THE TWO-BEAM FREE ELECTRON LASER OSCILLATOR

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Abstract

A one-dimensional model of a free-electron laser operating simultaneously with two electron beams of different energies [1] is extended to an oscillator configuration. The electron beam energies are chosen so that an harmonic of the lower energy beam is at the fundamental radiation wavelength of the higher energy beam. Potential benefits over a single-beam free-electron laser oscillator are discussed.

INTRODUCTION

A one-dimensional model of a single pass high-gain free-electron laser operating with co-propagating electron beams of different energies has been described elsewhere [1, 2]. The resonant beam energies are chosen such that the nth harmonic radiation wavelength of the lower energy beam is at the fundamental radiation wavelength of the higher energy beam. It was suggested that this concept may have certain advantages over a single beam FEL amplifier. For example, it may be possible to seed the lower energy beam at the fundamental wavelength and use the coupled interaction between the two beams to transfer the coherence properties of the seed field to the harmonic radiation field. In this paper the model is extended to a one-dimensional steady-state oscillator configuration. Numerical studies are done of the evolution of the system and the properties of the model are explored, including the effect of different energy spreads in the higher energy beam.

THE MODEL

The evolution of the two-beam system is given by a set of coupled equations whose derivation and underlying assumptions are outlined elsewhere [1]. For convenience the working equations are repeated here in their final form, specialised to the case where $n = 3$:

$$\frac{\mathrm{d}\vartheta_j}{\mathrm{d}\bar{z}} = p_j \tag{1}$$

$$\frac{\mathrm{d}\varphi_j}{\mathrm{d}\bar{z}} = \wp_j \tag{2}$$

$$\frac{\mathrm{d}p_j}{\mathrm{d}\bar{z}} = -\frac{1}{c_1}\sum_{h,odd}^{3} F_h(A_h e^{ih\vartheta_j} + c.c.) \tag{3}$$

$$\frac{\mathrm{d}\wp_j}{\mathrm{d}\bar{z}} = -(F_1 A_3 e^{i\varphi_j} + c.c.) \tag{4}$$

$$\frac{\mathrm{d}A_1}{\mathrm{d}\bar{z}} = S_\vartheta \tag{5}$$

$$\frac{\mathrm{d}A_3}{\mathrm{d}\bar{z}} = S_\varphi + S_{3\vartheta}. \tag{6}$$

Here p and ϑ represent the energy detuning and phase of the low energy electrons and $\wp$ and φ refer to the high energy beam. For resonant beams $p = \wp = 0$. The variable $j = 1 \ldots N$ where N is the total number of sample electrons in each beam and F_1 and F_3 are the usual difference of Bessel function factors for planar wigglers. Also

$$S_{k\vartheta} \equiv \frac{1}{c_2}F_k\langle e^{-ik\vartheta}\rangle, \quad S_\varphi \equiv F_1\langle e^{-i\varphi}\rangle \tag{7}$$

$$c_1 = \frac{\sqrt{R_3}}{3}, \quad c_2 = \frac{\sqrt{R_3}}{\sqrt{3}} \tag{8}$$

where R_3 is the ratio of currents in the high energy and low energy beams I_3/I_1.

In these equations the universal scaling is via the FEL parameter ρ_3 related to the higher energy beam. Hence the explicit appearance of the additional scaling factors R_3 and $n = 3$ via c_1 in equation (3), and via c_2 in equation (7). By changing the value of R_3 we assume the current of the lower energy beam is altered whereas that of the higher energy beam remains constant.

An important feature of the system is expressed in (6) - the harmonic field has two source terms S_φ and $S_{3\vartheta}$ which derive from the high and low energy beams respectively. This feature is exploited in the work in this paper to allow either beam to be ‘switched off’ by setting the appropriate source term to zero. This is useful when comparing the behaviour of the two-beam system to the behaviour with one or the other beam alone. An equivalent way of ‘switching off’ the lower energy beam is by setting $R_3 \gg 1$ making the low energy beam current negligibly small. From (8) and (7) it can be seen that this reduces the source term $|S_{3\vartheta}| \ll 1$.

The oscillator model is very simple. At the end of each pass the radiation intensities within the cavity are reduced by appropriate factors depending on the mirror reflectivities then used as the input intensities for the next pass. We assume mirror reflectivities r_1 and r_3 which can be varied independently for the two radiation wavelengths. We also assume that any other cavity losses are zero so that the outcoupled intensities are given by

$$|A_i|^2_{\mathrm{out}} = (1 - r_i)|A_i|^2, \quad i = 1, 3. \tag{9}$$

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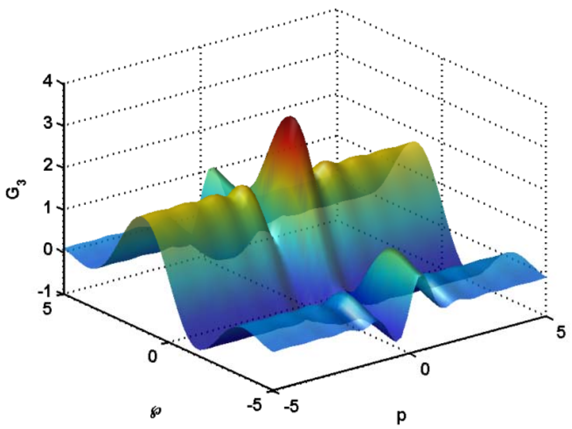

Figure 1: Both beams: third harmonic gain G_3 as a function of beam detunings in p and $\wp$. The current ratio R_3=5, so that the lower energy beam has one fifth the current of the higher energy beam: $G_{3,\text{peak}} \approx 3.1$.

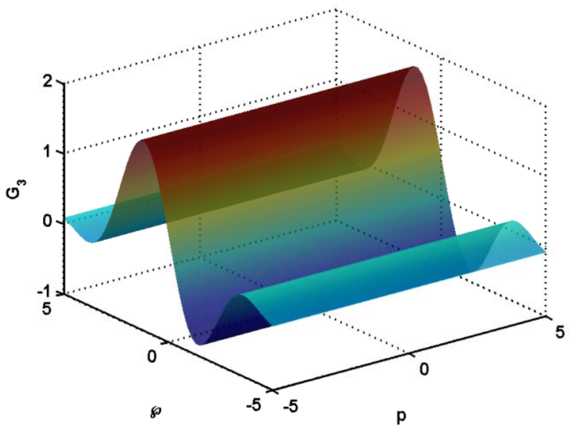

Figure 2: Low energy beam 'switched off': third harmonic gain G_3 as a function of detunings in p and $\wp$. There is now no p-dependence as expected. $G_{3,\text{peak}} \approx 1.7$.

NUMERICAL METHODS

The system equations (2–6) were solved numerically using MATLAB and Fortran codes. The MATLAB code used a variable step size intrinsic integration method and the Fortran code used the standard fixed step size 4th-order Runge-Kutta method. The codes were benchmarked against each other and agreed well.

NUMERICAL INVESTIGATIONS

In a high-gain 2-beam amplifier, and assuming cold, resonant beams, linear analysis gives a threshold condition in the beam currents for the harmonic gain to be greater than that of the fundamental [1]:

$$R_3 > 3\sqrt{3}\left(1 - \frac{3\,|F_3|^2}{|F_1|^2}\right). \qquad (10)$$

Here this gives the condition $R_3 > 2.13$ for the chosen wiggler parameter a_w=2. Although an intermediate gain of $\bar{z} = 2$ is assumed, and therefore the system is not high-gain, a current ratio of $R_3 = 5$ was chosen and this is subsequently shown in the following section to similarly result in a greater gain in the harmonic field than the fundamental.

Model Optimisation

The optimum beam detunings, $p(\bar{z} = 0)$ and $\wp(\bar{z} = 0)$, were determined by a numerical scan of the single pass gain at the third harmonic G_3. The result is shown in Fig. 1 where the peak gain is ≈ 3.1 (310%) at (p, $\wp$) ≈ (0.4, 1.2). For comparison the scan was repeated for the high energy beam only (Fig. 2), where the peak gain was found to be ≈ 1.7, and the low energy beam only (Fig. 3) where the peak gain was found to be ≈ 0.9.

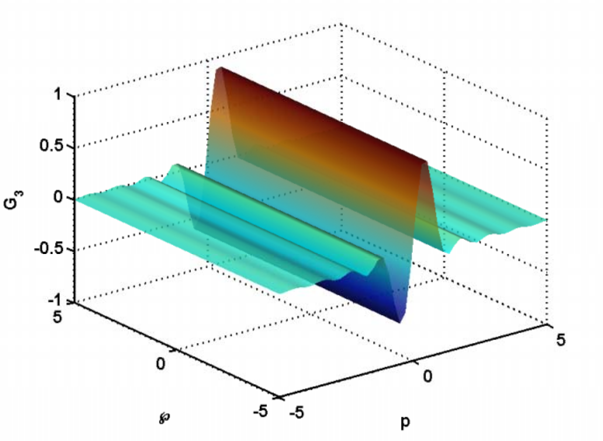

Figure 3: High energy beam 'switched off': third harmonic gain G_3 as a function of beam detunings in p and $\wp$. There is now no $\wp$-dependence. $G_{3,\text{peak}} \approx 0.89$.

The gain for the two-beam system is greater than the sum of the gains due to the individual beams, even though the current in the lower energy beam is only one fifth the current in the higher energy beam. A similar scan was carried out for $\bar{z} = 0.5$ and it was found that in this linear small signal gain regime the gain for the two-beam system was the sum of the individual gains. The small enhancement in the nonlinear ($\bar{z} = 2$) regime is due to the non-linear coupling between the two beams, there being no such coupling in the small signal/small gain regime.

It is clear from Fig. 1 that the gain is sensitive to the relative detuning between the two beams. Away from the maximum at (p, $\wp$) ≈ (0.4, 1.2) the coupling decreases and the surface evolves into a Madey gain curve as a function of the low energy beam detuning p and a more asymmetric higher gain curve as a function of the higher energy beam detuning $\wp$. These curves are seen more clearly in Fig. 2

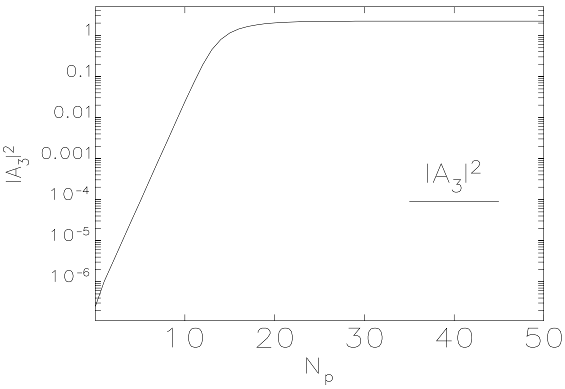

Figure 4: Evolution of harmonic radiation field intensity $|A_3|^2$. Current ratio $R_3 = 5$, $\bar{z} = 2$.

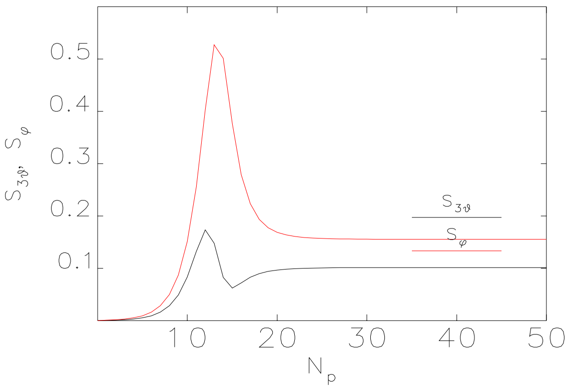

Figure 5: Evolution of harmonic radiation field source terms S_φ and $S_{3\vartheta}$. Current ratio $R_3 = 5$, $\bar{z} = 2$.

and Fig. 3 where the low and high energy beams respectively are 'switched off'.

The mirror reflectivities r_1 and r_3 were optimised via numerical scans to maximise the saturation intensity of the third harmonic, $|A_3|^2_{\mathrm{sat}}$.

The optimum reflectivity at the fundamental was found to be $r_1 < 0.45$, i.e. below threshold for the fundamental ($G_1 <$ cavity losses). Thus bunching of the lower energy beam at the fundamental disrupts its coupling to the harmonic field.

The optimum reflectivity for the harmonic was determined to be $r_3 \approx 0.7$.

For all further simulations the following values were therefore adopted: $r_1 = 0.0$, $r_3 = 0.7$. Thus the fundamental resonant field of the lower energy beam A_1 is not stored in the cavity and so has little effect upon the coupled interaction.

NUMERICAL EXAMPLES

A two-beam oscillator FEL with the above optimised parameters was simulated. A minimal gaussian energy spread of $\sigma_p = \sigma_\wp = 0.1$ was assumed. Fig. 4 shows the evolution of the harmonic output intensity over 50 passes. The fundamental shows no growth due to the zero reflectivity at that wavelength and so is not shown. The harmonic field saturates after around 20 passes. The steepest growth in the harmonic field (i.e. the maximum increase in intensity over a single pass) occurs around pass 12. It can be seen from Fig. 5 and Fig. 6 that this coincides with peaks in the evolution of the source terms S_φ and $S_{3\vartheta}$ and peaks in the bunching terms $|b_3|$ and $|b_\varphi|$. These latter terms are measures of the bunching in the lower energy beam at its third harmonic wavelength and in the higher energy beam at its fundamental wavelength. The harmonic radiation field is clearly being driven here by both electron beams with the lower current, lower energy beam being the dominant contribution. The simulation was repeated twice more, each time with one of the beams 'switched off'. For the low

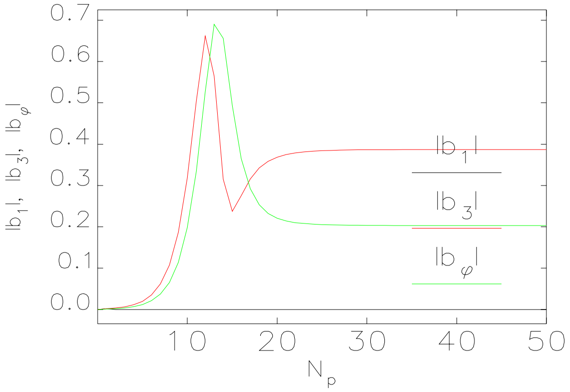

Figure 6: Evolution of electron beam bunching parameters $|b_1|$, $|b_3|$ and $|b_\varphi|$. Current ratio $R_3 = 5$, $\bar{z} = 2$.

energy beam only the source term is $S_{3\vartheta}$, the other source term S_φ having been set to zero, and *vice versa* for the case of the high energy beam only. The total source terms for the three cases are shown in Fig. 7. It is seen that for the coupled interaction the source term of the harmonic field is significantly greater than for either of the two beams individually.

EFFECT OF ENERGY SPREAD

The equations (2–6) were solved numerically with the same parameters as before, but now with a variation in the higher energy beam energy spread $\sigma_\wp$. This was done both for the case of the higher energy beam alone (by setting $R_3 = 1000$) and for the coupled two-beam oscillator, with $R_3 = 5$ as before. The results are shown in Fig. 8 and Fig. 9 respectively. The significance of the results is illustrated by considering first the performance due to the single electron beam, and then the change in performance by adding a lower energy beam of one fifth the current. For a minimal energy spread in the higher beam, $\sigma_\wp = 0.1$, the saturation intensity $|A_3|^2_{\mathrm{sat}}$ is increased by 18% from 1.9 to

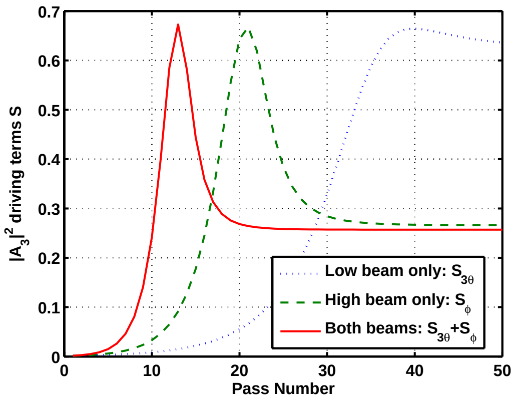

Figure 7: Source terms for the two beam oscillator and for the low energy and high energy beams individually.

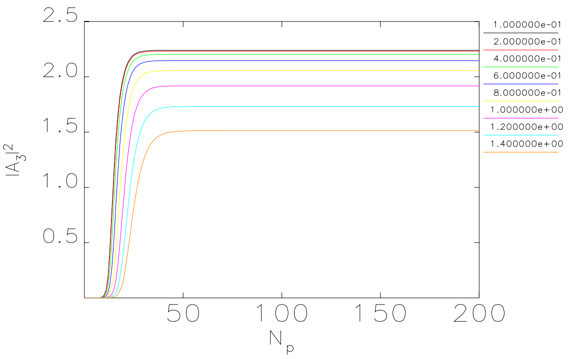

Figure 8: Evolution of the harmonic intensity in the two-beam oscillator for varying gaussian energy spreads $\sigma_\wp$ in the higher energy beam.

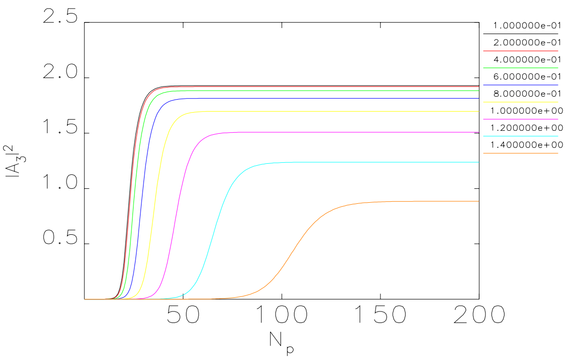

Figure 9: Evolution of the harmonic intensity with the lower energy beam switched off, for varying gaussian energy spreads $\sigma_\wp$ in the higher energy beam.

		$\sigma_\wp = \mathbf{0.1}$	$\sigma_\wp = \mathbf{1.4}$
$\lvert A_3\rvert^2_{\rm sat}$	High beam only	1.9	0.8
	Two beams	2.25	1.5
	Increase	18%	87%
$N_{\rm sat}$	High beam only	50	150
	Two beams	30	50
	Decrease	40%	66%

Table 1: Changes in saturation intensity $|A_3|^2_{\rm sat}$and saturation time $N_{\rm sat}$ due to the introduction of a lower energy beam of one fifth the current, for different energy spreads $\sigma_\wp$ in the higher energy beam.

2.25 and the number of passes to saturation, $N_{\rm sat}$, reduced by 40% from 50 passes to 30 passes. However as the energy spread in the higher beam is increased the effect of adding the lower energy beam becomes more significant—for $\sigma_\wp = 1.4$ the saturation intensity $|A_3|^2_{\rm sat}$ is increased by 87% from 0.8 to 1.5 and $N_{\rm sat}$ reduced by 66% from 150 passes to only 50 passes. These figures are summarised in Table 1.

These preliminary results examining the effect of energy spread in the higher energy beam deserve more study but give an initial indication of the the potential benefit of adding a smaller current, lower energy beam to the higher energy beam.

CONCLUSION

A model of a one-dimensional two-beam oscillator free-electron laser has been presented and investigated numerically in the steady-state regime for the case where the current in the lower energy beam is one-fifth of the current in the higher energy beam. It has been seen that in the mildly nonlinear regime of $\bar{z} = 2$ considered here, the gain of the coupled two-beam system is enhanced slightly over that obtained from a summation of the gains obtained from the individual beam interactions. It is seen that in the amplification phase before saturation the increase in intensity of the third harmonic is driven by both electron beams simultaneously. The effect of energy spread in the higher energy beam has been investigated and there is evidence that a two-beam oscillator may allow for a reduction in the beam quality requirements.

Further analysis is required over a wider range of the multi-parameter space defining the system before any definitive statements can be made as to whether the two-beam FEL oscillator offers real prospects of improvement over that available from a single beam interaction only.

REFERENCES

[1] B. W. J. McNeil, M.W. Poole and G.R.M. Robb, these proceedings.

[2] B. W. J. McNeil, G. R. M. Robb and M.W. Poole , Phys. Rev. E (in press)