THE HARMONICALLY COUPLED 2-BEAM FEL

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Abstract

A model of a Free Electron Laser amplifier operating simultaneously with two electron beams of different energy is presented. The electron beam energies are chosen so that the fundamental resonance of the higher energy beam is at an harmonic of the lower energy beam. By seeding the lower energy FEL interaction at its fundamental radiation wavelength, an improved coherence of the un-seeded higher energy FEL emission is predicted and may offer an alternative to current proposals for seeding XUV and x-ray FELs. An initial study of the effects of beam energy spread on the intearction is presented.

INTRODUCTION

It is now well established that high gain FELs operating in the SASE mode [1] are characterised by a noisy FEL output at saturation with relatively poor temporal coherence and large fluctuations [2]. The simplest conceptual method to resolve this problem is to inject a well-formed resonant coherent seed field at the beginning of the FEL interaction that dominates the intrinsic noise. The FEL output is then significantly improved over that of SASE at saturation. However, there are, as yet, no suitable seed sources available in the XUV and x-ray. Seeding at longer wavelengths can generate shorter wavelengths by using the two-wiggler harmonic method of [3]. Variations on this theme have been suggested and implemented experimantally [4]. Another self-seeding method proposes using a monochromator either at the early stages of the FEL interaction [5] or with some feedback [6] to improve temporal coherence. In this paper an alternative method of seeding, based on a two electron beam FEL interaction [7], is described and investigated in the 1-D limit. Preliminary results of the effects of electron beam quality on the two-beam FEL are also presented.

THE MODEL

A simple FEL wiggler system, of constant period λ_w, and field strength B_w, was proposed, through which two electron beams of different energy co-propogate [7]. The lower energy electron beam has a Lorentz factor of γ_1 and the higher energy γ_n. The higher energy electron beam is chosen so that its fundamental resonant wavelength is an harmonic resonant wavelength of the lower energy beam. From the FEL resonance relation, $\lambda = \lambda_w(1 + a_w^2)/2\gamma^2$, it may easily be shown that $\gamma_n = \sqrt{n}\gamma_1$. It should then be possible to seed the co-propagating electron beams with an externally injected seed radiation field at the fundamental of the lower energy electron beam. If such a seed field is significantly above the noise level then the lower energy electrons will begin to bunch at their fundamental resonant wavelength and retain the coherence properties of the seed. Such bunching at the fundamental also generates significant components of bunching at its harmonics which can also be expected to retain the coherence properties of the seed. In a planar FEL this also results in on-axis radiation emission at these harmonics. This process should couple strongly with the the co-propagating higher energy beam whose fundamental FEL interaction is at one of the lower energy beam's harmonics. This coupling between lower and higher energy FEL interactions may allow the transferal of the coherence properties of the longer wavelength seed field to the un-seeded shorter harmonic wavelength interaction.

Another coupling between the lower and higher energy electron beams, which has the potential to degrade beam quality, is the two-stream instability [8]. Using the results of [8], however, it can be shown that the instability is either below threshold or has an insignificant effect for electron beam currents ($\gtrsim 1$ kA) and energies ($\gtrsim 500$ MeV) typical to those used in the FEL interactions presented here.

The physics of the planar wiggler FEL in the 1-D Compton limit may be described by the coupled Maxwell/Lorentz equations which, under the simplifying assumptions, universal scaling and notation of [1, 9], are written:

$$\frac{d\vartheta_j}{d\bar{z}} = p_j \tag{1}$$

$$\frac{dp_j}{d\bar{z}} = -\sum_{h,odd} F_h \left(A_h e^{ih\vartheta_j} + c.c. \right) \tag{2}$$

$$\frac{dA_h}{d\bar{z}} = F_h \left\langle e^{-ih\vartheta} \right\rangle , \tag{3}$$

where $j = 1..N$ are the total number of electrons, $h = 1, 3, 5...$ are the odd harmonic components of the field and F_h are the usual difference of Bessel function factor associated with planar wiggler FELs. This set of equations (1..3) is used to describe the FEL interaction of the lower energy (γ_1) electron beam.

Strongest coupling between the lower and higher energy electron beams would be expected for the lowest harmonic interaction $h = n = 3$ so that the higher energy electron beam has energy $\gamma_3 = \sqrt{3}\gamma_1$. Higher harmonic interactions $h = n > 3$ may also be of interest, as those harmonics $h < n$ of the lower energy beam are not resonant with the higher energy beam and would not be expected to unduly disrupt the coupling to the higher harmonic.

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The equations describing the FEL interaction of the higher energy electron beam, with its fundamental resonant field only, may be written in the similar form:

$$\frac{d\varphi_j}{dz'} = \wp'_j \tag{4}$$
$$\frac{d\wp'_j}{dz'} = -F_1\left(A'_1 e^{i\varphi_j} + c.c.\right) \tag{5}$$
$$\frac{dA'_1}{dz'} = F_1\left\langle e^{-i\varphi}\right\rangle. \tag{6}$$

We shall neglect harmonics of the higher energy electron beam as these will have a weaker coupling to the lower energy beam , e.g. if $n = 3$ then the third harmonic of the higher energy beam will be the ninth harmonic of the lower energy beam.

In their universally scaled forms the two sets of equations (1..3) and (4..6) have different Pierce parameters [1], ρ, due to the different energy and current density of the electron beams. In [7], by using the usual relations between scaled and unscaled fields and lengths [1], equations (4..6) were written with the same scaling of equations (1..3) to give the final set of working equations describing the coupled FEL system. Here, we use the scaling of equations (4..6) to rescale equations (1..3). For purely notational convenience we rewrite $z' \to \bar{z}$ and $A'_h \to A_h$ in equations (1..6) after rescaling to give the final set of working equations:

$$\frac{d\vartheta_j}{d\bar{z}} = p_j \tag{7}$$
$$\frac{d\varphi_j}{d\bar{z}} = \wp_j \tag{8}$$
$$\frac{dp_j}{d\bar{z}} = -\frac{1}{c_1}\sum_{h,odd}^{n} F_h\left(A_h e^{ih\vartheta_j} + c.c.\right) \tag{9}$$
$$\frac{d\wp_j}{d\bar{z}} = -\left(F_1 A_n e^{i\varphi_j} + c.c.\right) \tag{10}$$
$$\frac{dA_h}{d\bar{z}} = S_{h\vartheta} \tag{11}$$
$$\frac{dA_n}{d\bar{z}} = S_\varphi + S_{n\vartheta}, \tag{12}$$

where

$$S_{k\vartheta} \equiv \frac{1}{c_2}F_k\left\langle e^{-ik\vartheta}\right\rangle, \quad S_\varphi \equiv F_1\left\langle e^{-i\varphi}\right\rangle \tag{13}$$
$$c_1 = \frac{1}{n^{1/4}}\left(\frac{\rho_n}{\rho_1}\right)^{3/2} = \frac{1}{n}\sqrt{\frac{I_n}{I_1}} \equiv \frac{\sqrt{R_n}}{n} \tag{14}$$
$$c_2 = n^{1/4}\left(\frac{\rho_n}{\rho_1}\right)^{3/2} = \frac{1}{\sqrt{n}}\sqrt{\frac{I_n}{I_1}} \equiv \frac{\sqrt{R_n}}{\sqrt{n}}, \tag{15}$$

and in (11) h refers to all odd harmonics $h < n$, I is the beam current and subscripts 1 (n) refers to the parameters of the lower (higher) energy beam. Note that all harmonic interactions have been assumed negligable for $h > n$ in equation (9). By assuming both beams have the same transverse cross section (or equivalently the same normalised transverse emittance in a common, matched focusing channel through the wiggler) then $\rho_{1,n} \propto I_{1,n}^{1/3}/\gamma_{1,n}$ and the second equalities of (14) and (15) are obtained in terms of the beam currents. This assumption is applied to the work presented hereafter. Note that the equations (3) and (6), describing evolution of the fields A_n and A'_1, refer to the *same* field, which, once (3) has been re-scaled, allows the two source terms to be combined into the single differential equation (12) for the harmonic field $A'_n \to A_n$.

The coupling of the low energy electrons to both fundamental and harmonic field is seen in equation (9). From equation (10), the higher energy electrons only couple to the harmonic field A_n (their fundamental) - the fields A_h are sub-harmonic to the higher energy electrons and are therefore not resonant. The fields A_h, are seen from equation (11), to be driven only by the lower energy electron beam (the higher energy electrons are not resonant with A_h.) Equation (12) demonstrates that the highest harmonic field has two driving sources, both the lower and higher energy electron beams. From these couplings it is seen that, whereas the shorter wavelength radiation field is directly coupled to both lower and higher energy electron beams, the longer wavelengths have no direct coupling with the higher energy beam. In this sense, the short wavelength harmonic interaction may be described as 'parasitic' as it may resonantly extract energy directly from both lower and higher energy electron beams, whereas the longer wavelengths may only directly extract energy from the lower energy beam.

The working equations readily yield a constant of motion corresponding to conservation of energy:

$$\sum_{h,odd}^{n} |A_h|^2 + \frac{\langle p\rangle}{\sqrt{n}} + \langle\wp\rangle. \tag{16}$$

It is seen from the definitions of the scaled electron energy parameters $p_j \equiv (\gamma_j - \gamma_1)/\rho_n\gamma_1$ and $\wp_j \equiv (\gamma_j - \gamma_n)/\rho_n\gamma_n$ that the electron beam energy relation $\gamma_n = \sqrt{n}\gamma_1$ accounts for the factor of $\sqrt{n}$ in (16).

A linear analysis of the system (7..12) was carried out using the method of collective variables [1]. Assuming resonant interactions for both electron beams, and that both beams are effectively 'cold' so that neither emittance nor energy spread have a deleterious effect upon the FEL interaction, this analysis yields a condition for the beam current ratio, $R_n \equiv I_n/I_1$, above which gain at the harmonic is greater than gain at the longer wavelength:

$$R_n > n\sqrt{n}\left(1 - \frac{n\,|F_n|^2}{|F_1|^2}\right). \tag{17}$$

A NUMERICAL EXAMPLE

The evolution of the coupled two-beam FEL system is demonstrated by numerically solving the working equations (7..12). Two (noiseless) electron beams were used with $n = 3$ and of current ratio $R_3 = 5$, co-propagating in

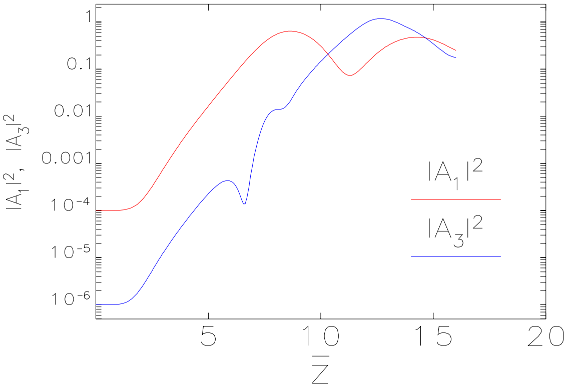

Figure 1: The scaled radiation intensities $|A_1|^2$ and $|A_3|^2$ as a function of scaled distance $\bar{z}$ through the FEL interaction region for $n = 3$, $R_3 = 5$, $\sigma_p = 0.1$ and $\sigma_\wp = 0.4$.

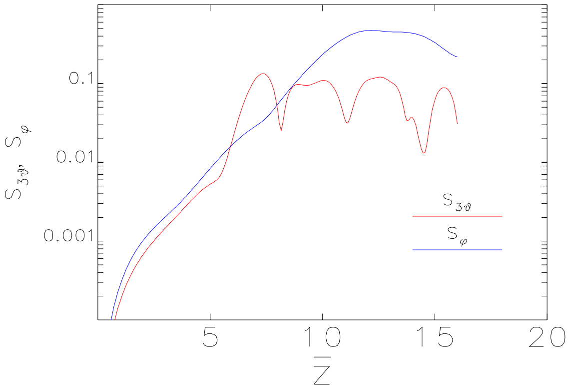

Figure 2: The source terms S_φ and $S_{3\vartheta}$ as a function of scaled distance through the FEL interaction region for $n = 3$, $R_3 = 5$, $\sigma_p = 0.1$ and $\sigma_\wp = 0.4$.

a wiggler of parameter $a_w = 2$. Both beams were given a Gaussian energy spread of scaled width in p of $\sigma_p = 0.1$, and in $\wp$ of $\sigma_\wp = 0.4$. The seed field at the longer wavelength is modelled by defining its initial scaled intensity at the beginning of the FEL interaction $|A_1(\bar{z} = 0)|^2$ to be two orders of magnitude greater than that of the harmonic. In figure (1) the scaled intensities $|A_1|^2$ and $|A_3|^2$ are plotted as a function of scaled distance, $\bar{z}$, through the FEL interaction region. The feature of interest from figure (1) is the rapid growth in the harmonic intensity $|A_3|^2$ by approximately two orders of magnitude in the interval $6.5 \lesssim \bar{z} \lesssim 8$. This period of more rapid growth (approximately $n = 3$ times the fundamental growth rate) is driven by the strong harmonic component of the bunching $|b_3|$ of the lower energy beam [9, 10, 11]. This harmonic bunching is caused, not by electrons bunching at the harmonic wavelength, but by the significant harmonic component of the strong, non-linear, bunching at the fundamental $|b_1|$ as the lower energy FEL interaction approaches its saturation at $\bar{z} \approx 8.5$. The harmonic bunching and subsequent harmonic emission of A_3 from the lower energy beam, can be expected to retain the coherence properties of the initial radiation seed field at the fundamental. This process should therefore act as a harmonic seed field with good coherence properties. Following this harmonic seeding by the lower energy electron beam it is seen that the harmonic intensity continues exponential growth by approximately another two orders of magnitude until saturation at $\bar{z} \approx 12.5$.

Further insight is gained by plotting individually the moduli of the source terms S_φ and $S_{3\vartheta}$ of the harmonic field evolution equation (12), as shown in figure (2). The contribution S_φ is due to the higher energy electron beam, and $S_{3\vartheta}$, the lower energy beam. The figure clearly shows the strong seeding phase between $6.5 \lesssim \bar{z} \lesssim 8$, as discussed above, where the harmonic field is strongly driven by the lower energy beam ($S_{3\vartheta} > S_\varphi$). This is followed by the amplification phase where the harmonic field is driven by the higher energy beam ($S_\varphi > S_{3\vartheta}$).

The above results also suggest that a hybrid HGHG scheme may be possible if the interaction is stopped at $\bar{z} \approx 8.0$ and the mildly bunched higher energy electron beam injected into another wiggler which would allow strong emission at one of its higher harmonics.

The equations have also been numerically solved, in the cold beam limit ($\sigma_p = \sigma_\wp = 0$) to demonstrate the same seeding mechanism for a fifth harmonic interaction [7]. A similar interaction to that for the above simulation was observed, except the intermediate third harmonic field had no resonant coupling with the higher energy beam. This results in the fifth harmonic having both a greater linear growth rate and saturation intensity than the third harmonic.

ENERGY SPREAD EFFECTS

An initial study of the effects of energy spread in a higher energy beam with $n = 3$, was conducted by solving the working equations for a range of values of initial Gaussian spread $\sigma_\wp$. The same lower energy beam as above ($R_3 = 5$ and $\sigma_p = 0.1$), wiggler parameter ($a_w = 2$) and initial field intensities ($|A_1(\bar{z} = 0)|^2 = 10^{-4}$ and $|A_3(\bar{z} = 0)|^2 = 10^{-6}$) were assumed. For comparative purposes, FEL evolution of the higher energy beam in the absence of the lower energy beam was also modelled by conducting a set of simulations with a large electron beam current ratio of $R_3 = 10^3$ and a reduced initial longer wavelength scaled intensity of $|A_1(\bar{z} = 0)|^2 = 10^{-16}$. With these parameters there is no effective FEL evolution of the lower energy beam and the higher energy harmonic interaction evolves as though the lower energy beam were absent.

Figure (3) plots the scaled harmonic saturation intensity, $|A_3|^2_{\rm sat}$, for a range of Gaussian energy spread parameter $\sigma_\wp$ and figure (4) plots the corresponding scaled harmonic field sataration lengths $\bar{z}_{sat}$.

From these figures one distinguishing difference be-

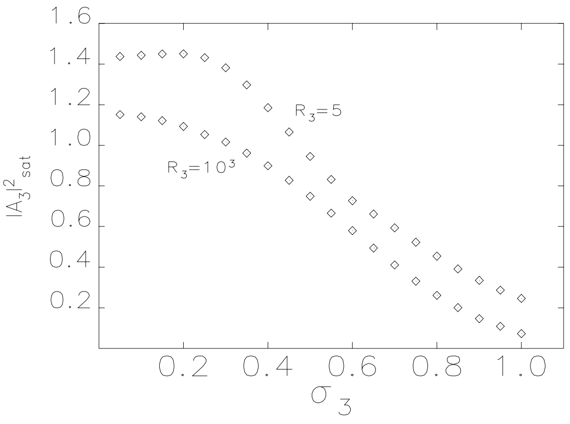

Figure 3: Saturation intensity $|A_3|^2_{\rm sat}$ as a function of energy spread $\sigma_\wp$ (σ_3 in plot) of the higher energy beam.

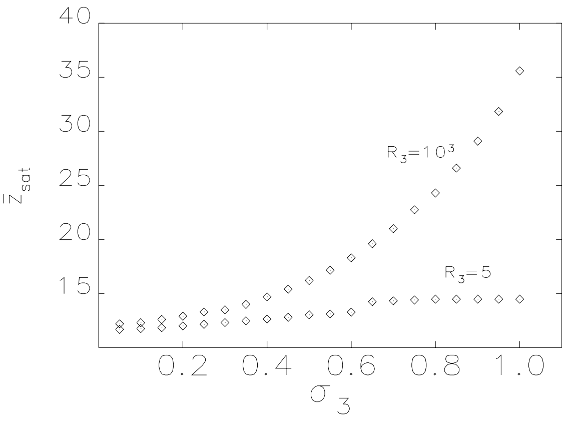

Figure 4: Scaled saturation length of harmonic intensity $\bar{z}_{sat}$ as a function of energy spread $\sigma_\wp$ (σ_3 in plot) of the higher energy beam.

tween the coupled two-beam interaction ($R_3 = 5$) and the higher energy beam only interaction ($R_3 = 10^3$) is apparent: for the coupled two-beam interaction case, both the harmonic saturation intensity$|A_3|^2_{\rm sat}$ and its sataration length are less sensative to the energy spread $\sigma_\wp$ of the higher energy beam. This encouraging preliminary result will be extended to a wider range of energy spread parameter space of the two beams in future work.

CONCLUSIONS

A simple 1-D model of a two-beam FEL has been presented. This concept introduces potentially interesting coupled FEL interactions some of which may have beneficial properties over single beam interactions. Indeed, one need not be limited to only two beams and can envisage more complex systems with more than two harmonically coupled beams - a multi-beam FEL equivalent of the cascaded HGHG scheme [4]. Such multi-beam FELs may offer the prospect of a reduction in overall length when compared with an equivalent HGHG scheme. One can also envisage possible hybrid schemes involving combinations of multibeam FELs with HGHG. Clearly, these suggestions are speculative at this stage and will require further research.

As an illustration of the type of interactions possible, numerical simulations of the coupled two-beam FEL interaction demonstrated that a seeded lower energy beam interaction may evolve to effectively seed that of the higher energy. The conjecture was made that the improved coherence properties of the seeded interaction at the longer wavelength would be inherited by the higher beam energy, shorter wavelength, interaction. Such seeding may be of interest to proposals for FELs operating at sub-VUV wavelengths where no ‘conventional’ seed sources are yet available. Further analysis is required to verify any predicted improvement in the coherence properties. Any analysis will need to model a minimum of two independent variables (e.g. (z, t) or (z, ω)) and include the effects of the relative slippage between the lower and higher energy electron pulses. This relative slippage must be less than that between the lower energy beam and the radiation field.

An initial study of the effects of an electron beam energy spread on the two-beam FEL system has shown no debilitating effects. Indeed, the results encourage a more complete investigation of parameter space.

No assessment has yet been made of the importance of other factors that will effect such two-beam interactions, e.g. accelerator physics issues such as electron pulse synchronism, beam focussing and emittance, the relative energy detuning between the electron beams and transverse modes. Some of these issues will undoubtedly require the development of new simulation models.

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