2}\right)+\frac{x'^2}{2}+\frac{y'^2}{2}\right) \quad (16)$$

Here $p=p_0(1+\delta)$, and we retain only terms up to the second order in δ, x, y, and their derivatives. The first and second terms on the right are the first-order terms. Simple estimations show that the third and the fourth term contributions are negligible in comparison with the fifth and the sixth ones. These last terms give the positive contribution which can be estimated as

$$\sigma_{c\Delta t}^2 \le \frac{\varepsilon_x^2}{2}\left(\int_0^s \gamma_x(\tau)d\tau\right)^2+\frac{\varepsilon_y^2}{2}\left(\int_0^s \gamma_y(\tau)d\tau\right)^2 \quad (17)$$

where ε_x, ε_y are the rms beam emittances, γ_x, γ_y are the Twiss parameters. This debunching can be compensated by the contribution of the first term in Eq.(16). For example, one of the second order matrix elements is

$$T_{511}(s)=\frac{1}{\beta_0}\int_0^s\left(\frac{C_x'^2}{2}+h(x\,|\,x_0^2)\right)ds' \quad (18)$$

where C_x, S_x are the basic trajectories, and $(x|x_0^2)=T_{111}$. Choosing the proper values of sextupoles one can adjust $(x|x_0^2)$ to make T_{511} zero.

ISOCHRONOUS BEND FOR 500 Å RING FEL

According to [2], the 500 Å ring FEL shall be driven by a 500MeV energy recovery LINAC. The parameters of the LINAC and the laser are shown in table 1.

Table 1. Soft X-ray ring FEL parameters.

Energy, GeV	0.5
Relative energy spread, %	0.05
Normalized rms emittance, micron	5
Peak current, kA	0.3
Undulator period, m	0.03
Undulator deflection parameter K	2
Radiation wavelength, Å	500
Undulator section length, m	12
Undulator first and last section length, m	5
Bend angle, degrees	60
Bend length, m	5

The proposed lattice of the 60˚ isochronous bend modeled in MAD is shown in fig.2. It consists of two 30˚ bending sections. Each section is essentially a symmetric isochronous three-bend achromat. Bends are proposed to be made with the field gradient in order to shorten the overall bend length. Bending sections are separated by quadruples. Sextupole correction is inserted into the negative bends of each bending section.

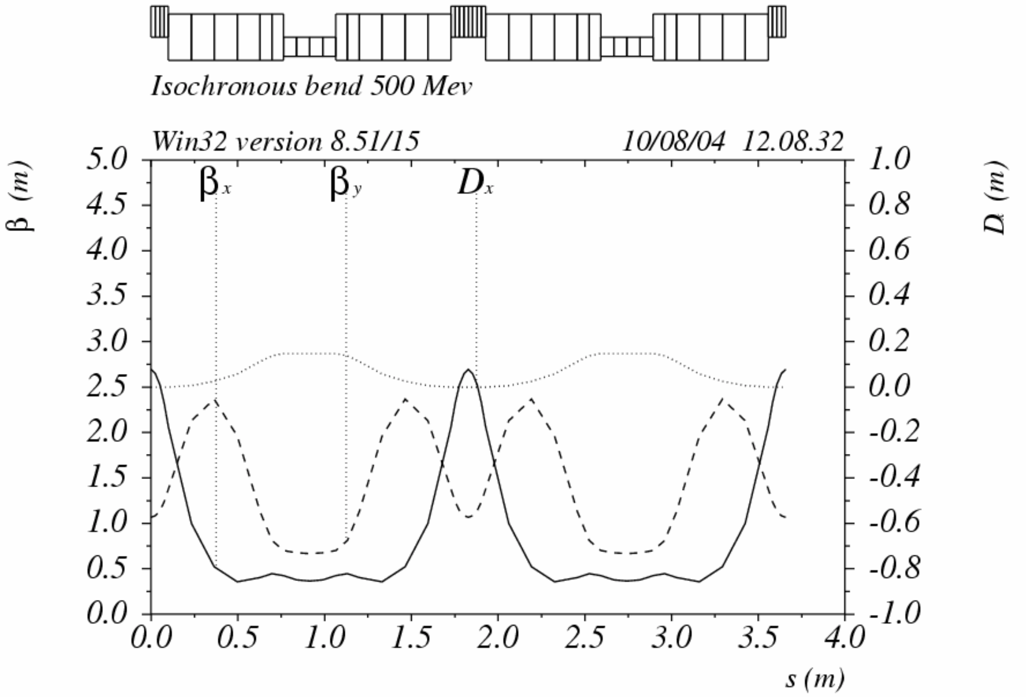

Figure 2. Linear lattice functions β_x, β_y, and D_x of the isochronous bend of the 500 Å ring FEL.

Contribution of quantum fluctuations of synchrotron radiation to the dispersion of the time of flight calculated according to Eq.(7)

$$\sigma_{c\Delta t} \sim 60\,\mathring{A} \tag{19}$$

Second order aberrations are calculated using the second order transport matrix T_{5ij} from MAD program in the following way.

$$c\Delta t^{(2)} = T_{5ij} x_i x_j \tag{20}$$

$$\sigma_{c\Delta t}^2 = \left\langle (c\Delta t)^2 \right\rangle - \left\langle c\Delta t \right\rangle^2 = T_{5ij} T_{5kl} \left(\left\langle x_i x_j x_k x_l \right\rangle - \left\langle x_i x_j \right\rangle \left\langle x_k x_l \right\rangle \right) \tag{21}$$

If we consider plane system and upright phase plane ellipses on the system entrance, the only nonzero elements remain

$$\begin{aligned} \sigma_{c\Delta t}^2 &= T_{511}^2 \cdot 2\beta_x^2 \varepsilon_{0x}^2 + T_{522}^2 \cdot 2\gamma_x^2 \varepsilon_{0x}^2 + T_{512}^2 \cdot 4\varepsilon_{0x}^2 + \\ &+ T_{533}^2 \cdot 2\beta_y^2 \varepsilon_{0y}^2 + T_{544}^2 \cdot 2\gamma_y^2 \varepsilon_{0y}^2 + T_{534}^2 \cdot 4\varepsilon_{0y}^2 + \\ &+ T_{566}^2 \cdot 2\beta_\Delta^2 \varepsilon_{0\Delta}^2 + T_{516}^2 \cdot 4\beta_x \varepsilon_{0x} \beta_\Delta \varepsilon_\Delta + T_{526}^2 \cdot 4\gamma_x \varepsilon_{0x} \beta_\Delta \varepsilon_\Delta \end{aligned} \tag{22}$$

where β, γ are the Twiss parameters at the entrance of the system, ε are the beam emittances. Indexes x, y, and Δ indicate transverse planes x and y, and longitudinal motion respectively.

For the system shown in Fig. 2.

$$\sigma_{c\Delta t} \sim 60\,\mathring{A} \tag{23}$$

Total dispersion then

$$\sigma_{c\Delta t} \sim 80\,\mathring{A} \approx \frac{\lambda}{2\pi} \tag{24}$$

CONCLUSION

The feasibility of the magnetic system for the ring FEL at the soft X-ray band is shown. Quantum fluctuations of synchrotron radiation and second order aberrations in the lattice are taken into account. The collective effects including CSR have to be considered yet.

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